Physics 21: Final (Winter 2013)

Exam is closed book, closed notes, no cell phones. Calculators are OK.

For question 1 (conceptual questions), do **6** of the 7 questions. For questions 2-5, do **3** of the 4 computational questions. **Do only the required number of questions. If you do more than that, only the first 6 conceptual and the first 3 computational problems will be graded.**

Rotational Motion

$v = \omega r;\;\; a = \alpha r$
$I_1 = Mh^2 + I_0$, where h is perpendicular distance between 2 parallel axes I_0, I_1
$I(\text{rod about center}) = \frac{1}{12}ML^2$; $I(\text{disk}) = \frac{2}{5}MR^2$
$L_z = I_0\omega + (\mathbf{r} \times m\mathbf{v})_z$
$\tau = \mathbf{r} \times \mathbf{F} = \frac{d\mathbf{L}}{dt} = I\alpha$
$K = \frac{1}{2}I_0\omega^2 + \frac{1}{2}Mv^2$; $U = Mgh$

Harmonic Motion

Simple Harmonic Oscillator: $m\ddot{x} + kx = 0$
has solution $x = A\cos(\omega t + \phi)$; where $\omega = \sqrt{\frac{k}{m}}$

Pendulum $\omega = \sqrt{g/l}$; $\omega = \frac{2\pi}{T}$

Damped Harmonic Oscilllator: $m\ddot{x} + b\dot{x} + kx = 0$. has solution $x = Ae^{-\gamma t/2}cos(\omega t + \phi)$ where $\omega = \sqrt{\frac{k}{m}}$, and energy $E = E_0 e^{-\gamma t}$; $\gamma = b/m$.

Quality factor $Q = E/(\Delta E \text{ in 1 radian}) \approx \omega/\gamma$

Fluids

Pressure=Force/Area
Continuity $R = \rho v A$ =const
Bernoulli $p + \frac{1}{2}\rho v^2 + \rho gh$ =const

1. [**28 points**] Conceptual questions: **short** answers, not more than 3 lines. Do **6** of the 7 questions below.

a) [**4 points**] Astronauts go back to the moon, which has gravity 1/6 that of earth. (i) The astronauts bring a pendulum which has a frequency of 1 Hz on earth. What is its frequency on the moon? (ii) The spring suspension of the lunar buggy has a frequency of 1 Hz on earth. What is its frequency on the moon?

b) [**4 points**] Consider a boat floating in your bathtub. Suppose the boats springs a leak, and starts taking in water, though it is still floating. How does the water level in the bathtub change? Why? Suppose the boat sinks to the bottom. How does the water level in the bathtub change? Why?

c)[**4 points**] The word is there's gold just offshore on Campus Point. You don't have money for scuba, so you build a 10m long snorkel and head out. Your really annoying Physics 21 prof yells out to you, "Don't do it—even if you can breath through it, the blood vessels in your lungs could burst!" Should you believe him? Why?

d)[**4 points**] Consider the uniform U-tube shown in Fig 1. Imagine the diaphragm at point 2 is cut, so that liquid flows from left to right. Show that application of Bernoulli's equation to points 1 & 3 leads to a contradiction. Why is Bernoulli's principle not applicable?

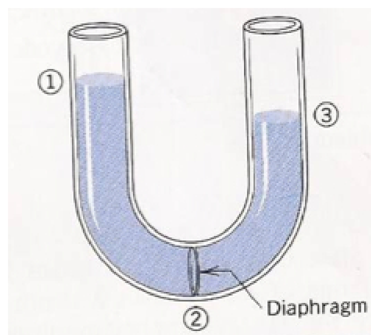

Fig. 1.— Question 1d.

e)[**4 points**] Consider 2 surfaces A & B, identical in all respects except that the coefficient of static friction is double on surface B compared to surface A. Objects rolling on surface B will stop in half the distance compared to surface A. True or false? Why?

f)[**4 points**] Suppose a helicopter with 3 rotors is rotating with angular velocity ω_0. Now you nip in and add a 4th rotor (you have *very* fast reflexes). What is the ratio of the new angular velocity ω_1 to the old angular velocity ω_0? What is the ratio of the new energy E_1 to the old energy E_0?

g)[**4 points**] Mighty Mouse is stuck in a bowl with a heavy ball (see Fig 2). He wants to get rid of the ball, but is too weak to push it out all the way–he can only roll it a little. Can he ever get it out? How?

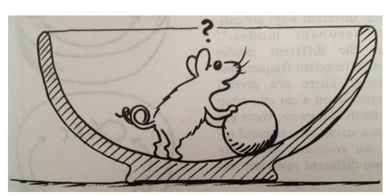

Fig. 2.— Question 1g.

Computational Questions

Do **3** of the 4 problems below.

2. [**10 points**] 10 kg of mercury is poured into a glass U-tube, as shown in Fig 3. The tube's inner radius is $r = 1$ cm, and the mercury oscillates freely up and down about its position of equilibrium ($x = 0$). The density of mercury is $\rho_{\rm M} = 13\,600\,{\rm kg\,m^{-3}}$. Ignore friction and surface tension, and assume small oscillations. Compute:
(a) [**4 points**] The effective spring constant k for the system (give in terms of N/m).
(b) [**3 points**] The period of vibration (give in s).
(c) [**3 points**] Suppose this motion damps by a factor of 2 in 10 seconds. What is the quality factor Q?

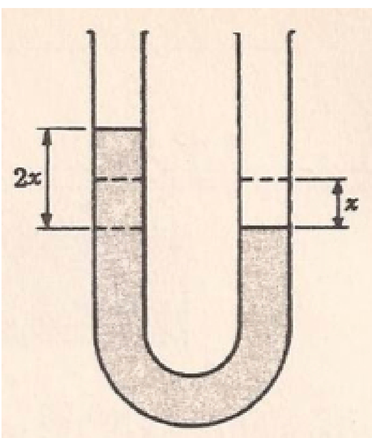

Fig. 3.— Question 2.

3. [**10 points**] See Fig 4. Water, which has density $\rho_{\rm W}$, is pumped at high pressure at rate R into the left end of a cylindrical pipe. R is the mass flux, i.e. the mass pumped per unit time into the pipe. The radius of the left end of the pipe is r_1. The pipe gradually curves and narrows. At the mouth at the other end of the pipe, the radius is r_2. The mouth of the pipe is at the same height as point B. Express your answers symbolically in terms of $\rho_{\rm W}, R, r_1, r_2, g$, where g is the gravitational constant:

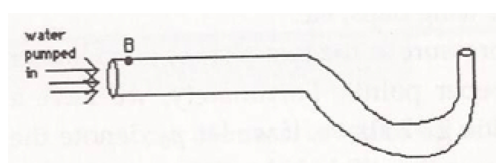

Fig. 4.— Question 3.

(a) [**5 points**] How high into the air, above the mouth of the pipe, does the water shoot?

(b) [**5 points**] If the air pressure is p_0, what is the pressure exerted by the water on the wall of the pipe at point B?

4. [**10 points**] See Fig 5. A long thin rod (mass M, length L) has a very small block of mass m glued to its right end. The left end is attached to a wall with a pin joint, which acts as a pivot around which the rod can rotate.
(a) [**5 points**] What is the moment of inertia of the rod + block about the pivot?
(b) [**5 points**] The rod is released from rest from a horizontal configuration. It swings down and crashes into the wall. What is its angular speed immediately before crashing?

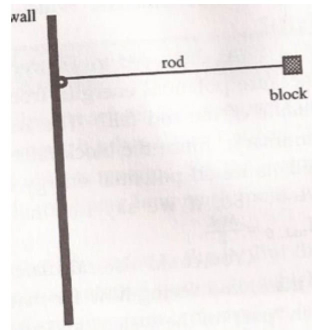

Fig. 5.— Question 4.

5. [**10 points**] See Fig 6. A uniform solid disk, of mass M and radius R, is mounted on an axle through its center. A spring with spring constant k is attached to an essentially massless peg that sticks out of the disk at a distance $R/2$ from the axle. The other end is attached to a wall. In the diagram, the spring is at its equilibrium length. If the disk is rotated only a small distance away from equilibrium, the spring extends by the arc length traced out by the peg, and the force it exerts is almost exactly perpendicular to the line connecting the peg and axle.
(a) [**3 points**] Someone displaces the disk through 1°, lets go, and times the period of oscillation. She then displaces the disk through 0.5°, and again times the period. Which period is longer?
(b) [**7 points**] When it is displaced by 1° and released, what is the disk's period of oscillation?

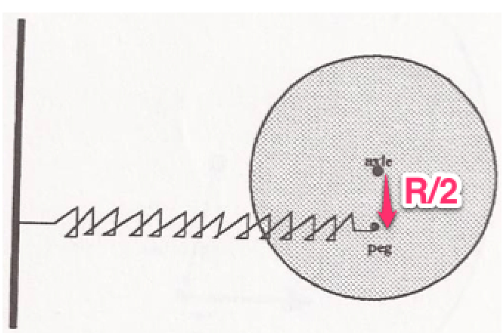

Fig. 6.— Question 5.