Phys 150 HW 7 solutions

Nelson, Problem 7.1, pg 290 (*4 pts*)

a) Water came out of the strawberries. The strawberries are made of cells whose membranes are permeable to water but not to sugar. The powdered sugar dissolved in water was already on the surface of the strawberries (from washing or condensation) and formed a concentrated sugar solution on the outside of the cells. This solution was higher in osmolarity than the cytoplasm (solution inside the strawberry cells), so there was a net flux of water out of the cells that lowered the osmotic pressure mismatch across the membranes.

b) Osmosis is the term for diffusion of a *solvent* through a semipermeable membrane. Permiation is the term for diffusion of a solvent through a solid. Permeation is perhaps a more apt analogy for learning than osmosis since the "solvent", knowledge in this case, must pass through the learner's skull - which is a solid.

Nelson, Problem 7.3, pg 290 (*6 pts*)

a) The fact that 1mM is good does not mean that 2mM is twice as good. Certainly, the "cells" won't burst in 2mM salt solution. Instead, they will crumple. That is, they will lose volume, but not surface area. This means they will no longer be spherical. But it is not clear that a sphere is the best shape for an artificial blood cell. It may be that, by increasing the number of hemoglobin molecules near the membrane, the excess surface area to volume will actually improve the rate at which oxygen and carbon dioxide to diffuse in and out of the "cells".

b) Salt dissociates completely in water. A 1mM salt solution actually has 2mM of solute: 1mM of Na^+ ions and 1mM of Cl^- ions. To achieve the same osmotic pressure with glucose, which does not dissociate in water, you would need to use 2mM (twice as much).

Nelson, Problem 7.4 pg 291 (*8 pts*)

a) To estimate the average molar mass M in g/mole, assume blood plasma is sufficiently dilute that the osmotic pressure is given by $p = ck_BT$, and take care to convert the units.

$$M = \frac{60 \text{ g L}^{-1}}{c \text{ mole L}^{-1}} = 60\frac{k_BT}{p} = 60\frac{1.38\cdot 10^{-23}\text{J K}^{-1}\cdot 303 \text{ K}}{28 \text{ mm Hg}}\cdot\frac{752 \text{ mm Hg}}{10^5\text{J m}^{-3}}\cdot\frac{1000 \text{ L}}{\text{m}^3}\cdot\frac{6\cdot 10^{23}}{\text{mole}} = 40,400 \text{ g mole}^{-1} \quad (1)$$

b) In part a) we are told that the osmotic pressure of blood proteins is about 28 mm Hg ·1 atm/760 mm Hg = 0.037 atm. A 10% drop implies a pressure difference $\Delta p = 0.0037$ atm. Given the filtration coefficient, the resulting volume flux (volume per unit time per unit area) is $L_p\Delta p = 7\cdot 10^{-6}\cdot 0.0037$ cm/s $= 2.6\cdot 10^{-8}$ cm/s $= 2.6\cdot 10^{-10}$ m/s. Multiplying this by the total surface area of the capillaries and converting the units gives

$$\frac{2.6\cdot 10^{-10}\text{m}\cdot 250 \text{ m}^2}{\text{s}}\cdot\frac{10^3\text{L}}{\text{m}^3}\cdot\frac{3.6\cdot 10^3\text{s}}{\text{h}}\cdot\frac{24 \text{ h}}{\text{day}} = \frac{5.6 \text{ L}}{\text{day}} \quad (2)$$

It would seem that starving children might suffer from a low level of blood protein, which would result in water leaving their capillaries until the protein concentration in the capillaries is increased to the point that its osmotic pressure matches the osmotic pressure in the surrounding tissue. Meanwhile the lost water might accumulate in body cavities, swelling their bellies.

Nelson, Problem 7.5, pg 291 (*8 pts*)

a) An object of radius r is excluded from a region of depth r immediately adjacent to another object. Inside the cell, globular proteins of radius $r = 10$ nm are excluded from a region 10 nm thick around any other object. When the separation between two objects $d \leq 2r$, the excluded regions surrounding the two objects overlap and the overall volume accessible to the globular proteins increases. If the two objects are large and compatible in shape, this decrease in excluded volume can correspond to a significant increase in entropy (due to proteins having more translational freedom). The associated decrease in the free energy of the system drives the two large objects together - a phenomenon known as depletion attraction.

b) The total decrease in excluded volume once the surfaces touch is

$$\Delta V = 2rA = 2\cdot 10\text{nm}\cdot 10\mu\text{m}^2\cdot 10^6\text{nm}^2/\mu\text{m}^2 = 2\cdot 10^8\text{nm}^3 \quad (3)$$

The associated reduction in free energy is $p\Delta V$, where p is the osmotic pressure of the excluded objects.

Given that an individual protein occupies a volume of $4\pi(10 \text{ nm})^3/3 = 4.2\cdot 10^3$ nm^3, and that proteins are at a concentration such that they occupy 30% of the total volume,

$$c = \frac{1 \text{ protein molecule}}{4.2\cdot 10^3 \text{ nm}^3/0.3} = 7.2\cdot 10^{-5}\text{nm}^{-3} \quad (4)$$

we can calculate the osmotic pressure of the protein solution inside the cell using the van't Hoff relation

$$p = ck_BT = 7.2 \cdot 10^{-5}\text{nm}^{-3} \cdot 4.1 \text{ pN nm} = 3 \cdot 10^{-4}\text{pN nm}^{-2} \tag{5}$$

and the resulting free energy change

$$p\Delta V = 3 \cdot 10^{-4}\text{pN nm}^{-2} \times 2 \cdot 10^{8}\text{nm}^3 = 6 \cdot 10^4 \text{ pN nm} \approx 15,000 k_BT \tag{6}$$

which is *very* significant!

Nelson, Your Turn 7D, pg 261 (*4 pts*)
For $R = 1\mu$m:

$$\left(\frac{q}{e}\right)^2 = \left(\frac{10^3\text{kg}}{\text{m}^3}\frac{6 \cdot 10^{23}}{0.018 \text{ kg}} \times \frac{4\pi}{3}(10^{-6}\text{m})^3 \times 0.01\right)^2 = 1.9 \cdot 10^{18}. \tag{7}$$

multiplying by $2.3 \cdot 10^{-28}$ J m and dividing by $2 \cdot 10^{-6}$ m yields about $2 \cdot 10^{-4}$ J, which is still much bigger than $k_BT_r \approx 4.1 \cdot 10^{-21}$ J.

For $R = 1$ nm:

$$\left(\frac{q}{e}\right)^2 = \left(\frac{10^3\text{kg}}{\text{m}^3}\frac{6 \cdot 10^{23}}{0.018 \text{ kg}} \times \frac{4\pi}{3}(10^{-9}\text{m})^3 \times 0.01\right)^2 = 1.9, \tag{8}$$

multiplying by $2.3 \cdot 10^{-28}$ J m and dividing by $2 \cdot 10^{-9}$ m yields about $2 \cdot 10^{-19}$ J, which only about 50 times bigger than $k_BT_r \approx 4.1 \cdot 10^{-21}$ J.

Nelson, Your Turn 7H, pg 279 (*4 pts*)

From the bottom of the first paragraph on page 279, the estimated free energy cost per surface area is

$$3k_BT_r\text{nm}^{-2} = 3(4.1 \cdot 10^{-21}\text{J}) \text{ nm}^{-2} = 12.3 \cdot 10^{-3}\text{J m}^{-2} = 0.012 \text{ J m}^{-2} \tag{9}$$

which is of the same order of magnitude as the measured bulk oil-water surface tension $\Sigma \approx 0.04$ – 0.05 J m^{-2}.

Nelson, Problem C7.12, pg 588 (*2 pts*)
The difference in volatility between benzene and glucose is also due to the polarizability of the molecule. Polar glucose molecules can interact with one another via hydrogen bonding, which means to vaporize (*i.e.*, separate) the molecules, energy must be added to overcome that interaction. Non-polar benzene molecules have no such interaction, so it requires less energy to vaporize benzene than it does to vaporize glucose.

Nelson, Problem C7.16, pg 590 (*2 pts*)
a) In the absence of toxin, cells in 150 mM NaCl are stable because the cell membrane is impermeable to NaCl just as it is impermeable to protein. So, the Na^+ and Cl^- ions on the outside of the cell generate an osmotic pressure that balances the osmotic pressure generated by proteins on the inside. When the toxin is added, it creates small pores in the cell membranes that allow NaCl to pass, but not protein. As a result, Na^+ and Cl^- ions permeate the cell and there is no osmotic pressure from outside the cell to balance the osmotic pressure of the proteins on the inside. The excess interior pressure swells the cells until they burst (*i.e.* lyse).
b) The toxin makes pores in the cell membrane of a certain size. If the polymer has a radius of gyration that is small compared to the pore size, it will permeate the cells. But if the polymer's radius of gyration is large compared to the pore size, the membrane will remain impermeable to the polymer, even in the presence of the toxin. As we learned in Chapter 4, radius of gyration increases with the molecular weight of a polymer. Apparently, the osmotic pressure provided by 30mM polymer outside the cell is enough to balance the osmotic pressure inside the cell and prevent lysis.
c) The time for lysis depends on the time it takes for the polymer to pass through the pores created by the toxin and permeate the cell. Larger polymers will spend a longer time diffusing into the cell, because they have to distort and thread their way through the small hole created by the toxin.