Physics 21 Final Solutions

1 a) Spring suspension: $\omega = \sqrt{\frac{k}{m}}$ stays the same (2)

Pendulum: $\omega = \sqrt{\frac{g}{L}} \Rightarrow \omega = \sqrt{1/6}$ Hz. (2)

b) Leaking but floating: water level stays the same (1)
(bcos boat displaces same vol of ~~mass~~ water
= weight of boat)
the additional water it takes on has the same effect
inside ~~or~~ outside the boat (1)

Sunk: water level falls (1)
Now boat only displaces its volume. Since boat is denser
than water, this is less than when it was floating.
(1)

c) (1) Yes, it is dangerous.
(1) Your blood pressure will match the higher water pressure (= 2 atms) at 10 ~~mm~~ m underwater.
(1) But you will be breathing air at atmospheric pressure.
(1) The pressure difference can make blood vessels rupture.

d) Point 1 moves downward and Point 3 moves upward at the same velocity

At 1 : $p_0 + \frac{1}{2}\rho v^2 + \rho g y_1$

At 2 : $p_0 + \frac{1}{2}\rho v^2 + \rho g y_2$

But $y_2 \neq y_1$

(2)

Bernoulli does not apply because flow is not time steady (2)

e) False (2)

Coefficient of static friction applies to "no slip" rolling, and no work is done. (2)
(no energy loss).

f) Suppose each rotor has a moment of inertia I_o.

$$3 I_o \omega_o = 4 I_o \omega_1$$

$$\Rightarrow \frac{\omega_1}{\omega_o} = \frac{3}{4} \quad (2)$$

$$E_1 = \frac{1}{2}(4 I_o)\omega_1^2 = 2 I_o \left(\frac{3}{4}\right)^2 \omega_o^2$$

$$= \frac{9}{8} I_o \omega_o^2$$

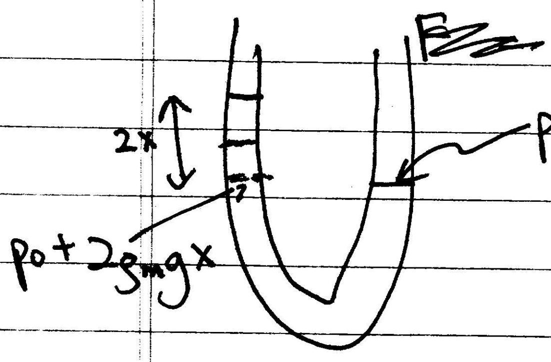
$$E_o = \frac{1}{2}(3 I_o)\omega_o^2 = \frac{3}{2} I_o \omega_o^2$$

$$\Rightarrow \frac{E_1}{E_o} = \frac{9}{8} \cdot \frac{2}{3} = \frac{3}{4} \quad (2)$$

g) <u>Yes</u>, he can get it out. (2)

He just has to consistently push in phase with the motion of the ball to achieve resonance — just like pumping a swing. (2)

2. (a) The ~~restoring~~ net force on the system



$$F = A(p_0 - (p_0 + 2\rho_m g x))$$

$$= -2\rho_m g A x$$

(k = $2\rho_m g A$) (3)

$$\Rightarrow k = 2\rho_m g A$$

$$= 2(13600)(9.8)(\pi(10^{-2})^2)$$

$$= 84 \text{ N/m} \quad (1)$$

(b) $$\omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$ (3)

$$= 2\pi\sqrt{\frac{10}{84}} = 2.2\,s$$

(c) $$Q = \frac{\omega}{\gamma}, \quad \omega = \frac{2\pi}{T} = 2.9\,s^{-1}.$$

$$e^{-\gamma t} = \frac{1}{2}$$

$$\Rightarrow \gamma t = \ln 2 \Rightarrow \gamma = \frac{1}{10}\ln 2$$

$$Q = \frac{\omega}{\gamma} = \frac{29}{\ln 2} = 42$$

(4)

3. (a) $R = \rho v_1 A_1 = \rho v_2 A_2$

$$\Rightarrow v_2 = \frac{R}{\rho \pi r_2^2}$$

$$\frac{1}{2}\rho v_2^2 = \rho g h$$

$$\Rightarrow h = \frac{1}{2g} v_2^2$$

$$= \frac{1}{2g}\left[\frac{R}{\rho \pi r_2^2}\right]^2$$

$$= \frac{R^2}{2\pi^2 g r_2^4 \rho^2}$$

(5)

(b) Bernoulli

$$p_0 + \frac{1}{2}\rho v_2^2 = p_1 + \frac{1}{2}\rho v_1^2$$

$$\Rightarrow p_1 = p_0 + \frac{1}{2}\rho\left[v_2^2 - v_1^2\right]$$

$$= p_0 + \frac{1}{2}\rho\left[\left(\frac{R}{\rho\pi r_2^2}\right)^2 - \left(\frac{R}{\rho\pi r_1^2}\right)^2\right]$$

$$= p_0 + \frac{R^2}{2\rho\pi^2}\left[\frac{1}{r_2^4} - \frac{1}{r_1^4}\right]$$

(5)

4. Using parallel axis theorem,

(a) $I_{rod} = \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3} ML^2.$

$$I = I_{rod} + I_{block} = \frac{1}{3} ML^2 + mL^2$$

$$= \left(\frac{1}{3}M + m\right) L^2.$$

(b) Use conservation of energy

$$\Delta U = mgL + Mg\frac{L}{2}$$

$$K = \frac{1}{2} I \omega^2$$

$$\Rightarrow \omega^2 = \frac{2}{I}\left(mgL + Mg\frac{L}{2}\right)$$

$$= \frac{gL}{I}(2m + M)$$

$$\Rightarrow \omega = \left[\frac{gL(2m+M)}{\left(\frac{1}{3}M + m\right)L^2}\right]^{1/2}$$

$$= \left[\frac{g(6m + 3M)}{(M + 3m)L}\right]^{1/2}$$

5. (a) The period is the same.
The frequency of a SHO is independent of amplitude.

(3)

(b) $\tau = I\alpha = \vec{r} \times \vec{F} = \frac{R}{2} F$ (since force is $\perp$ to $\vec{r}$).

$$= \frac{R}{2}(-kx)$$

$$= \frac{R}{2}\left(-k\theta\left(\frac{R}{2}\right)\right)$$

$$= -\left(\frac{R}{2}\right)^2 k\theta$$

$$\Rightarrow I\ddot{\theta} + \left(\frac{R}{2}\right)^2 k\theta = 0$$

$$\Rightarrow \ddot{\theta} + \left(\frac{R}{2}\right)^2 \frac{k}{I}\theta = 0$$

$$\Rightarrow \omega^2 = \left(\frac{R}{2}\right)^2 \frac{k}{I} = \left(\frac{R}{2}\right)^2 \frac{k}{\frac{1}{2}MR^2}$$

$$\Rightarrow \omega = \sqrt{\frac{k}{2M}}$$

(7)

Period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2M}{k}}$