7. Suppose we have a block of unknown mass and a spring of unknown force constant. Show how we can predict the period of oscillation of this block–spring system simply by measuring the extension of the spring produced by attaching the block to it.
8. Any real spring has mass. If this mass is taken into account, explain qualitatively how this will affect the period of oscillation of a spring–block system.
9. Can one have an oscillator that even for small amplitudes is not simple harmonic? That is, can one have a nonlinear restoring force in an oscillator even at arbitrarily small amplitudes?
10. How are each of the following properties of a simple harmonic oscillator affected by doubling the amplitude: period, force constant, total mechanical energy, maximum velocity, maximum acceleration?
11. What changes could you make in a harmonic oscillator that would double the maximum speed of the oscillating object?
12. A person stands on a bathroom-type scale, which rests on a platform suspended by a large spring. The whole system executes simple harmonic motion in a vertical direction. Describe the variation in scale reading during a period of motion.
13. Could we ever construct a true simple pendulum? Explain your answer.
14. Could standards of mass, length, and time be based on properties of a pendulum? Explain.
15. Considering the elastic and the inertial aspects involved, explain the fact that whereas when an object of mass m oscillates vertically on a spring the period depends on m but is independent of g, the reverse is true for a simple pendulum.
16. Predict by qualitative arguments whether a pendulum oscillating with large amplitude will have a period longer or shorter than the period for oscillations with small amplitude. (Consider extreme cases.)
17. As the amplitude θ_m in Eq. 25 approaches 180°, what value do you expect the period to approach? Explain in physical terms.
18. What happens to the frequency of a swing as its oscillations die down from large amplitude to small?
19. How is the period of a pendulum affected when its point of suspension is (*a*) moved horizontally in the plane of oscillation with acceleration a; (*b*) moved vertically upward with acceleration a; (*c*) moved vertically downward with acceleration $a < g$; with acceleration $a > g$? Which case, if any, applies to a pendulum mounted on a cart rolling down an inclined plane?
20. Why was an axis through the center of mass excluded in using Eq. 29 to determine I? Does this equation apply to such an axis? How can you determine I for such an axis using physical pendulum methods?
21. A hollow sphere is filled with water through a small hole in it. It is hung by a long thread and, as the water flows out of the hole at the bottom, one finds that the period of oscillation first increases and then decreases. Explain.
22. (*a*) The effect of the mass, m, of the cord attached to the bob, of mass M, of a pendulum is to increase the period over that for a simple pendulum in which $m = 0$. Make this plausible. (*b*) Although the effect of the mass of the cord on the pendulum is to increase its period, a cord of length L swinging without anything on the end ($M = 0$) has a period less than that of a simple pendulum of length L. Make that plausible.
23. If taken to the Moon, will there be any change in the frequency of oscillation of a torsional pendulum? A simple pendulum? A spring–block oscillator? A physical pendulum?
24. How can a pendulum be used so as to trace out a sinusoidal curve?
25. What component simple harmonic motions would give a figure 8 as the resultant motion?
26. Is there a connection between the F versus x relation at the molecular level and the macroscopic relation between F and x in a spring? Explain your answer.
27. (*a*) Under what circumstances would the reduced mass of a two-body system be equal to the mass of one of the bodies? Explain. (*b*) What is the reduced mass if the bodies have equal mass? (*c*) Do cases (*a*) and (*b*) give the extreme values of the reduced mass?
28. Why is the tub of a washing machine often mounted on springs?
29. Why are damping devices often used on machinery? Give an example.
30. Give some examples of common phenomena in which resonance plays an important role.
31. The lunar ocean tide is much more important than the solar ocean tide. The opposite is true for tides in the Earth's atmosphere, however. Explain this, using resonance ideas, given the fact that the atmosphere has a natural period of oscillation of nearly 12 hours.
32. In Fig. 20, what value does the amplitude of the forced oscillations approach as the driving frequency ω'' approaches (*a*) zero and (*b*) infinity?
33. Buildings of different heights sustain different amounts of damage in an earthquake. Explain why.
34. A singer, holding a note of the right frequency, can shatter a glass if the glassware is of high quality. This cannot be done if the glassware quality is low. Explain why.

PROBLEMS

Section 15-3 Simple Harmonic Motion

1. A 3.94-kg block extends a spring 15.7 cm from its unstretched position. The block is removed and a 0.520-kg object is hung from the same spring. Find the period of its oscillation.
2. An oscillator consists of a block of mass 512 g connected to a spring. When set into oscillation with amplitude 34.7 cm, it is observed to repeat its motion every 0.484 s. Find (*a*) the period, (*b*) the frequency, (*c*) the angular frequency, (*d*) the

force constant, (*e*) the maximum speed, and (*f*) the maximum force exerted on the block.

3. The vibration frequencies of atoms in solids at normal temperatures are of the order of 10.0 THz. Imagine the atoms to be connected to one another by "springs." Suppose that a single silver atom vibrates with this frequency and that all the other atoms are at rest. Compute the effective force constant. One mole of silver has a mass of 108 g and contains 6.02×10^{23} atoms.
4. A loudspeaker produces a musical sound by the oscillation of a diaphragm. If the amplitude of oscillation is limited to 1.20×10^{-3} mm, what frequencies will result in the acceleration of the diaphragm exceeding g?
5. A 5.22-kg object is attached to the bottom of a vertical spring and set vibrating. The maximum speed of the object is 15.3 cm/s and the period is 645 ms. Find (*a*) the force constant of the spring, (*b*) the amplitude of the motion, and (*c*) the frequency of oscillation.
6. In an electric shaver, the blade moves back and forth over a distance of 2.00 mm. The motion is simple harmonic, with frequency 120 Hz. Find (*a*) the amplitude, (*b*) the maximum blade speed, and (*c*) the maximum blade acceleration.
7. An automobile can be considered to be mounted on four springs as far as vertical oscillations are concerned. The springs of a certain car of mass 1460 kg are adjusted so that the vibrations have a frequency of 2.95 Hz. (*a*) Find the force constant of each of the four springs (assumed identical). (*b*) What will be the vibration frequency if five persons, averaging 73.2 kg each, ride in the car?
8. A body oscillates with simple harmonic motion according to the equation

$$x = (6.12\text{ m}) \cos [(8.38\text{ rad/s})t + 1.92\text{ rad}].$$

Find (*a*) the displacement, (*b*) the velocity, and (*c*) the acceleration at the time $t = 1.90$ s. Find also (*d*) the frequency and (*e*) the period of the motion.

9. The scale of a spring balance reading from 0 to 50.0 lb is 4.00 in. long. A package suspended from the balance is found to oscillate vertically with a frequency of 2.00 Hz. How much does the package weigh?
10. The piston in the cylinder head of a locomotive has a stroke of 76.5 cm. What is the maximum speed of the piston if the drive wheels make 193 rev/min and the piston moves with simple harmonic motion?
11. Figure 24 shows an astronaut on a Body Mass Measurement Device (BMMD). Designed for use on orbiting space vehicles, its purpose is to allow astronauts to measure their mass in the weightless conditions in Earth orbit. The BMMD is a spring-mounted chair; an astronaut measures his or her period of oscillation in the chair; the mass follows from the formula for the period of an oscillating block–spring system. (*a*) If M is the mass of the astronaut and m the effective mass of that part of the BMMD that also oscillates, show that

$$M = (k/4\pi^2)T^2 - m,$$

where T is the period of oscillation and k is the force constant. (*b*) The force constant is $k = 605.6$ N/m for the BMMD on Skylab Mission Two; the period of oscillation of the empty chair is 0.90149 s. Calculate the effective mass of the chair. (*c*) With an astronaut in the chair, the period of oscillation becomes 2.08832 s. Calculate the mass of the astronaut.

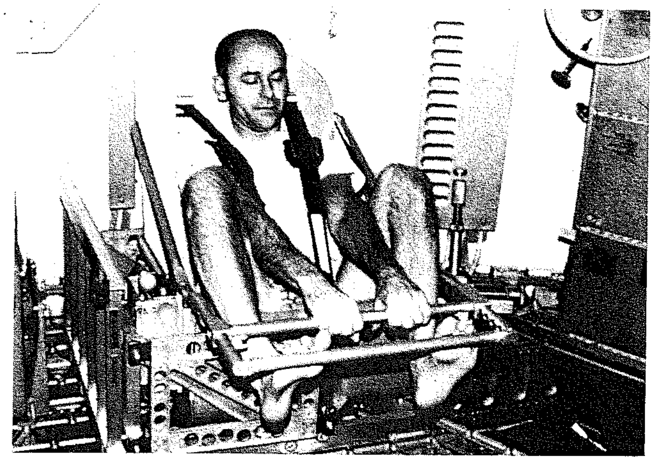

Figure 24 Problem 11.

12. A 2.14-kg object hangs from a spring. A 325-g body hung below the object stretches the spring 1.80 cm farther. The 325-g body is removed and the object is set into oscillation. Find the period of the motion.
13. At a certain harbor, the tides cause the ocean surface to rise and fall in simple harmonic motion, with a period of 12.5 h. How long does it take for the water to fall from its maximum height to one-half its maximum height above its average (equilibrium) level?
14. Two blocks ($m = 1.22$ kg and $M = 8.73$ kg) and a spring ($k = 344$ N/m) are arranged on a horizontal, frictionless surface as shown in Fig. 25. The coefficient of static friction between the blocks is 0.42. Find the maximum possible amplitude of the simple harmonic motion if no slippage is to occur between the blocks.

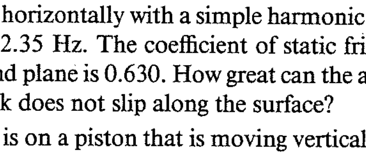

Figure 25 Problem 14.

15. A block is on a horizontal surface (a shake table) that is moving horizontally with a simple harmonic motion of frequency 2.35 Hz. The coefficient of static friction between block and plane is 0.630. How great can the amplitude be if the block does not slip along the surface?
16. A block is on a piston that is moving vertically with simple harmonic motion. (*a*) At what amplitude of motion will the block and the piston separate if the period of the piston's motion is 1.18 s? (*b*) If the piston has an amplitude of 5.12 cm in its motion, find the maximum frequency for which the block and piston will be in contact continuously.
17. The force of interaction between two atoms in certain diatomic molecules can be represented by $F = -a/r^2 + b/r^3$, in which a and b are positive constants and r is the separation distance of the atoms. Make a graph of F versus r. Then (*a*) show that the separation at equilibrium is b/a; (*b*) show

that for small oscillations about this equilibrium separation the force constant is a^4/b^3; (*c*) find the period of this motion.

18. An oscillator consists of a block attached to a spring ($k = 456$ N/m). At some time *t*, the position (measured from the equilibrium location), velocity, and acceleration of the block are $x = 0.112$ m, $v = -13.6$ m/s, $a = -123$ m/s^2. Calculate (*a*) the frequency, (*b*) the mass of the block, and (*c*) the amplitude of oscillation.

19. Two particles oscillate in simple harmonic motion along a common straight line segment of length *L*. Each particle has a period of 1.50 s but they differ in phase by 30.0°. (*a*) How far apart are they (in terms of *L*) 0.500 s after the lagging particle leaves one end of the path? (*b*) Are they moving in the same direction, toward each other, or away from each other at this time?

20. Two particles execute simple harmonic motion of the same amplitude and frequency along the same straight line. They pass one another when going in opposite directions each time their displacement is half their amplitude. Find the phase difference between them.

21. Two springs are attached to a block of mass *m*, free to slide on a frictionless horizontal surface, as shown in Fig. 26. Show that the frequency of oscillation of the block is

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} = \sqrt{\nu_1^2 + \nu_2^2},$$

where ν_1 and ν_2 are the frequencies at which the block would oscillate if connected only to spring 1 or spring 2. (The electrical analog of this system is a series combination of two capacitors.)

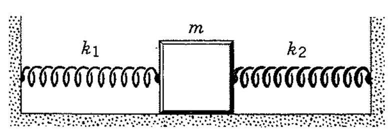

Figure 26 Problem 21.

22. Two springs are joined and connected to a block of mass *m* as shown in Fig. 27. The surfaces are frictionless. If the springs separately have force constants k_1 and k_2, show that the frequency of oscillation of the block is

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}} = \frac{\nu_1 \nu_2}{\sqrt{\nu_1^2 + \nu_2^2}},$$

where ν_1 and ν_2 are the frequencies at which the block would oscillate if connected only to spring 1 or spring 2. (The electrical analog of this system is a parallel combination of two capacitors.)

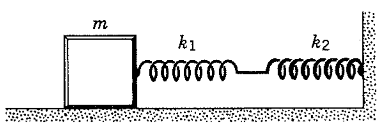

Figure 27 Problem 22.

23. Three 10,000-kg ore cars are held at rest on a 26.0° incline on a mine railway using a cable that is parallel to the incline (Fig. 28). The cable is observed to stretch 14.2 cm just before a coupling breaks, detaching one of the cars. Find (*a*) the frequency of the resulting oscillations of the remaining two cars and (*b*) the amplitude of the oscillations.

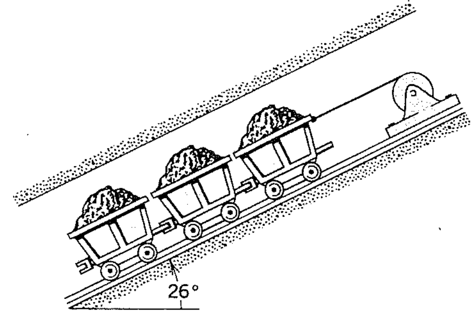

Figure 28 Problem 23.

24. A massless spring of force constant 3.60 N/cm is cut into halves. (*a*) What is the force constant of each half? (*b*) The two halves, suspended separately, support a block of mass *M* (see Fig. 29). The system vibrates at a frequency of 2.87 Hz. Find the value of the mass *M*.

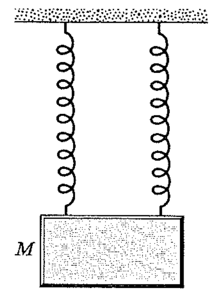

Figure 29 Problem 24.

25. If the mass of a spring m_s is not negligible but is small compared to the mass *m* of the object suspended from it, the period of motion is $T = 2\pi\sqrt{(m + m_s/3)/k}$. Derive this result. (*Hint:* The condition $m_s \ll m$ is equivalent to the assumption that the spring stretches proportionally along its length.) (See H. L. Armstrong, *American Journal of Physics*, Vol. 37, p. 447, 1969, for a complete solution of the general case.)

Section 15-4 Energy Considerations in Simple Harmonic Motion

26. An oscillating block–spring system has a mechanical energy of 1.18 J, an amplitude of 9.84 cm, and a maximum speed of 1.22 m/s. Find (*a*) the force constant of the spring, (*b*) the mass of the block, and (*c*) the frequency of oscillation.

27. A (hypothetical) large slingshot is stretched 1.53 m to launch a 130-g projectile with speed sufficient to escape from the Earth (11.2 km/s). (*a*) What is the force constant of the device, if all the potential energy is converted to kinetic energy? (*b*) Assume that an average person can exert a force of 220 N. How many people are required to stretch the slingshot?

28. (*a*) When the displacement is one-half the amplitude x_m, what fraction of the total energy is kinetic and what fraction is potential in simple harmonic motion? (*b*) At what displacement is the energy half kinetic and half potential?

29. A 12.3-kg particle is undergoing simple harmonic motion with an amplitude of 1.86 mm. The maximum acceleration experienced by the particle is 7.93 km/s^2. (*a*) Find the period of the motion. (*b*) What is the maximum speed of the particle? (*c*) Calculate the total mechanical energy of this simple harmonic oscillator.

30. A 5.13-kg object moves on a horizontal frictionless surface under the influence of a spring with force constant 9.88 N/cm. The object is displaced 53.5 cm and given an initial velocity of 11.2 m/s back toward the equilibrium position. Find (*a*) the frequency of the motion, (*b*) the initial potential energy of the system, (*c*) the initial kinetic energy, and (*d*) the amplitude of the motion.

31. Show that the general relationships between the two initial values of position $x(0)$ and velocity $v(0)$, and the amplitude x_m and phase angle ϕ of Eq. 6, are

$$x_m = \sqrt{[x(0)]^2 + [v(0)/\omega]^2}, \qquad \tan\phi = -v(0)/\omega x(0).$$

32. Solve Eq. 16, which expresses conservation of energy, for dt and integrate the result. Assume that $x = x_m$ at $t = 0$, and show that Eq. 6 (with $\phi = 0$), the displacement as a function of time, is obtained.

33. An object of mass 1.26 kg attached to a spring of force constant 5.38 N/cm is set into oscillation by extending the spring 26.3 cm and giving the object a velocity of 3.72 m/s toward the equilibrium position of the spring. Using the results obtained in Problem 31, calculate (*a*) the amplitude and (*b*) the phase angle of the resulting simple harmonic motion.

34. A block of mass M, at rest on a horizontal, frictionless table, is attached to a rigid support by a spring of force constant k. A bullet of mass m and speed v strikes the block as shown in Fig. 30. The bullet remains embedded in the block. Determine the amplitude of the resulting simple harmonic motion, in terms of m, M, v, and k.

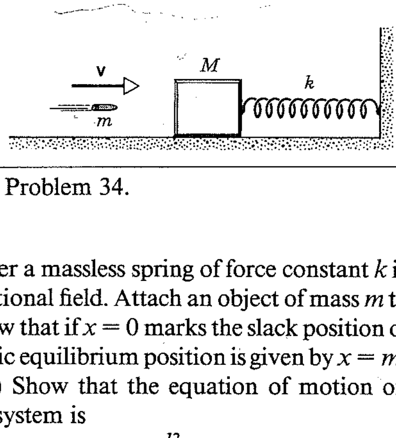

Figure 30 Problem 34.

35. Consider a massless spring of force constant k in a uniform gravitational field. Attach an object of mass m to the spring. (*a*) Show that if $x = 0$ marks the slack position of the spring, the static equilibrium position is given by $x = mg/k$ (see Fig. 31). (*b*) Show that the equation of motion of the mass–spring system is

$$m\frac{d^2x}{dt^2} + kx = mg$$

and that the solution for the displacement as a function of time is $x = x_m \cos(\omega t + \phi) + mg/k$, where $\omega = \sqrt{k/m}$ as before. (*c*) Show therefore that the system has the same ω, v, a, ν, and T in a uniform gravitational field as in the absence of such a field, with the one change that the equilibrium position has been displaced by mg/k. (*d*) Now consider the energy of the system, $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 + mg(h - x) =$ constant, and show that time differentiation leads to the equation of motion of part (*b*). (*e*) Show that when the object

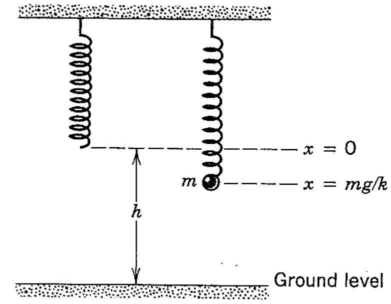

Figure 31 Problem 35.

falls from $x = 0$ to the static equilibrium position, $x = mg/k$, the loss in gravitational potential energy goes half into a gain in elastic potential energy and half into a gain in kinetic energy. (*f*) Finally, consider the system in motion about the static equilibrium position. Compute separately the change in gravitational potential energy and in elastic potential energy when the object moves *up* through a displacement x_m, and when the object moves *down* through a displacement x_m. Show that the *total* change in potential energy is the same in each case, namely, $\frac{1}{2}kx_m^2$. In view of the results (*c*) and (*f*), one can simply ignore the uniform gravitational field in the analysis merely by shifting the reference position from $x = 0$ to $x_0 = x - mg/k = 0$. The new potential energy curve $[U(x_0) = \frac{1}{2}kx_0^2 + \text{constant}]$ has the same parabolic shape as the potential energy curve in the absence of a gravitational field $[U(x) = \frac{1}{2}kx^2]$.

36. A 4.00-kg block is suspended from a spring with a force constant of 5.00 N/cm. A 50.0-g bullet is fired into the block from below with a speed of 150 m/s and comes to rest in the block. (*a*) Find the amplitude of the resulting simple harmonic motion. (*b*) What fraction of the original kinetic energy of the bullet appears as mechanical energy in the oscillator?

37. A solid cylinder is attached to a horizontal massless spring so that it can *roll without slipping* along a horizontal surface, as in Fig. 32. The force constant k of the spring is 2.94 N/cm. If the system is released from rest at a position in which the spring is stretched by 23.9 cm, find (*a*) the translational kinetic energy and (*b*) the rotational kinetic energy of the cylinder as it passes through the equilibrium position. (*c*) Show that under these conditions the center of mass of the cylinder executes simple harmonic motion with a period

$$T = 2\pi\sqrt{3M/2k},$$

where M is the mass of the cylinder.

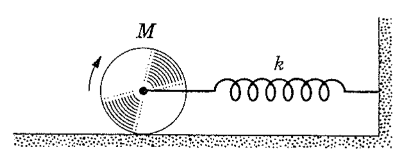

Figure 32 Problem 37.

38. (*a*) Prove that in simple harmonic motion the average potential energy equals the average kinetic energy when the average is taken with respect to time over one period of the motion, and that each average equals $\frac{1}{4}kx_m^2$. (*b*) Prove that when the average is taken with respect to position over one cycle, the average potential energy equals $\frac{1}{6}kx_m^2$ and the average kinetic energy equals $\frac{1}{3}kx_m^2$. (*c*) Explain physically why the two results above (*a* and *b*) are different.

Section 15-5 Applications of Simple Harmonic Motion

39. Find the length of a simple pendulum whose period is 1.00 s at a location where $g = 9.82\ \text{m/s}^2$.

40. A simple pendulum of length 1.53 m makes 72.0 complete oscillations in 180 s at a certain location. Find the acceleration due to gravity at this point.

41. A 2500-kg demolition ball swings from the end of a crane, as shown in Fig. 33. The length of the swinging segment of cable is 17.3 m. Find the period of swing, assuming that the system can be treated as a simple pendulum.

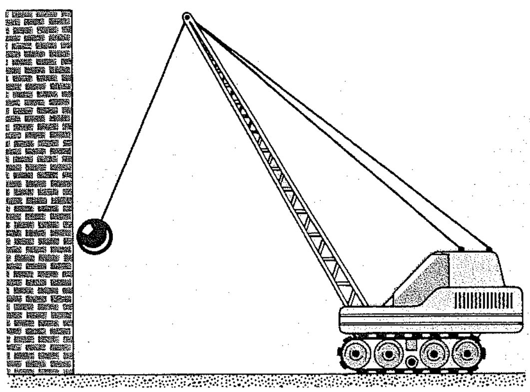

Figure 33 Problem 41.

42. There is an interesting relation between the block–spring system and the simple pendulum. Suppose that you hang an object of mass *M* on the end of a spring, and when the object is in equilibrium the spring is stretched a distance *h*. Show that the frequency of this block–spring system is the same as that of a simple pendulum of mass *m* and length *h*, even if $m \neq M$; see Fig. 34.

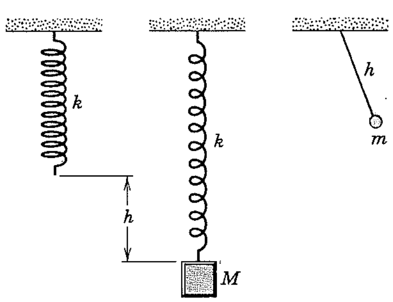

Figure 34 Problem 42.

43. A circular hoop of radius 65.3 cm and mass 2.16 kg is suspended on a horizontal nail. (*a*) Find its frequency of oscillation for small displacements from equilibrium. (*b*) What is the length of the equivalent simple pendulum?

44. An engineer wants to find the rotational inertia of an odd-shaped object of mass 11.3 kg about an axis through its center of mass. The object is supported with a wire through its center of mass and along the desired axis. The wire has a torsional constant $\kappa = 0.513\ \text{N}\cdot\text{m}$. The engineer observes that this torsional pendulum oscillates through 20.0 complete cycles in 48.7 s. What value of rotational inertia is calculated?

45. A physical pendulum consists of a uniform solid disk of mass $M = 563$ g and radius $R = 14.4$ cm supported in a vertical plane by a pivot located a distance $d = 10.2$ cm from the center of the disk, as shown in Fig. 35. The disk is displaced by a small angle and released. Find the period of the resulting simple harmonic motion.

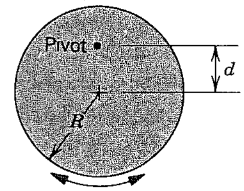

Figure 35 Problem 45.

46. A 95.2-kg solid sphere with a 14.8-cm radius is suspended by a vertical wire attached to the ceiling of a room. A torque of $0.192\ \text{N}\cdot\text{m}$ is required to twist the sphere through an angle of 0.850 rad. Find the period of oscillation when the sphere is released from this position.

47. A physical pendulum consists of a meter stick that is pivoted at a small hole drilled through the stick a distance *x* from the 50.0-cm mark. The period of oscillation is observed to be 2.50 s. Find the distance *x*.

48. A pendulum consists of a uniform disk with radius 10.3 cm and mass 488 g attached to a 52.4-cm long uniform rod with mass 272 g; see Fig. 36. (*a*) Calculate the rotational inertia of the pendulum about the pivot. (*b*) What is the distance between the pivot and the center of mass of the pendulum? (*c*) Calculate the small-angle period of oscillation.

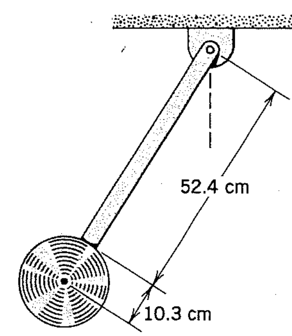

Figure 36 Problem 48.

49. A pendulum is formed by pivoting a long thin rod of length *L* and mass *m* about a point on the rod which is a distance *d* above the center of the rod. (*a*) Find the small-amplitude period of this pendulum in terms of *d*, *L*, *m*, and *g*. (*b*) Show that the period has a *minimum* value when $d = L/\sqrt{12} = 0.289L$.

50. A wheel is free to rotate about its fixed axle. A spring is attached to one of its spokes a distance *r* from the axle, as shown in Fig. 37. Assuming that the wheel is a hoop of mass *M* and radius *R*, obtain the angular frequency of small oscil-

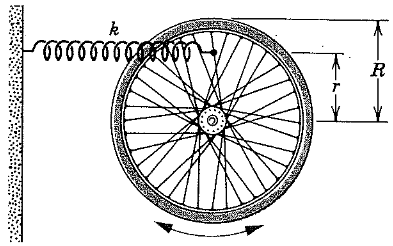

Figure 37 Problem 50.

lations of this system in terms of M, R, r, and the force constant k. Discuss the special cases $r = R$ and $r = 0$.

51. A meter stick swinging from one end oscillates with a frequency ν_0. What would be the frequency, in terms of ν_0, if the bottom third of the stick were cut off?

52. A particle is released from rest at a point P inside a frictionless hemispherical bowl of radius R. (*a*) Show that when P is near the bottom of the bowl the particle undergoes simple harmonic motion. (*b*) Find the length of the equivalent simple pendulum.

53. A physical pendulum has two possible pivot points; one has a fixed position and the other is adjustable along the length of the pendulum, as shown in Fig. 38. The period of the pendulum when suspended from the fixed pivot is T. The pendulum is then reversed and suspended from the adjustable pivot. The position of this pivot is moved until, by trial and error, the pendulum has the same period as before, namely, T. Show that the free-fall acceleration g is given by

$$g = \frac{4\pi^2 L}{T^2},$$

in which L is the distance between the two pivot points. Note that g can be measured in this way without needing to know the rotational inertia of the pendulum or any of its other dimensions except L.

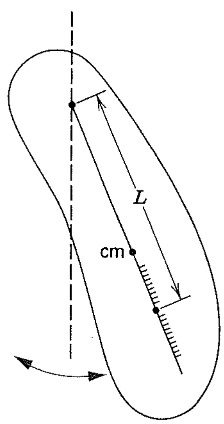

Figure 38 Problem 53.

54. A 2.50-kg disk, 42.0 cm in diameter, is supported by a light rod, 76.0 cm long, which is pivoted at its end, as shown in Fig. 39. (*a*) The light, torsional spring is initially not connected. What is the period of oscillation? (*b*) The torsional spring is now connected so that, in equilibrium, the rod hangs vertically. What should be the torsional constant of

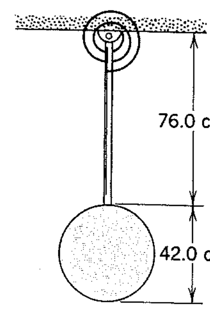

Figure 39 Problem 54.

the spring so that the new period of oscillation is 500 ms shorter than before?

55. A simple pendulum of length L and mass m is suspended in a car that is traveling with a constant speed v around a circle of radius R. If the pendulum undergoes small oscillations in a radial direction about its equilibrium position, what will its frequency of oscillation be?

56. Figure 40 shows a physical pendulum constructed from equal-length sections of identical pipe. The inner radius of the pipe is 10.2 cm and the thickness is 6.40 mm. (*a*) Calculate the period of oscillation about the pivot shown. (*b*) Suppose that a new physical pendulum is constructed by rotating the bottom section 90° about a vertical axis through its center. Show that the new period of oscillation about the same pivot is about 2% less than the period of the original pendulum.

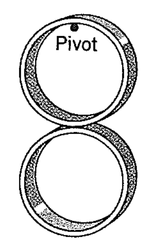

Figure 40 Problem 56.

Section 15-7 Combinations of Harmonic Motions

57. Sketch the path of a particle that moves in the xy plane according to $x = x_m \cos(\omega t - \pi/2)$, $y = 2x_m \cos(\omega t)$.

58. The diagram shown in Fig. 41 is the result of combining the two simple harmonic motions $x = x_m \cos \omega_x t$ and $y =$

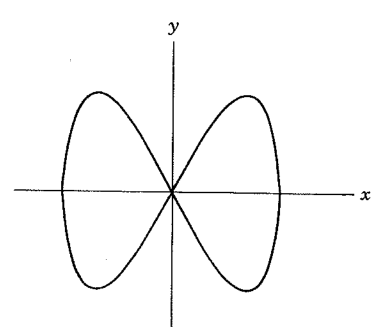

Figure 41 Problem 58.

$y_m \cos(\omega_y t + \phi_y)$. (*a*) What is the value of x_m/y_m? (*b*) What is the value of ω_x/ω_y? (*c*) What is the value of ϕ_y?

59. Electrons in an oscilloscope are deflected by two mutually perpendicular electric fields in such a way that at any time t the displacement is given by

$$x = A\cos\omega t, \qquad y = A\cos(\omega t + \phi_y).$$

Describe the path of the electrons and determine their equation when (*a*) $\phi_y = 0°$, (*b*) $\phi_y = 30°$, and (*c*) $\phi_y = 90°$.

60. A particle of mass m moves in a fixed plane along the trajectory $\mathbf{r} = \mathbf{i}A\cos\omega t + \mathbf{j}A\cos 3\omega t$. (*a*) Sketch the trajectory of the particle. (*b*) Find the force acting on the particle. Also find (*c*) its potential energy and (*d*) its total energy as functions of time. (*e*) Is the motion periodic? If so, find the period.

61. When oscillations at right angles are combined, the frequencies for the motion of the particle in the x and y directions need not be equal, so that in the general case Eqs. 36 become

$$x = x_m\cos(\omega_x t + \phi_x) \quad \text{and} \quad y = y_m\cos(\omega_y t + \phi_y).$$

The path of the particle is no longer an ellipse but is called a *Lissajous curve*, after Jules Antoine Lissajous who first demonstrated such curves in 1857. (*a*) If ω_x/ω_y is a rational number, so that the angular frequencies ω_x and ω_y are "commensurable," then the curve is closed and the motion repeats itself at regular intervals of time. Assume $x_m = y_m$ and $\phi_x = \phi_y$ and draw the Lissajous curve for $\omega_x/\omega_y = \frac{1}{2}, \frac{1}{3}$, and $\frac{2}{3}$. (*b*) Let ω_x/ω_y be a rational number, either $\frac{1}{2}, \frac{1}{3}$, or $\frac{2}{3}$, say, and show that the shape of the Lissajous curve depends on the phase difference $\phi_x - \phi_y$. Draw curves for $\phi_x - \phi_y = 0, \pi/4$, and $\pi/2$ rad. (*c*) If ω_x/ω_y is not a rational number, then the curve is "open." Convince yourself that after a long time the curve will have passed through every point lying in the rectangle bounded by $x = \pm x_m$ and $y = \pm y_m$, the particle never passing twice through a given point with the same velocity. For definiteness, assume $\phi_x = 0$ throughout.

Section 15-8 Damped Harmonic Motion

62. For the system shown in Fig. 18, the block has a mass of 1.52 kg and the force constant is 8.13 N/m. The frictional force is given by $-b(dx/dt)$, where $b = 227$ g/s. Suppose that the block is pulled aside a distance 12.5 cm and released. (*a*) Calculate the time interval required for the amplitude to fall to one-third of its initial value. (*b*) How many oscillations are made by the block in this time?

63. Verify, by taking derivatives, that Eq. 38 is a solution of Eq. 37 for the damped oscillator, provided that the frequency ω' is given by Eq. 39.

64. A damped harmonic oscillator involves a block ($m = 1.91$ kg), a spring ($k = 12.6$ N/m), and a damping force $F = -bv$. Initially, it oscillates with an amplitude of 26.2 cm; because of the damping, the amplitude falls to three-fourths of this initial value after four complete cycles. (*a*) What is the value of b? (*b*) How much energy has been "lost" during these four cycles?

65. Assume that you are examining the characteristics of a suspension system of a 2000-kg automobile. The suspension "sags" 10 cm when the weight of the entire automobile is placed on it. In addition, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of k and b for the spring and shock absorber system of each wheel. Assume each wheel supports 500 kg.

Section 15-9 Forced Oscillations and Resonance

66. Consider the forced oscillations of a damped block–spring system. Show that at resonance (*a*) the amplitude of oscillation is $x_m = F_m/b\omega$, and (*b*) the maximum speed of the oscillating block is $v_{max} = F_m/b$.

67. A 2200-lb car carrying four 180-lb people is traveling over a rough "washboard" dirt road. The corrugations in the road are 13 ft apart. The car is observed to bounce with maximum amplitude when its speed is 10 mi/h. The car now stops and the four people get out. By how much does the car body rise on its suspension owing to this decrease in weight?

68. Starting from Eq. 42, find the velocity $v\ (= dx/dt)$ in forced oscillatory motion. Show that the velocity amplitude is $v_m = F_m/[(m\omega'' - k/\omega'')^2 + b^2]^{1/2}$. The equations of Section 15-9 are identical in form with those representing an electrical circuit containing a resistance R, an inductance L, and a capacitance C in series with an alternating emf $V = V_m\cos\omega'' t$. Hence b, m, k, and F_m are analogous to R, L, $1/C$, and V_m, respectively, and x and v are analogous to electric charge q and current i, respectively. In the electrical case the current amplitude i_m, analogous to the velocity amplitude v_m above, is used to describe the quality of the resonance.

Section 15-10 Two-Body Oscillations

69. Suppose that the spring in Fig. 22*a* has a force constant $k = 252$ N/m. Let $m_1 = 1.13$ kg and $m_2 = 3.24$ kg. Calculate the period of oscillation of the two-body system.

70. (*a*) Show that when $m_2 \to \infty$ in Eq. 46, $m \to m_1$. (*b*) Show that the effect of a noninfinite wall ($m_2 < \infty$) on the oscillations of a body of mass m_1 at the end of a spring attached to the wall is to reduce the period, or increase the frequency, of oscillation compared to (*a*). (*c*) Show that when $m_2 = m_1$ the effect is as though the spring were cut in half, each body oscillating independently about the center of mass at the middle.

71. (*a*) Calculate the reduced mass of each of the following diatomic molecules: O_2, HCl, and CO. Express your answers in unified atomic mass units, the mass of a hydrogen atom being 1.00 u. (*b*) An HCl molecule is known to vibrate at a fundamental frequency of $\nu = 8.7 \times 10^{13}$ Hz. Find the effective "force constant" k for the coupling forces between the atoms. In terms of your experience with ordinary springs, would you say that this "molecular spring" is relatively stiff or not?

72. Show that the kinetic energy of the two-body oscillator of Fig. 22*a* is given by $K = \frac{1}{2}mv^2$, where m is the reduced mass and $v\ (= v_1 - v_2)$ is the relative velocity. It may help to note that linear momentum is conserved while the system oscillates.

Computer Projects

73. Write a computer program or design a spreadsheet to calculate the amplitude and phase of a simple harmonic motion when you supply the force constant k, the mass m, the initial coordinate x_0, and the initial velocity v_0. Write the coordinate as $x(t) = x_m \cos(\omega t + \phi)$ and use $\omega = \sqrt{k/m}$, $x_m = \sqrt{x_0^2 + v_0^2/\omega^2}$, and $\phi = \tan^{-1}(-v_0/\omega x_0)$. Be sure to check that the value calculated for ϕ is correct by verifying that $\cos\phi$ has the same sign as x_0 and $\sin\phi$ has the same sign as v_0. If they do not, add 180° (or π rad) to the calculated value. Also be careful of division by zero. If $x_0 = 0$ automatically set $\phi = +90°$ or $-90°$ without attempting to calculate $v_0/\omega x_0$. Which angle you choose, of course, depends on the sign of v_0. Write the program so that once it has finished a calculation it returns to the beginning and requests data for the next problem. Here are some oscillations to try. All involve a mass of 250 g on a spring with a force constant of 200 N/m. (*a*) $x_0 = 2.8$ cm, $v_0 = 0$. (*b*) $x_0 = -2.8$ cm, $v_0 = 0$. (*c*) $x_0 = 0$, $v_0 = 56$ cm/s. (*d*) $x_0 = 0$, $v_0 = -56$ cm/s. (*e*) $x_0 = 2.8$ cm, $v_0 = 56$ cm/s. (*f*) $x_0 = 2.8$ cm, $v_0 = -56$ cm/s. (*g*) $x_0 = -2.8$ cm, $v_0 = 56$ cm/s. (*h*) $x_0 = -2.8$ cm, $v_0 = -56$ cm/s.

74. You can use a computer to study damped oscillations. Consider a mass m on the end of a spring with force constant k, subject to a drag force that is proportional to its velocity. Newton's second law yields $m\,d^2x/dt^2 = -kx - bv$. Write a computer program or design a spreadsheet to compute the coordinate x, the velocity v, and the total mechanical energy E at the end of every time interval of duration Δt from $t = 0$ to $t = t_f$. See Section 6-6 and the computer projects at the end of Chapter 6. Use the program to solve the following problems. In each case take $m = 2.0$ kg, $k = 350$ N/m, $x_0 = 0.070$ m, $v_0 = 0$, $t_f = 1.0$ s. Use an integration interval of 0.001 s. (*a*) Take $b = 2.8$ kg/s and on separate graphs plot $x(t)$ and $E(t)$. Notice the decrease in amplitude as time goes on. The decrease is intimately associated with a loss in energy via the drag force. Notice that the energy graph has small oscillations and that there are short regions where the energy is nearly constant. Where in the oscillatory motion do these regions occur? Give a physical explanation for their occurrence. Does the drag force change the period of the oscillation? Use the graph to estimate the time between successive maxima and compare the result with $2\pi\sqrt{m/k}$. (*b*) If the drag force is increased sufficiently, no oscillations occur and the motion is said to be *overdamped*. Take $b = 110$ kg/s and use your program to plot $x(t)$ and $E(t)$.

75. If a sinusoidal force is applied to an object on the end of a spring, Newton's second law becomes $m\,d^2x/dt^2 = -kx - bv + F_m \cos\omega'' t$. Write a computer program or design a spreadsheet to compute the coordinate x, velocity v, and total mechanical energy E of the oscillator at the end of every time interval of duration Δt from $t = 0$ to $t = t_f$. See Section 6-6 and the computer projects at the end of Chapter 6. For the following problems take $m = 2.0$ kg, $k = 350$ N/m, $x_0 = 0.070$ m, $v_0 = 0$, $t_f = 2.0$ s. Use an integration interval of 0.001 s. (*a*) Neglect damping by setting $b = 0$ and take $F_m = 18$ N and $\omega'' = 35$ rad/s. Use your program to plot $x(t)$ and $E(t)$ on separate graphs. Notice that the applied force causes slight deviations from a sinusoidal shape. Also notice that the applied force transfers energy to the oscillator during some portions of the motion and removes it during others. As a result, the amplitude changes slightly with time. Use the graph to estimate the average amplitude. Also estimate the period and use its value to calculate the angular frequency. Is it closer to 35 rad/s or to $\omega = \sqrt{k/m}$? (*b*) Again set $b = 0$ and take $F_m = 18$ N but now take $\omega'' = 15$ rad/s, much closer to $\sqrt{k/m}$. Plot $x(t)$ and $E(t)$ and note the increasing amplitude and energy. The applied force puts energy into the oscillator over much longer periods than it takes energy out. (*c*) Now take damping into account by setting $b = 15$ kg/s. Again let $F_m = 18$ N and $\omega'' = 35$ rad/s, far from $\sqrt{k/m}$. Plot $x(t)$ and $E(t)$. Use your graph to find the angular frequency near $t = 0$ and near $t = 2$ s. You might measure half a period and double your result. Notice that at first the motion is close to the natural motion, the motion in the absence of an applied force. By about 1 s the natural motion has been damped considerably and the subsequent motion is that which is applied to the mass by the external force. Estimate the amplitude near $t = 2$ s. (*d*) Repeat part (*c*) but take $\omega'' = 15$ rad/s. Estimate the amplitude. (*e*) For which of the situations considered is the amplitude near $t = 2$ s the greatest? the least?