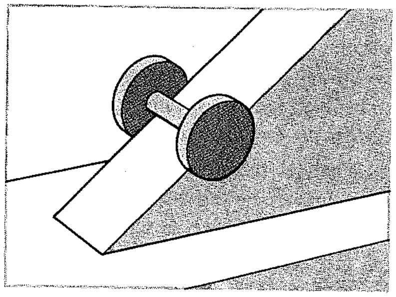

Figure 31 Question 24.

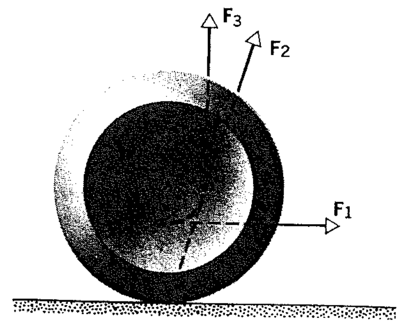

Figure 32 Question 26.

with greatly increased translational speed. Explain why. (*b*) If this system raced a hoop (of any radius) down the ramp, which would reach the bottom first?

25. A yo-yo falls to the bottom of its cord and then climbs back up. Does it reverse its direction of rotation at the bottom? Explain your answer.

26. A yo-yo is resting on a horizontal table and is free to roll (see Fig. 32). If the string is pulled by a horizontal force such as $\mathbf{F}_1$, which way will the yo-yo roll? What happens when the force $\mathbf{F}_2$ is applied (its line of action passes through the point of contact of the yo-yo and table)? If the string is pulled vertically with the force $\mathbf{F}_3$, what happens?

27. A solid flanged wheel consists of two joined concentric disks, the larger of radius R and the smaller of radius r. The wheel is to roll along the two-level rail, as shown in Fig. 33. However, in making one rotation, the center of the wheel moves a distance $2\pi r$ according to the smaller disk and $2\pi R$ according to the larger disk. Explain the apparent discrepancy.

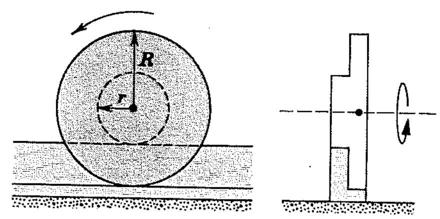

Figure 33 Question 27.

28. State Newton's three laws of motion in words suitable for rotating bodies.

PROBLEMS

Section 12-2 Kinetic Energy of Rotation and Rotational Inertia

1. The masses and coordinates of four particles are as follows: 50 g, $x = 2.0$ cm, $y = 2.0$ cm; 25 g, $x = 0$, $y = 4.0$ cm; 25 g, $x = -3.0$ cm, $y = -3.0$ cm; 30 g, $x = -2.0$ cm, $y = 4.0$ cm. Calculate the rotational inertia of this collection with respect to the (*a*) x, (*b*) y, and (*c*) z axes.

2. A molecule has a rotational inertia of 14,000 $\text{u} \cdot \text{pm}^2$ and is spinning at an angular speed of 4.30×10^{12} rad/s. (*a*) Express the rotational inertia in $\text{kg} \cdot \text{m}^2$. (*b*) Calculate the rotational kinetic energy in eV.

3. The oxygen molecule has a total mass of 5.30×10^{-26} kg and a rotational inertia of 1.94×10^{-46} $\text{kg} \cdot \text{m}^2$ about an axis through the center perpendicular to the line joining the atoms. Suppose that such a molecule in a gas has a mean speed of 500 m/s and that its rotational kinetic energy is two-thirds of its translational kinetic energy. Find its average angular velocity.

Section 12-3 Rotational Inertia of Solid Bodies

4. A communications satellite is a uniform cylinder with mass 1220 kg, diameter 1.18 m, and length 1.72 m. Prior to launching from the shuttle cargo bay, it is set spinning at 1.46 rev/s about the cylinder axis; see Fig. 34. Calculate the satellite's rotational kinetic energy.

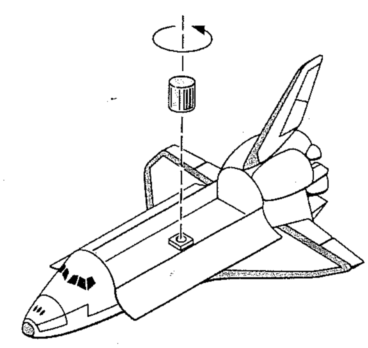

Figure 34 Problem 4.

5. Each of three helicopter rotor blades shown in Fig. 35 is 5.20 m long and has a mass of 240 kg. The rotor is rotating at 350 rev/min. (*a*) What is the rotational inertia of the rotor assembly about the axis of rotation? (Each blade can be

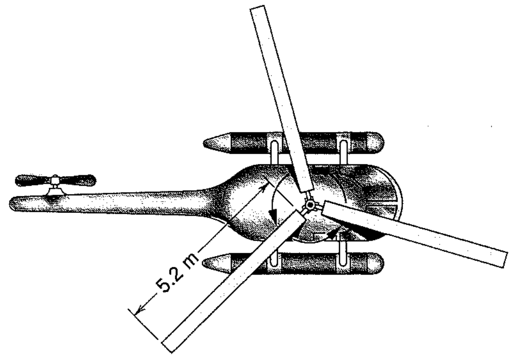

Figure 35 Problem 5.

considered a thin rod.) (*b*) What is the kinetic energy of rotation?

6. Figure 36 shows a uniform block of mass M and edge lengths a, b, and c. Calculate its rotational inertia about an axis through one corner and perpendicular to the large face of the block. (*Hint:* See Fig. 9.)

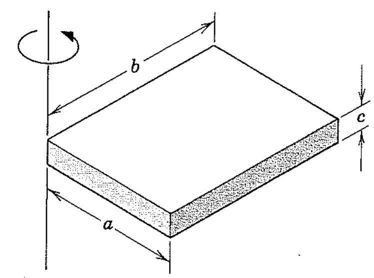

Figure 36 Problem 6.

7. Calculate the rotational inertia of a meter stick, with mass 0.56 kg, about an axis perpendicular to the stick and located at the 20-cm mark.

8. Two particles, each with mass m, are fastened to each other and to a rotation axis by two rods, each with length L and mass M, as shown in Fig. 37. The combination rotates around the rotation axis with angular velocity ω. Obtain algebraic expressions for (*a*) the rotational inertia of the combination about O and (*b*) the kinetic energy of rotation about O.

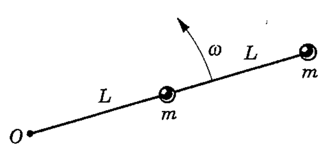

Figure 37 Problem 8.

9. (*a*) Show that the sum of the rotational inertias of a plane laminar body about any two perpendicular axes in the plane of the body is equal to the rotational inertia of the body about an axis through their point of intersection perpendicular to the plane. (*b*) Apply this to a circular disk to find its rotational inertia about a diameter as axis.

10. Delivery trucks that operate by making use of energy stored in a rotating flywheel have been used in Europe. The trucks are charged by using an electric motor to get the flywheel up to its top speed of 624 rad/s. One such flywheel is a solid, homogeneous cylinder with a mass of 512 kg and a radius of 97.6 cm. (*a*) What is the kinetic energy of the flywheel after charging? (*b*) If the truck operates with an average power requirement of 8.13 kW, for how many minutes can it operate between chargings?

11. (*a*) Show that a solid cylinder of mass M and radius R is equivalent to a thin hoop of mass M and radius $R/\sqrt{2}$, for rotation about a central axis. (*b*) The radial distance from a given axis at which the mass of a body could be concentrated without altering the rotational inertia of the body about that axis is called the *radius of gyration*. Let k represent the radius of gyration and show that

$$k = \sqrt{I/M}.$$

This gives the radius of the "equivalent hoop" in the general case.

12. Figure 38 shows the solid rod considered in Section 12-3 (see also Fig. 6) divided into an arbitrary number N of pieces. (*a*) What is the mass m_i of each piece? (*b*) Show that the distance of each piece from the axis of rotation can be written $r_i = (i-1)L/N + (\frac{1}{2})L/N = (i-\frac{1}{2})L/N$. (*c*) Use Eq. 5 to evaluate the rotational inertia of this rod, and show that it reduces to Eq. 6. You may need the following sums:

$$\sum_{i=1}^{n} 1 = n, \qquad \sum_{i=1}^{n} i = n(n+1)/2,$$

$$\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6.$$

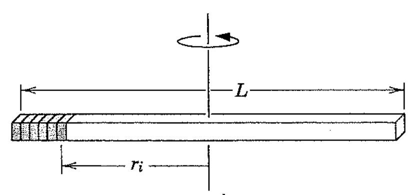

Figure 38 Problem 12.

13. In this problem we seek to compute the rotational inertia of a disk of mass M and radius R about an axis through its center and perpendicular to its surface. Consider a mass element dm in the shape of a ring of radius r and width dr (see Fig. 39). (*a*) What is the mass dm of this element, expressed as a fraction of the total mass M of the disk? (*b*) What is the rotational inertia dI of this element? (*c*) Integrate the result of part (*b*) to find the rotational inertia of the entire disk.

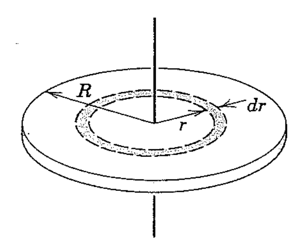

Figure 39 Problem 13.

14. In this problem, we use the result of the previous problem for the rotational inertia of a disk to compute the rotational inertia of a uniform solid sphere of mass M and radius R about an axis through its center. Consider an element dm of the sphere in the form of a disk of thickness dz at a height z above the center (see Fig. 40). (*a*) Expressed as a fraction of the total mass M, what is the mass dm of the element? (*b*) Considering the element as a disk, what is its rotational inertia dI? (*c*) Integrate the result of (*b*) over the entire sphere to find the rotational inertia of the sphere.

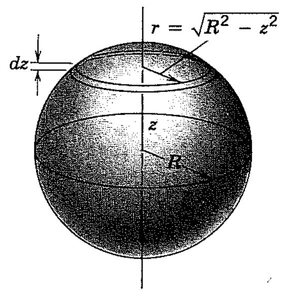

Figure 40 Problem 14.

Section 12-4 Torque Acting on a Particle

15. Figure 41 shows the lines of action and the points of application of two forces about the origin O. Imagine these forces to be acting on a rigid body pivoted at O, all vectors being in the plane of the figure. (*a*) Find an expression for the magnitude of the resultant torque on the body. (*b*) If $r_1 = 1.30$ m, $r_2 = 2.15$ m, $F_1 = 4.20$ N, $F_2 = 4.90$ N, $\theta_1 = 75.0°$, and $\theta_2 = 58.0°$, what are the magnitude and direction of the resultant torque?

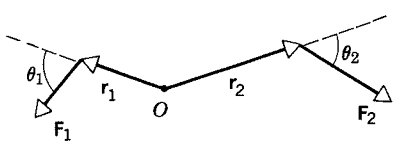

Figure 41 Problem 15.

16. Redraw Fig. 12 under the following transformations: (*a*) $\mathbf{F} \to -\mathbf{F}$, (*b*) $\mathbf{r} \to -\mathbf{r}$, and (*c*) $\mathbf{F} \to -\mathbf{F}$ and $\mathbf{r} \to -\mathbf{r}$, in each case showing the new direction of the torque. Check for consistency with the right-hand rule.

17. The object shown in Fig. 42 is pivoted at O. Three forces act on it in the directions shown on the figure: $F_A = 10$ N at point A, 8.0 m from O; $F_B = 16$ N at point B, 4.0 m from O; and $F_C = 19$ N at point C, 3.0 m from O. What are the magnitude and direction of the resultant torque about O?

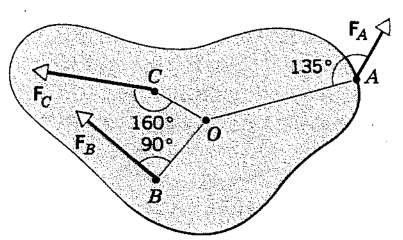

Figure 42 Problem 17.

18. (*a*) Given that $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$ and $\mathbf{F} = \mathbf{i}F_x + \mathbf{j}F_y + \mathbf{k}F_z$, find the torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$. (*b*) Show that if $\mathbf{r}$ and $\mathbf{F}$ lie in a given plane, then $\boldsymbol{\tau}$ has no component in that plane.

Section 12-5 Rotational Dynamics of a Rigid Body

19. A cylinder having a mass of 1.92 kg rotates about its axis of symmetry. Forces are applied as shown in Fig. 43: $F_1 = 5.88$ N, $F_2 = 4.13$ N, and $F_3 = 2.12$ N. Also, $R_1 = 4.93$ cm and $R_2 = 11.8$ cm. Find the magnitude and direction of the angular acceleration of the cylinder.

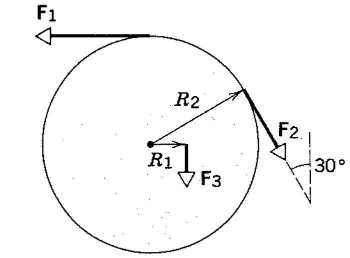

Figure 43 Problem 19.

20. A thin spherical shell has a radius of 1.88 m. An applied torque of 960 N·m imparts an angular acceleration equal to 6.23 rad/s^2 about an axis through the center of the shell. Calculate (*a*) the rotational inertia of the shell about the axis of rotation and (*b*) the mass of the shell.

21. In the act of jumping off a diving board, a diver changed his angular velocity from zero to 6.20 rad/s in 220 ms. The diver's rotational inertia is 12.0 kg·m^2. (*a*) Find the angular acceleration during the jump. (*b*) What external torque acted on the diver during the jump?

22. An automobile engine develops 133 hp (= 99.18 kW) when rotating at 1820 rev/min. How much torque does it deliver?

23. A 31.4-kg wheel with radius 1.21 m is rotating at 283 rev/min. It must be brought to a stop in 14.8 s. Find the required average power. Assume the wheel to be a thin hoop.

24. If $R = 12.3$ cm, $M = 396$ g, and $m = 48.7$ g in Fig. 18*a*, find the speed of the block after it has descended 54.0 cm starting from rest. Solve the problem using energy-conservation principles.

25. Assume the Earth to be a sphere of uniform density. (*a*) Calculate its rotational kinetic energy. (*b*) Suppose that this energy could be harnessed for our use. For how long could the Earth supply 1.00 kW of power to each of the 4.20×10^9 persons on the Earth?

26. Figure 44 shows the massive shield door at a neutron test facility at Lawrence Livermore Laboratory; this is the world's heaviest hinged door. The door has a mass of 44,000 kg, a rotational inertia about its hinge line of 8.7×10^4 kg·m^2, and a width of 2.4 m. What steady force, applied at its outer edge at right angles to the door, can move it from rest through an angle of 90° in 30 s?

27. A pulley having a rotational inertia of 1.14×10^{-3} kg·m^2 and a radius of 9.88 cm is acted on by a force, applied tangentially at its rim, that varies in time as $F = 0.496t + 0.305t^2$, where F is in newtons and t is in seconds. If the pulley was initially at rest, find its angular speed after 3.60 s.

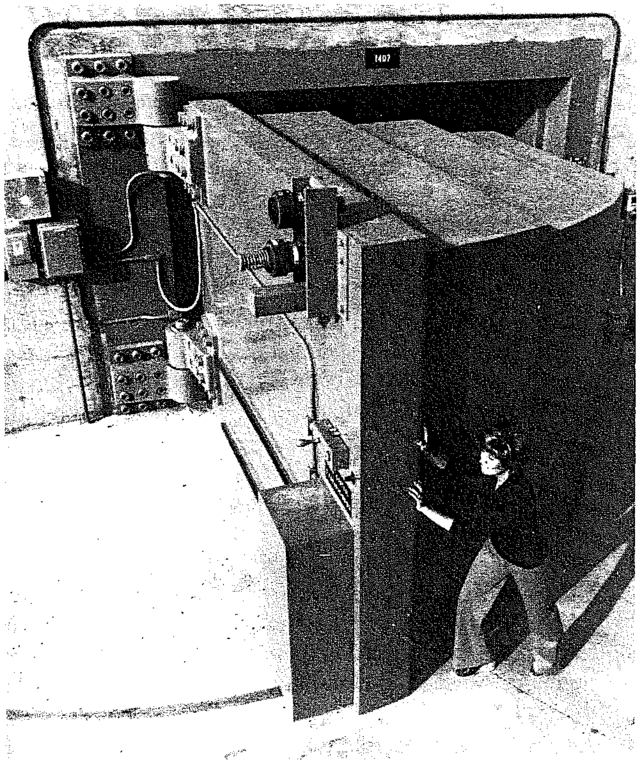

Figure 44 Problem 26.

28. Figure 45 shows two blocks each of mass m suspended from the ends of a rigid weightless rod of length $L_1 + L_2$, with $L_1 = 20.0$ cm and $L_2 = 80.0$ cm. The rod is held in the horizontal position shown in the figure and then released. Calculate the linear accelerations of the two blocks as they start to move.

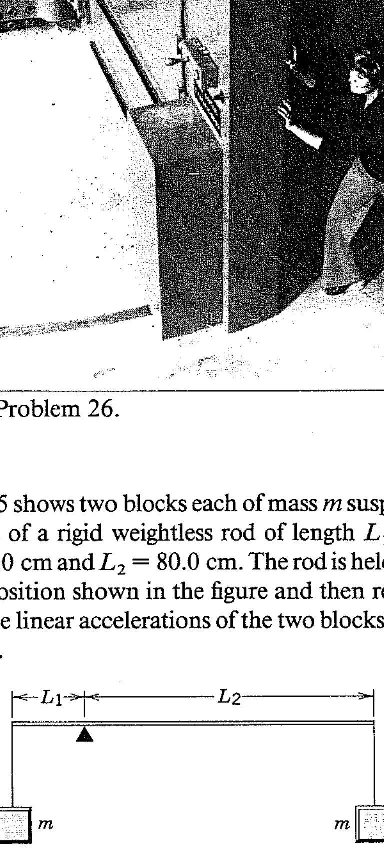

Figure 45 Problem 28.

29. Two identical blocks, each of mass M, are connected by a light string over a frictionless pulley of radius R and rotational inertia I (Fig. 46). The string does not slip on the pulley, and it is not known whether or not there is friction between the plane and the sliding block. When this system is released, it is found that the pulley turns through an angle θ in time t and the acceleration of the blocks is constant. (a) What is the angular acceleration of the pulley? (b) What is the acceleration of the two blocks? (c) What are the tensions in the upper and lower sections of the string? All answers are to be expressed in terms of M, I, R, θ, g, and t.

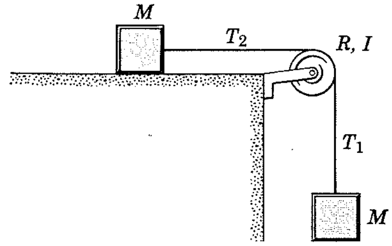

Figure 46 Problem 29.

30. A wheel of mass M and radius of gyration k (see Problem 11) spins on a fixed horizontal axle passing through its hub. Assume that the hub rubs the axle of radius a at only the topmost point, the coefficient of kinetic friction being μ_k. The wheel is given an initial angular velocity ω_0. Assume uniform deceleration and find (a) the elapsed time and (b) the number of revolutions before the wheel comes to a stop.

31. In an Atwood's machine one block has a mass of 512 g and the other a mass of 463 g. The pulley, which is mounted in horizontal frictionless bearings, has a radius of 4.90 cm. When released from rest, the heavier block is observed to fall 76.5 cm in 5.11 s. Calculate the rotational inertia of the pulley.

32. A wheel in the form of a uniform disk of radius 23.0 cm and mass 1.40 kg is turning at 840 rev/min in frictionless bearings. To stop the wheel, a brake pad is pressed against the rim of the wheel with a radially directed force of 130 N. The wheel makes 2.80 revolutions in coming to a stop. Find the coefficient of friction between the brake pad and the rim of the wheel.

33. A stick 1.27 m long is held vertically with one end on the floor and is then allowed to fall. Find the speed of the other end when it hits the floor, assuming that the end on the floor does not slip.

34. A uniform spherical shell rotates about a vertical axis on frictionless bearings (Fig. 47). A light cord passes around the equator of the shell, over a pulley, and is attached to a small object that is otherwise free to fall under the influence of gravity. What is the speed of the object after it has fallen a distance h from rest?

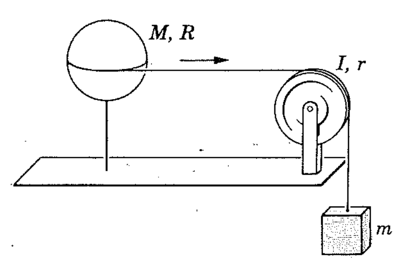

Figure 47 Problem 34.

35. A uniform steel rod of length 1.20 m and mass 6.40 kg has attached to each end a small ball of mass 1.06 kg. The rod is constrained to rotate in a horizontal plane about a vertical axis through its midpoint. At a certain instant, it is observed to be rotating with an angular speed of 39.0 rev/s. Because of axle friction, it comes to rest 32.0 s later. Compute, assuming a constant frictional torque, (a) the angular acceleration, (b) the retarding torque exerted by axle friction, (c) the energy dissipated by the axle friction, and (d) the number of revolutions executed during the 32.0 s. (e) Now suppose that the frictional torque is known not to be constant.

Which, if any, of the quantities (*a*), (*b*), (*c*), or (*d*) can still be computed without requiring any additional information? If such exists, give its value.

36. A rigid body is made of three identical thin rods fastened together in the form of a letter H (Fig. 48). The body is free to rotate about a horizontal axis that passes through one of the legs of the H. The body is allowed to fall from rest from a position in which the plane of the H is horizontal. What is the angular speed of the body when the plane of the H is vertical?

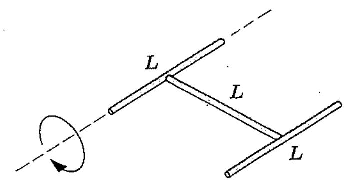

Figure 48 Problem 36.

37. A helicopter rotor blade is 7.80 m long and has a mass of 110 kg. (*a*) What force is exerted on the bolt attaching the blade to the rotor axle when the rotor is turning at 320 rev/min? (*Hint:* For this calculation the blade can be considered to be a point mass at the center of mass. Why?) (*b*) Calculate the torque that must be applied to the rotor to bring it to full speed from rest in 6.70 s. Ignore air resistance. (The blade cannot be considered to be a point mass for this calculation. Why not? Assume the distribution of a uniform rod.)

38. A tall chimney cracks near its base and falls over. Express (*a*) the radial and (*b*) the tangential linear acceleration of the top of the chimney as a function of the angle θ made by the chimney with the vertical. (*c*) Can the resultant linear acceleration exceed g? (*d*) The chimney cracks during the fall. Explain how this can happen. (See "More on the Falling Chimney," by Albert A. Bartlett, *The Physics Teacher*, September 1976, p. 351.)

39. The length of the day is increasing at the rate of about 1 ms/century. This is primarily due to frictional forces generated by movement of water in the world's shallow seas as a response to the tidal forces exerted by the Sun and Moon. (*a*) At what rate is the Earth losing rotational kinetic energy? (*b*) What is its angular acceleration? (*c*) What tangential force, exerted at latitudes 60°N and 60°S, is applied by the seas on the near-coastal seabed?

40. A uniform disk of radius R and mass M is spinning with angular speed ω_0. It is placed on a flat horizontal surface; the coefficient of kinetic friction between disk and surface is μ_k. (*a*) Find the frictional torque on the disk. (*b*) How long will it take for the disk to come to rest?

41. A car is fitted with an energy-conserving flywheel, which in operation is geared to the driveshaft so that it rotates at 237 rev/s when the car is traveling at 86.5 km/h. The total mass of the car is 822 kg, the flywheel weighs 194 N, and it is a uniform disk 1.08 m in diameter. The car descends a 1500-m long, 5.00° slope, from rest, with the flywheel engaged and no power supplied from the motor. Neglecting friction and the rotational inertia of the wheels, find (*a*) the speed of the car at the bottom of the slope, (*b*) the angular acceleration of the flywheel at the bottom of the slope, and (*c*) the power being absorbed by the rotation of the flywheel at the bottom of the slope.

Section 12-6 Combined Rotational and Translational Motion

42. A solid sphere of radius 4.72 cm rolls up an inclined plane of inclination angle 34.0°. At the bottom of the incline the center of mass of the sphere has a translational speed of 5.18 m/s. (*a*) How far does the sphere travel up the plane? (*b*) How long does it take to return to the bottom? (*c*) How many rotations does the sphere make during the round trip?

43. A hoop rolling down an inclined plane of inclination angle θ keeps pace with a block sliding down the same plane. Show that the coefficient of kinetic friction between block and plane is given by $\mu_k = \frac{1}{3} \tan \theta$.

44. A hoop of radius 3.16 m has a mass of 137 kg. It rolls along a horizontal floor so that its center of mass has a speed of 0.153 m/s. How much work must be done on the hoop to stop it?

45. An automobile traveling 78.3 km/h has tires of 77.0 cm diameter. (*a*) What is the angular speed of the tires about the axle? (*b*) If the car is brought to a stop uniformly in 28.6 turns of the tires (no skidding), what is the angular acceleration of the wheels? (*c*) How far does the car advance during this braking period?

46. A 1040-kg car has four 11.3-kg wheels. What fraction of the total kinetic energy of the car is due to rotation of the wheels about their axles? Assume that the wheels have the same rotational inertia as disks of the same mass and size. Explain why you do not need to know the radius of the wheels.

47. A yo-yo (see Sample Problem 7) has a rotational inertia of 950 $g \cdot cm^2$ and a mass of 120 g. Its axle radius is 3.20 mm and its string is 134 cm long. The yo-yo rolls from rest down to the end of the string. (*a*) What is its acceleration? (*b*) How long does it take to reach the end of the string? (*c*) If the yo-yo "sleeps" at the bottom of the string in pure rotary motion, what is its angular speed, in rev/s? (*d*) Repeat (*c*), but this time assume that the yo-yo was thrown down with an initial speed of 1.30 m/s.

48. A uniform sphere rolls down an incline. (*a*) What must be the incline angle if the linear acceleration of the center of the sphere is to be $0.133g$? (*b*) For this angle, what would be the acceleration of a frictionless block sliding down the incline?

49. A homogeneous sphere starts from rest at the upper end of the track shown in Fig. 49 and rolls without slipping until it rolls off the right-hand end. If $H = 60$ m and $h = 20$ m and the track is horizontal at the right-hand end, determine the distance to the right of point A at which the ball strikes the horizontal base line.

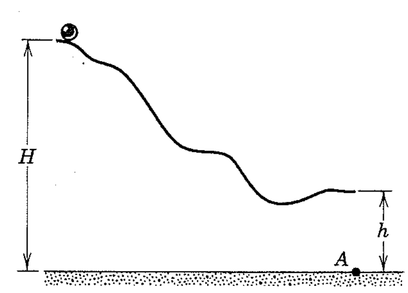

Figure 49 Problem 49.

50. A small solid marble of mass m and radius r rolls without slipping along the loop-the-loop track shown in Fig. 50, having been released from rest somewhere on the straight section of track. (*a*) From what minimum height above the bottom of the track must the marble be released in order that it just stay on the track at the top of the loop? (The radius of the loop-the-loop is R; assume $R \gg r$.) (*b*) If the marble is released from height $6R$ above the bottom of the track, what is the horizontal component of the force acting on it at point Q?

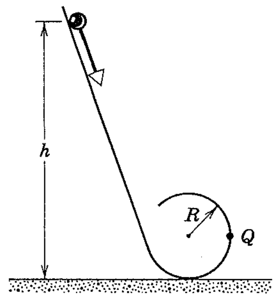

Figure 50 Problem 50.

51. A solid cylinder of length L and radius R has a weight W. Two cords are wrapped around the cylinder, one near each end, and the cord ends are attached to hooks on the ceiling. The cylinder is held horizontally with the two cords exactly vertical and is then released (Fig. 51). Find (*a*) the tension in each cord as they unwind and (*b*) the linear acceleration of the cylinder as it falls.

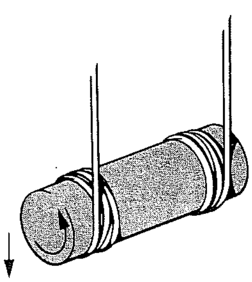

Figure 51 Problem 51.

52. A length L of flexible tape is tightly wound. It is then allowed to unwind as it rolls down a steep incline that makes an angle θ with the horizontal, the upper end of the tape being tacked down (Fig. 52). Show that the tape unwinds completely in a time $T = \sqrt{3L/g \sin \theta}$.

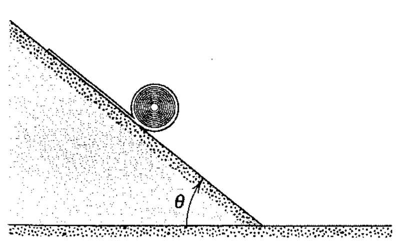

Figure 52 Problem 52.

53. Show that a cylinder will slip on an inclined plane of inclination angle θ if the coefficient of static friction between plane and cylinder is less than $\frac{1}{3} \tan \theta$.

54. A body is rolling horizontally without slipping with speed v. It then rolls up a hill to a maximum height h. If $h = 3v^2/4g$, what might the body be?

55. A uniform disk, of mass M and radius R, lies on one side initially at rest on a frictionless horizontal surface. A constant force **F** is then applied tangentially at its perimeter by means of a string wrapped around its edge. Describe the subsequent (rotational and translational) motion of the disk.

56. An apparatus for testing the skid resistance of automobile tires is constructed as shown in Fig. 53. The tire is initially motionless and is held in a light framework that is freely pivoted at points A and B. The rotational inertia of the wheel about its axis is 0.750 kg·m², its mass is 15.0 kg, and its radius is 30.0 cm. The tire is placed on the surface of a conveyor belt that is moving with a surface velocity of 12.0 m/s, such that AB is horizontal. (*a*) If the coefficient of kinetic friction between the tire and the conveyor belt is 0.600, what time will be required for the wheel to achieve its final angular velocity? (*b*) What will be the length of the skid mark on the conveyor surface?

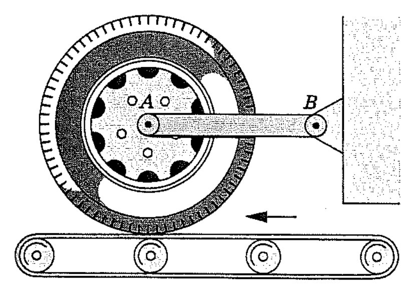

Figure 53 Problem 56.

57. A solid cylinder of radius 10.4 cm and mass 11.8 kg starts from rest and rolls without slipping a distance of 6.12 m down a house roof that is inclined at 27.0°. (*a*) What is the angular speed of the cylinder about its center as it leaves the house roof? (*b*) The outside wall of the house is 5.16 m high. How far from the wall does the cylinder hit the level ground? See Fig. 54.

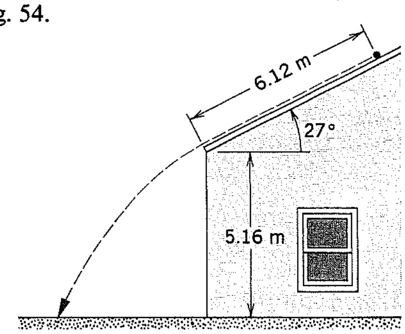

Figure 54 Problem 57.

58. A solid cylinder of mass 23.4 kg and radius 7.60 cm has a light thin tape wound around it. The tape passes over a light,

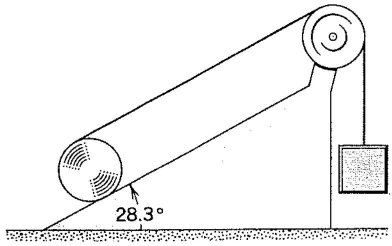

Figure 55 Problem 58.

frictionless pulley to an object of mass 4.48 kg, hanging vertically (see Fig. 55). The plane on which the cylinder moves is inclined 28.3° to the horizontal. Find (*a*) the linear acceleration of the cylinder down the incline and (*b*) the tension in the tape, assuming no slipping.

59. A student throws a stick of length L up into the air. At the moment it leaves her hand the speed of the stick's closest end is zero. The stick completes N turns just as it is caught by the student at the initial release point. Show that the height h that the center of mass rose is $h = \pi NL/4$.

60. A billiard ball is struck by a cue as in Fig. 56. The line of action of the applied impulse is horizontal and passes through the center of the ball. The initial velocity $\mathbf{v}_0$ of the ball, its radius R, its mass M, and the coefficient of friction μ_k between the ball and the table are all known. How far will the ball move before it ceases to slip on the table?

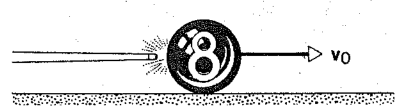

Figure 56 Problem 60.