

Phys 150 HW 1 Solutions ①

1.3 (a) The energy yield per liter oxygen for a given food type is just the

$$\frac{\text{energy released}}{\text{amount food}} \cdot \frac{\text{amount food}}{\text{vol oxygen consumed}}$$

carbohydrates $4.1 \frac{\text{kcal}}{\text{g}} \cdot \frac{9}{0.81 \text{ L}_{\text{O}_2}} = \boxed{5.1 \frac{\text{kcal}}{\text{L}_{\text{O}_2}}}$

fats $9.3 \frac{\text{kcal}}{\text{g}} \cdot \frac{9}{1.96 \text{ L}_{\text{O}_2}} = \boxed{4.7 \frac{\text{kcal}}{\text{L}_{\text{O}_2}}}$

proteins $4.0 \frac{\text{kcal}}{\text{g}} \cdot \frac{9}{0.94 \text{ L}_{\text{O}_2}} = \boxed{4.3 \frac{\text{kcal}}{\text{L}_{\text{O}_2}}}$

alcohols $7.1 \frac{\text{kcal}}{\text{g}} \cdot \frac{9}{1.46 \text{ L}_{\text{O}_2}} = \boxed{4.9 \frac{\text{kcal}}{\text{L}_{\text{O}_2}}}$

For the energy yield per L_{O_2} to be roughly constant, the standard deviation of the values for the different food types must be small compared to the average value.

The average value, $\mu = \left(\frac{5.1 + 4.7 + 4.3 + 4.9}{4} \right) \frac{\text{kcal}}{\text{L}_{\text{O}_2}} = \boxed{4.8 \frac{\text{kcal}}{\text{L}_{\text{O}_2}}}$

The standard deviation, $\sigma = \left(\langle (x - \mu)^2 \rangle \right)^{1/2}$

$$= \left(\frac{(5.1 - 4.8)^2 + (4.7 - 4.8)^2 + (4.3 - 4.8)^2 + (4.9 - 4.8)^2}{4} \right)^{1/2} = \boxed{0.6}$$

Then, to compare the standard deviation to the mean, we take the ratio, $\frac{\sigma}{\mu} = \frac{0.6}{4.8} \sim \boxed{0.12}$

Then the standard deviation is about 12% of the mean which is fairly small.

So we can say that the energy yield/ L_{O_2} is roughly constant for different food types.

We can do the same calculation, this time using the $\frac{\text{carbon dioxide released}}{\text{amount food}}$ to get the energy released per L_{CO_2} released:

carbohydrates $\sim 5.1 \frac{\text{kcal}}{\text{L}_{\text{CO}_2}}$, fats $\sim 6.7 \frac{\text{kcal}}{\text{L}_{\text{CO}_2}}$, proteins $\sim 5.3 \frac{\text{kcal}}{\text{L}_{\text{CO}_2}}$, alcohols $\sim 7.3 \frac{\text{kcal}}{\text{L}_{\text{CO}_2}}$

Similarly we find the mean, $\mu \approx 6.1 \frac{\text{kcal}}{\text{L}_{\text{CO}_2}}$ and standard deviation, $\sigma = 1.9 \frac{\text{kcal}}{\text{L}_{\text{CO}_2}}$

Then $\frac{\sigma}{\mu} = \frac{1.9}{6.1} \sim 0.31$. So the $\frac{\text{energy yield}}{\text{L}_{\text{CO}_2}}$ is not roughly constant for the different food types.

1.3 continued

Nelson suggests looking at the $\frac{\text{CO}_2 \text{ output}}{\text{O}_2 \text{ input}}$ to get an idea of which foods are

used as an energy source. We can approximate the number of molecules to be proportional to the volume of gas. We might expect to yield 1 CO₂ molecule per 1 O₂ molecule.

$$\text{carbohydrates} \quad \frac{0.81 \text{ L}_{\text{CO}_2}}{\text{g}} \cdot \frac{\text{g}}{0.81 \text{ L}_{\text{O}_2}} = 1.0 \frac{\text{L}_{\text{CO}_2}}{\text{L}_{\text{O}_2}}$$

$$\text{fats} \quad \frac{1.39 \text{ L}_{\text{CO}_2}}{\text{g}} \cdot \frac{\text{g}}{1.96 \text{ L}_{\text{O}_2}} = 0.709 \frac{\text{L}_{\text{CO}_2}}{\text{L}_{\text{O}_2}}$$

$$\text{proteins} \quad \frac{0.75 \text{ L}_{\text{CO}_2}}{\text{g}} \cdot \frac{\text{g}}{0.94 \text{ L}_{\text{O}_2}} = 0.80 \frac{\text{L}_{\text{CO}_2}}{\text{L}_{\text{O}_2}}$$

$$\text{alcohols} \quad \frac{0.97 \text{ L}_{\text{CO}_2}}{\text{g}} \cdot \frac{\text{g}}{1.46 \text{ L}_{\text{O}_2}} = 0.66 \frac{\text{L}_{\text{CO}_2}}{\text{L}_{\text{O}_2}}$$

Carbohydrates appear to be the best energy source.

(b) The basal metabolic rate, BMR, is the heat released by an adult at rest in a given time. We are given that the average adult at rest uses $16 \frac{\text{L}_{\text{O}_2}}{\text{hr}}$

From part (a) we know that the average energy released from food is $\sim 4.8 \frac{\text{kcal}}{\text{L}_{\text{O}_2}}$

$$\text{Then the BMR} = 4.8 \frac{\text{kcal}}{\text{L}_{\text{O}_2}} \cdot \frac{16 \text{ L}_{\text{O}_2}}{\text{hr}} = \boxed{77 \frac{\text{kcal}}{\text{hr}}} \cdot \frac{24 \text{ hr}}{\text{day}} = \boxed{1800 \frac{\text{kcal}}{\text{day}}}$$

(c) Power is work per unit time. A watt is a $\frac{\text{joule}}{\text{second}}$

$$P = 77 \frac{\text{kcal}}{\text{hr}} \cdot \frac{1000 \text{ cal}}{\text{kcal}} \cdot \frac{\text{J}}{0.24 \text{ cal}} \cdot \frac{\text{hr}}{3600 \text{ s}} = \boxed{89 \text{ W}}$$

(d) CO₂ output rate is $13.4 \frac{\text{L}}{\text{hr}}$. Remember from (b) that the corresponding O₂ consumption rate $\sim 16 \text{ L/hr}$

Then $\frac{13.4 \text{ L}_{\text{CO}_2}}{\text{hr}} \cdot \frac{\text{hr}}{16 \text{ L}_{\text{O}_2}} \sim 0.84 \frac{\text{L}_{\text{CO}_2}}{\text{L}_{\text{O}_2}}$ This ratio is closest to that of proteins, so we might guess that the food is high in proteins.

Physics 150 HW 1 solutions (3)

1.3 continued

(d) Hard labor : 50 W in 10 hr , requires 3500 $\frac{\text{kcal}}{\text{day}}$

Efficiency, Σ , is $\frac{\text{mechanical work done}}{\text{Excess energy consumed}}$

We remember from the BMR calculation in (b) that the typical energy intake is 1800 $\frac{\text{kcal}}{\text{day}}$ for an adult at rest.

$$\Sigma = \frac{\overbrace{50 \frac{\text{J}}{\text{s}} \cdot \frac{3600 \text{s}}{\text{hr}} \cdot 10 \text{ hr}}^{\text{work done}}}{\underbrace{(3500 \text{ kcal} - 1800 \text{ kcal}) \cdot \frac{1000 \text{ cal}}{\text{kcal}} \cdot \frac{\text{J}}{0.24 \text{ cal}}}_{\text{excess energy intake}}} \sim \boxed{0.25}$$

1.4 (a) Using the fact that the Earth is in steady state, we can set the incident energy flux equal to the radiated heat flux to solve for the Earth's temperature.

The incident energy is $I_e \cdot \alpha \cdot \pi R^2$

$$\pi R^2 \alpha I_e = 4 \pi R^2 \alpha \sigma T^4 \Rightarrow T = \left(\frac{I_e}{\sigma} \right)^{1/4} = \left(\frac{1.4 \text{ kW}}{\text{m}^2} \cdot \frac{1}{4} \cdot \frac{\text{m}^2 \text{ K}^4}{5.7 \cdot 10^{-8} \text{ W}} \cdot \frac{1000 \text{ W}}{\text{kW}} \right)^{1/4} \sim \boxed{280 \text{ K}}$$

This is close to the actual value of 289 K.

(b) To find how far a planet must be from the sun as a multiple of d_e , the earth's distance from the sun, and still be above freezing, we use the same formula replacing I_e with $I_e \left(\frac{d_e}{d} \right)^2$ to find the incident energy flux at the new distance, d .

We will define "above freezing" to be 1°C above the freezing point of water @ 273 K

$$\pi R^2 \alpha I_e \left(\frac{d_e}{d} \right)^2 = 4 \pi R^2 \alpha \sigma T^4 \quad \text{Rearranging, } d = \left(\frac{I_e d_e^2}{4 \sigma T^4} \right)^{1/2}$$

$$\text{Then } d = \left(\frac{1.4 \text{ kW}}{\text{m}^2} \cdot \frac{\text{m}^2 \text{ K}^4}{5.7 \cdot 10^{-8} \text{ W}} \cdot \frac{1000 \text{ W}}{\text{kW}} \cdot \frac{1}{4} \cdot \frac{1}{(274 \text{ K})^4} \right)^{1/2} d_e \sim \boxed{1.0 d_e}$$

(c) We will define "~~above~~ boiling" to be 372 K, one degree celsius below the boiling pt of H_2O .

$$d = \left(\frac{1.4 \text{ kW}}{\text{m}^2} \cdot \frac{\text{m}^2 \text{ K}^4}{5.7 \cdot 10^{-8} \text{ W}} \cdot \frac{1000 \text{ W}}{\text{kW}} \cdot \frac{1}{4} \cdot \frac{1}{(372 \text{ K})^4} \right)^{1/2} d_e \sim \boxed{0.57 d_e}$$

(d) Only Venus is a candidate for water based life using this estimate

Phys 150 HW 1 solutions (4)

1.5 Franklin put a certain amount of oil on a pond and measured the area over which it spread. Assuming that the layer of oil is one molecule thick, we can estimate the height of one oil molecule:

$$h = \frac{\text{volume used}}{\text{area spread}} = \frac{V}{A}$$

We can approximate each oil molecule to be a cube with side of length h . Then each molecule has a volume, $V_m = h^3$

From the molecular mass of the oil and the density, we can calculate the volume of a mole of oil molecules:

$$V_{\text{mol}} = \frac{\text{Molecular mass of oil}}{\text{density of oil}} = \frac{M_w}{\rho}$$

Then we can calculate the number of molecules in a mole of oil, by dividing the volume of a mole of oil by the volume of one oil molecule.

$$N_A = \frac{M_w}{\rho} \cdot \frac{1}{h^3} = \frac{M_w}{\rho} \left(\frac{A}{V} \right)^3$$

We are given that oleic acid has a molecular mass of $282 \frac{\text{g}}{\text{mol}}$ and that olive oil has a molecular mass about three times this. Then I will use a $M_w \sim 850 \frac{\text{g}}{\text{mol}}$

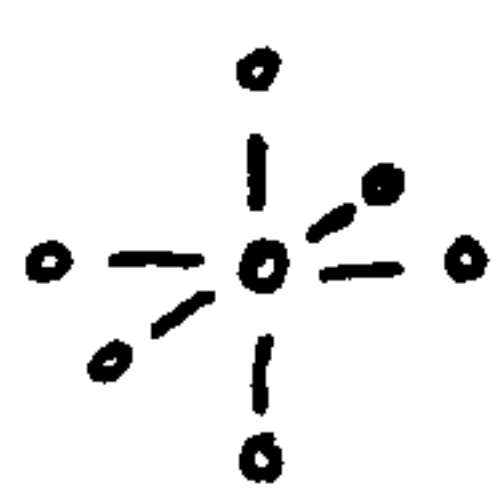
We are given a density of $0.9 \frac{\text{g}}{\text{cm}^3}$ for olive oil.

We are also given (in Section 1.5) that $\sim 5 \text{ cm}^3$ oil spread to $\sim 2000 \text{ m}^2$

$$N_A = \frac{850 \text{ g}}{\text{mol}} \cdot \frac{10 \text{ cm}^3}{9 \text{ g}} \cdot \left(\frac{2 \cdot 10^3 (10^2 \text{ cm})^2}{5 \text{ cm}^3} \right)^3 = \boxed{6 \cdot 10^{22} \frac{\text{molecules}}{\text{mol}}}$$

Phys 150 HW1 solutions (5)

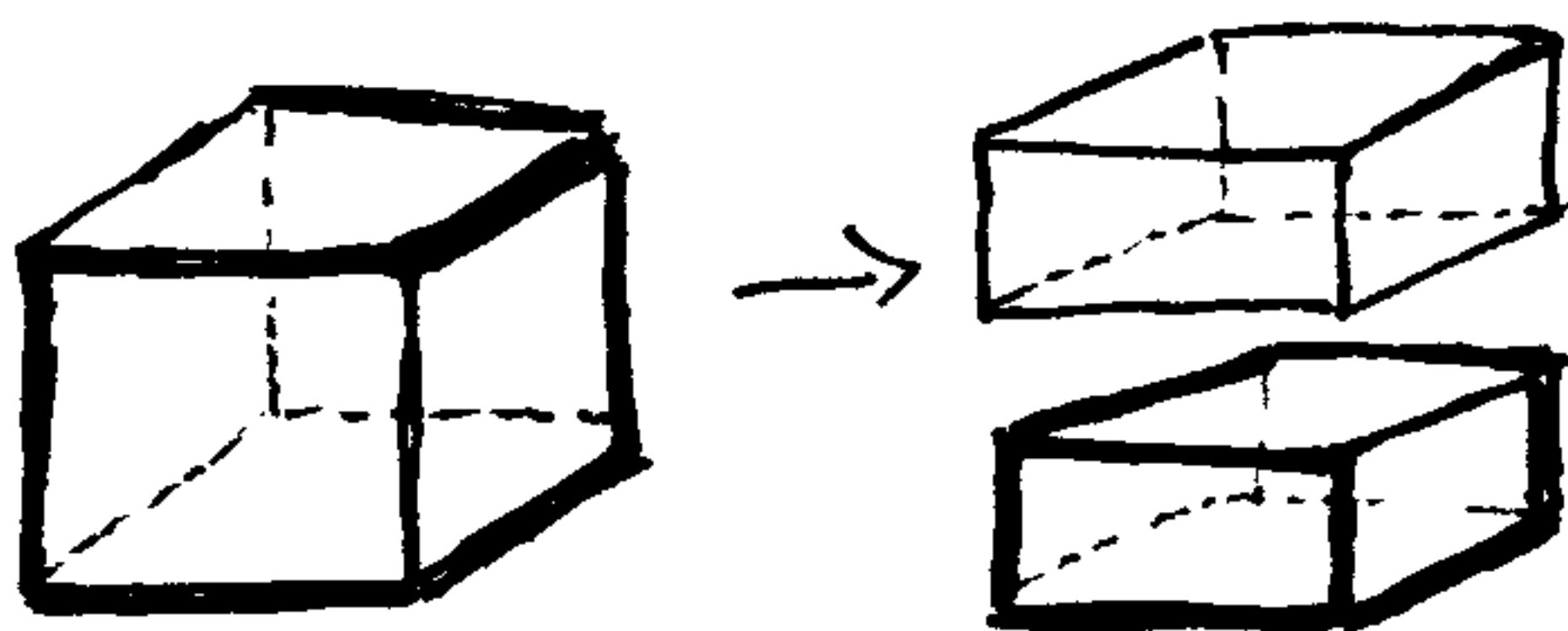
1.6 (a)



Each molecule in a cubic array shares bonds with its 6 nearest neighbors. Each bond is shared by 2 molecules, so there are $\sim 3N^3$ bonds in a volume with N molecules in each dimension.

→ We could refine our number of bonds by accounting for the N^2 fewer bonds that the molecules on each surface share compared to the molecules in the volume. With a cube as large as 1cm^3 on a side, the N^3 term dominates, so I will neglect the $-6N^2$. It is fine if you carry this term.

Then $Q_{\text{vap}} \cdot V = \text{energy to break every bond in the volume}$, so $Q_{\text{vap}} \cdot \frac{V}{3N^3}$ is ϵ , the energy to break on bond



Each molecule in a plane shares one bond with a molecule in a neighboring plane, to rip off one plane requires breaking N^2 bonds. If we break apart 1cm^2 surface, we create 2cm^2 new surface.

$\Sigma \cdot A = \text{Energy to break every bond between the surfaces}$, then $\Sigma \cdot \frac{A}{N^2} = \epsilon$

We can combine these to solve for N , the number of molecules in one dimension.

$$Q_{\text{vap}} \cdot \frac{V}{3N^3} = \Sigma \cdot \frac{A}{N^2} \Rightarrow N = \frac{Q_{\text{vap}}}{\Sigma} \cdot \frac{V}{3A}$$

For water, given that:

$$Q_{\text{vap}} = 2.3 \cdot 10^9 \frac{\text{J}}{\text{m}^3}, \quad \Sigma = 0.072 \frac{\text{J}}{\text{m}^2}$$

For a 1cm^3 surface, and 2cm^2 created surface

$$N = \frac{2.3 \cdot 10^9 \text{ J}}{(10^2 \text{ cm})^3} \cdot \frac{(10^2 \text{ cm})^2}{0.072 \text{ J}} \cdot \frac{1 \text{ cm}^3}{3 \cdot 2 \text{ cm}^2} = \boxed{5.3 \cdot 10^7} \text{ molecules in } 1 \text{ cm}$$

(b) The approximate molecular diameter is just the linear distance divided by the number of molecules in that distance, assuming that the molecules are close packed:

$$\frac{d}{N} = \frac{1.0 \text{ cm}}{5.3 \cdot 10^7 \text{ molecules}} = \boxed{0.19 \text{ nm}} \text{ molecule}$$

(c) To estimate avogadro's number, we can take the volume of one mole of water (from the molecular mass and the density) and divide it by the volume occupied by one water molecule. We will approximate a water molecule to be a cube with a side of 0.19 nm .

$$N_A = \frac{18 \text{ g}}{\text{mol}} \cdot \frac{\text{cm}^3}{1 \text{ g}} \cdot \frac{\text{molecule}}{(0.19 \cdot 10^{-7} \text{ cm})^3} = \boxed{2.6 \cdot 10^{24} \frac{\text{molecules}}{\text{mol}}}$$

Phys 150 HW1 solutions (6)

- C1.8 Suppose the concentration of particles is 10mM
Find the average # of particles in a region of volume $(10\text{ nm})^3$

$$1\text{ mole} = 6.02 \cdot 10^{23} \text{ particles} \quad 1\text{ cm}^3 = 1\text{ mL} \leftarrow \text{useful conversion}$$

$$(10 \cdot 10^{-7} \text{ cm})^3 \cdot \frac{1\text{ mL}}{\text{cm}^3} \cdot \frac{1\text{ L}}{1000\text{ mL}} \cdot \frac{0.01\text{ mol}}{\text{L}} \cdot \frac{6.02 \cdot 10^{23} \text{ particles}}{\text{mol}} = \boxed{6} \text{ particles in a } (10\text{ nm})^3 \text{ region}$$

- C1.9^(a) Find an approximate formula for the speed of a wave on the surface of the ocean. Use only mass density, ρ , wavelength, λ , and the acceleration due to gravity, g .

This is an exercise in dimensional analysis. What are the dimensions of our quantities?

$$[\rho] = \frac{\mathcal{M}}{\mathcal{L}^3} \quad [\lambda] = \mathcal{L} \quad [g] = \frac{\mathcal{L}}{\mathcal{T}^2}$$

We want a speed: $[v] = \frac{\mathcal{L}}{\mathcal{T}}$

The only way to get the dimensions of a speed from the given variables is

$$[(\lambda \cdot g)^{1/2}] = \left(\mathcal{L} \cdot \frac{\mathcal{L}}{\mathcal{T}^2} \right)^{1/2} = \frac{\mathcal{L}}{\mathcal{T}}$$

So an approximate formula for the speed of a wave is $v = (\lambda g)^{1/2}$

- (b) Find speed for a wave with $\lambda = 1\text{ m}$

$$v = \left(1\text{ m} \cdot 10 \frac{\text{m}}{\text{s}^2} \right)^{1/2} \sim \boxed{3 \frac{\text{m}}{\text{s}}} \quad \text{This seems reasonable.}$$