ing from chaos theory is truly *nonperiodic,* not simply the combination of a large number of periodic motions. There is a critical distinction between these two cases. If the transition from laminar to turbulent flow takes place through a succession of orderly periodic motions, then two particles of fluid that in the laminar flow are moving similarly will remain in closely related states of motion throughout the transition into turbulent flow. However, if the intermediate condition can be described as chaotic, then the motion loses its predictability, and the two particles can be found in the turbulent flow in very different states of motion. Chaos theory, which is applicable to a wide variety of physical systems, provides an alternative theoretical basis for understanding complex systems such as the turbulent motion of fluids. ■

QUESTIONS

1. Briefly describe what is meant by each of the following and illustrate with an example: (*a*) steady fluid flow; (*b*) nonsteady fluid flow; (*c*) rotational fluid flow; (*d*) irrotational fluid flow; (*e*) compressible fluid flow; (*f*) incompressible fluid flow; (*g*) viscous fluid flow; (*h*) nonviscous fluid flow.
2. Explain the pressure variations in your blood as it circulates through your body.
3. Explain how a physician can measure your blood pressure.
4. In steady flow, the velocity vector at any point is constant. Can there then be accelerated motion of the fluid particles? Explain.
5. Describe the forces acting on an element of fluid as it flows through a pipe of nonuniform cross section.
6. In a lecture demonstration, a Ping-Pong ball is kept in midair by a vertical jet of air. Is the equilibrium stable, unstable, or neutral? Explain.
7. The height of the liquid in the standpipes of Fig. 25 indicates that the pressure drops along the channel, even though the channel has a uniform cross section and the flowing liquid is incompressible. Explain.

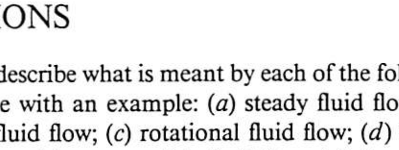

Figure 25 Question 7.

8. Explain why a taller chimney creates a better draft for taking the smoke out of a fireplace. Why doesn't the smoke pour into the room containing the fireplace?
9. (*a*) Explain how a baseball pitcher can make the baseball curve to his right or left. Can we justify applying Bernoulli's equation to such a spinning baseball? (See "Bernoulli and Newton in Fluid Mechanics," by Norman F. Smith, *The Physics Teacher,* November 1972, p. 451.) (*b*) Why is it easier to throw a curve with a tennis ball than with a baseball?
10. Not only a ball with a rough surface but also a smooth ball can be made to curve when thrown, but these balls will curve in *opposite* directions. Why? (See "Effect of Spin and Speed on the Curve of a Baseball and the Magnus Effect for Smooth Spheres," by Lyman J. Briggs, *American Journal of Physics,* November 1959, p. 589.)
11. Two rowboats moving parallel to one another in the same direction are pulled toward one another. Two automobiles moving parallel are also pulled together. Explain such phenomena on the basis of Bernoulli's equation.
12. In building "skyscrapers," what forces produced by the movement of air must be counteracted? How is this done? (See "The Wind Bracing of Buildings," by Carl W. Condit, *Scientific American,* February 1974, p. 92.)
13. Explain the action of a parachute in retarding free fall using Bernoulli's equation.
14. Why does a stream of water from a faucet become narrower as it falls?
15. Can you explain why water flows in a continuous stream down a vertical pipe, whereas it breaks into drops when falling freely?
16. How does the flush toilet work? Really. (See *Flushed with Pride: The Story of Thomas Crapper,* by W. Reyburn, Prentice-Hall, Englewood Cliffs, N.J., 1969.)
17. Sometimes people remove letters from envelopes by cutting a sliver from a narrow end, holding it firmly, and blowing toward it. Explain, using Bernoulli's equation, why this procedure is successful.
18. On takeoff would it be better for an airplane to move into the wind or with the wind? On landing?
19. Explain how the difference in pressure between the lower and upper surfaces of an airplane wing depend on the altitude of the moving plane.
20. The accumulation of ice on an airplane wing may greatly reduce its lift. Explain. (The *weight* of the ice is not the issue here.)
21. How is an airplane able to fly upside down?
22. "The characteristic banana-like shape of most returning boomerangs has hardly anything to do with their ability to return. . . . The essential thing is the cross section of the arms, which should be more convex on one side than on the other, like the wing profile of an airplane." (From "The Aerodynamics of Boomerangs," by Felix Hess, *Scientific American,* November 1968, p. 124.) Explain.
23. What powers the flight of soaring birds? (See "The Soaring Flight of Birds," by C. D. Cone, Jr., *Scientific American,* April 1962, p. 130.)
24. Why does the factor "2" appear in Eq. 15, rather than "1"? One might naively expect that the thrust would simply be the pressure difference times the area, that is, $A_0(p - p_0)$.
25. Explain why the destructive effect of a tornado is greater near the center of the disturbance than near the edge.

26. When a stopper is pulled from a filled basin, the water drains out while circulating like a small whirlpool. The angular velocity of a fluid element about a vertical axis through the orifice appears to be greatest near the orifice. Explain.

27. Is it true that in bathtubs in the northern hemisphere the water drains out with a counterclockwise rotation and in those in the southern hemisphere with a clockwise rotation? If so, explain and predict what would happen at the equator. (See "Bath-Tub Vortex," by Ascher H. Shapiro, *Nature,* December 15, 1962, p. 1080.)

28. Explain why you cannot remove the filter paper from the funnel of Fig. 26 by blowing into the narrow end.

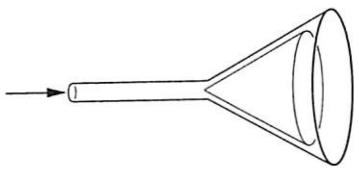

Figure 26 Question 28.

29. According to Bernoulli's equation, an increase in velocity should be associated with a decrease in pressure. Yet, when you put your hand outside the window of a moving car, increasing the speed at which the air flows by, you sense an *increase* in pressure. Why is this not a violation of Bernoulli's equation?

30. Why is it that the presence of the atmosphere reduces the maximum range of some objects (for example, tennis balls) but increases the maximum range of others (for example, Frisbees and golf balls)?

31. A discus can be thrown farther *against* a 25-mi/h wind than with it. What is the explanation? (*Hint:* Think about dynamic lift and drag.)

32. Explain why golf balls are dimpled.

33. The longer the board and the shallower the water, the farther will a surf board skim across the water. Explain. (See "The Surf Skimmer," by R. D. Edge, *American Journal of Physics,* July 1968, p. 630.)

34. When poured from a teapot, water has a tendency to run along the underside of the spout. Explain. (See "The Teapot Effect . . . a Problem," by Markus Reiner, *Physics Today,* September 1956, p. 16.)

35. Prairie dogs live in large colonies in complex interconnected burrow systems. They face the problem of maintaining a sufficient air supply to their burrows to avoid suffocation. They avoid this by building conical earth mounds about some of their many burrow openings. In terms of Bernoulli's equation, how does this air conditioning scheme work? Note that because of viscous forces the wind speed over the prairie is less close to the ground level than it is even a few inches higher up. (See *New Scientist,* January 27, 1972, p. 191.)

36. Viscosity is an example of a transport phenomenon. What property is being transported? Can you think of other transport phenomena and their corresponding properties?

37. Why do auto manufacturers recommend using "multi-viscosity" engine oil in cold weather?

38. Why is it more important to take viscosity into account for a fluid flowing in a narrow channel than in a relatively unconfined channel?

39. Viscosity can delay the onset of turbulence in fluid flow; that is, it tends to stabilize the flow. Consider syrup and water, for example, and make this plausible.

PROBLEMS

Section 18-2 Streamlines and the Equation of Continuity

1. A pipe of diameter 34.5 cm carries water moving at 2.62 m/s. How long will it take to discharge 1600 m^3 of water?

2. A garden hose having an internal diameter of 0.75 in. is connected to a lawn sprinkler that consists merely of an enclosure with 24 holes, each 0.050 in. in diameter. If the water in the hose has a speed of 3.5 ft/s, at what speed does it leave the sprinkler holes?

3. Figure 27 shows the confluence of two streams to form a river. One stream has a width of 8.2 m, depth of 3.4 m, and current speed of 2.3 m/s. The other stream is 6.8 m wide, 3.2 m deep, and flows at 2.6 m/s. The width of the river is 10.7 m and the current speed is 2.9 m/s. What is its depth?

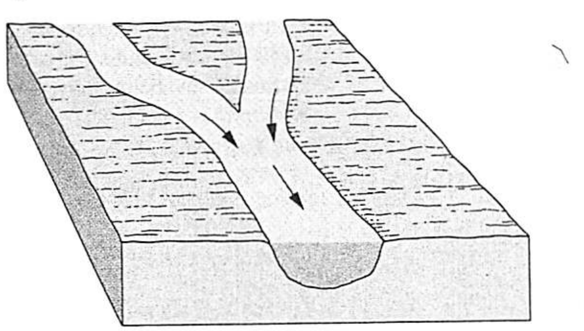

Figure 27 Problem 3.

4. Water is pumped steadily out of a flooded basement at a speed of 5.30 m/s through a uniform hose of radius 9.70 mm. The hose passes out through a window 2.90 m above the water line. How much power is supplied by the pump?

5. A river 21 m wide and 4.3 m deep drains a 8500-km^2 land area in which the average precipitation is 48 cm/y. One-fourth of this subsequently returns to the atmosphere by evaporation, but the remainder ultimately drains into the river. What is the average speed of the river current?

6. Tidal currents in narrow channels connecting coastal bays with the ocean can be very swift. Water must flow into the bay as the tide rises and back out to the sea as the tide falls. Consider the rectangular bay shown in Fig. 28*a*. The bay is connected to the sea by a channel 190 m wide and 6.5 m deep at mean sea level. The graph (Fig. 28*b*) shows the diurnal variation of the water level in the bay. Calculate the average speed of the tidal current in the channel.

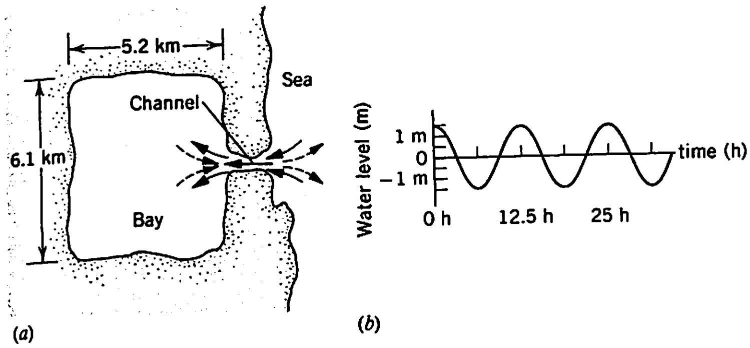

Figure 28 Problem 6.

Section 18-3 Bernoulli's Equation

7. How much work is done by pressure in forcing 1.4 m^3 of water through a 13-mm internal diameter pipe if the difference in pressure at the two ends of the pipe is 1.2 atm?

8. A water intake at a storage reservoir (see Fig. 29) has a cross-sectional area of 7.60 ft^2. The water flows in at a speed of 1.33 ft/s. At the generator building 572 ft below the intake point, the water flows out at 31.0 ft/s. (*a*) Find the difference in pressure, in lb/in.^{2}, between inlet and outlet. (*b*) Find the area of the outlet pipe. The weight density of water is 62.4 lb/ft^3.

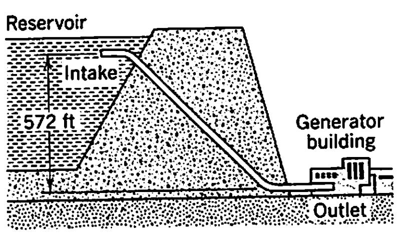

Figure 29 Problem 8.

9. Models of torpedoes are sometimes tested in a horizontal pipe of flowing water, much as a wind tunnel is used to test model airplanes. Consider a circular pipe of internal diameter 25.5 cm and a torpedo model, aligned along the axis of the pipe, with a diameter of 4.80 cm. The torpedo is to be tested with water flowing past it at 2.76 m/s. (*a*) With what speed must the water flow in the unconstricted part of the pipe? (*b*) Find the pressure difference between the constricted and unconstricted parts of the pipe.

10. Water is moving with a speed of 5.18 m/s through a pipe with a cross-sectional area of 4.20 cm^2. The water gradually descends 9.66 m as the pipe increases in area to 7.60 cm^2. (*a*) What is the speed of flow at the lower level? (*b*) The pressure at the upper level is 152 kPa; find the pressure at the lower level.

11. Suppose that two tanks, 1 and 2, each with a large opening at the top, contain different liquids. A small hole is made in the side of each tank at the same depth h below the liquid surface, but the hole in tank 1 has half the cross-sectional area of the hole in tank 2. (*a*) What is the ratio ρ_1/ρ_2 of the densities of the fluids if it is observed that the mass flux is the same for the two holes? (*b*) What is the ratio of the flow rates (volume flux) from the two tanks? (*c*) It is desired to equalize the two flow rates by adding or draining fluid in tank 2. What should be the new height of the fluid above the hole in tank 2 to make the flow rate in tank 2 equal to that of tank 1?

12. In a hurricane, the air (density 1.2 kg/m^3) is blowing over the roof of a house at a speed of 110 km/h. (*a*) What is the pressure difference between inside and outside that tends to lift the roof? (*b*) What would be the lifting force on a roof of area 93 m^2?

13. The windows in an office building are 4.26 m by 5.26 m. On a stormy day, air is blowing at 28.0 m/s past a window on the 53rd floor. Calculate the net force on the window. The density of the air is 1.23 kg/m^3.

14. A liquid flows through a horizontal pipe whose inner radius is 2.52 cm. The pipe bends upward through a height of 11.5 m where it widens and joins another horizontal pipe of inner radius 6.14 cm. What must the volume flux be if the pressure in the two horizontal pipes is the same?

15. Figure 30 shows liquid discharging from an orifice in a large tank at a distance h below the liquid surface. The tank is open at the top. (*a*) Apply Bernoulli's equation to a streamline connecting points 1, 2, and 3, and show that the speed of efflux is

$$v = \sqrt{2gh}.$$

This is known as *Torricelli's law.* (*b*) If the orifice were curved directly upward, how high would the liquid stream rise? (*c*) How would viscosity or turbulence affect the analysis?

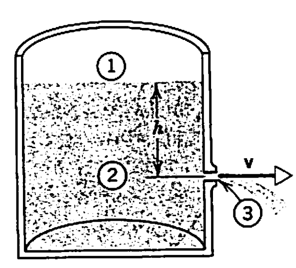

Figure 30 Problem 15.

16. A tank is filled with water to a height H. A hole is punched in one of the walls at a depth h below the water surface (Fig. 31). (*a*) Show that the distance x from the foot of the wall at which the stream strikes the floor is given by $x = 2\sqrt{h(H-h)}$. (*b*) Could a hole be punched at another depth so that this second stream would have the same range? If so, at what depth? (*c*) At what depth should the hole be

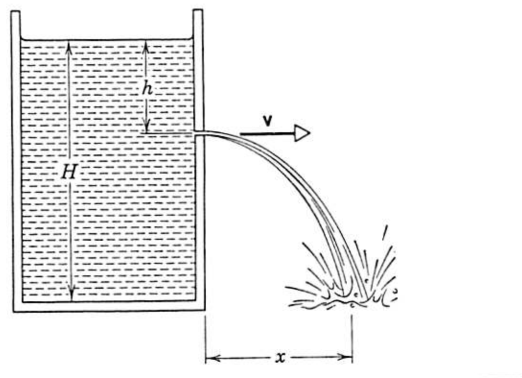

Figure 31 Problem 16.

placed to make the emerging stream strike the ground at the maximum distance from the base of the tank? What is this maximum distance?

17. A sniper fires a rifle bullet into a gasoline tank, making a hole 53.0 m below the surface of the gasoline. The tank was sealed and is under 3.10-atm absolute pressure, as shown in Fig. 32. The stored gasoline has a density of 660 kg/m³. At what speed does the gasoline begin to shoot out of the hole?

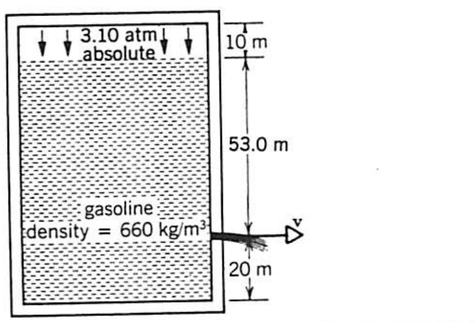

Figure 32 Problem 17.

18. Consider a uniform U-tube with a diaphragm at the bottom and filled with a liquid to different heights in each arm (see Fig. 33). Now imagine that the diaphragm is punctured so that the liquid flows from left to right. (*a*) Show that the application of Bernoulli's equation to points 1 and 3 leads to a contradiction. (*b*) Explain why Bernoulli's equation is not applicable here. (*Hint:* Is the flow steady?)

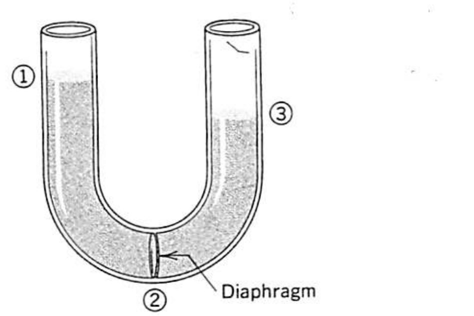

Figure 33 Problem 18.

19. If a person blows air with a speed of 15.0 m/s across the top of one side of a U-tube containing water, what will be the difference between the water levels on the two sides? Assume the density of air is 1.20 kg/m³.

20. The fresh water behind a reservoir dam is 15.2 m deep. A horizontal pipe 4.30 cm in diameter passes through the dam 6.15 m below the water surface, as shown in Fig. 34. A plug secures the pipe opening. (*a*) Find the frictional force between plug and pipe wall. (*b*) The plug is removed. What volume of water flows out of the pipe in 3.00 h?

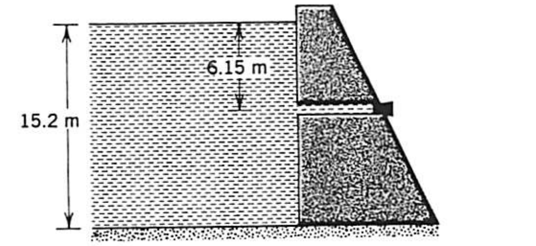

Figure 34 Problem 20.

21. A siphon is a device for removing liquid from a container that is not to be tipped. It operates as shown in Fig. 35. The tube must initially be filled, but once this has been done the liquid will flow until its level drops below the tube opening at A. The liquid has density ρ and negligible viscosity. (*a*) With what speed does the liquid emerge from the tube at C? (*b*) What is the pressure in the liquid at the topmost point B? (*c*) What is the greatest possible height h_1 that a siphon may lift water?

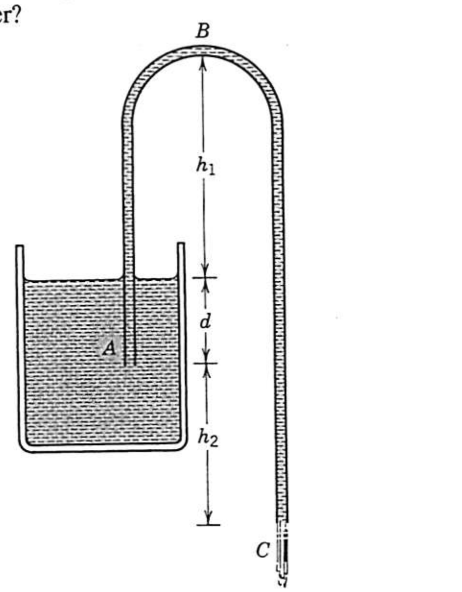

Figure 35 Problem 21.

22. (*a*) Consider a stream of fluid of density ρ with speed v_1 passing *abruptly* from a cylindrical pipe of cross-sectional area a_1 into a wider cylindrical pipe of cross-sectional area a_2 (see Fig. 36). The jet will mix with the surrounding fluid and, after the mixing, will flow on almost uniformly with an average speed v_2. Without referring to the details of the mixing, use momentum ideas to show that the increase in pressure due to the mixing is approximately

$$p_2 - p_1 = \rho v_2(v_1 - v_2).$$

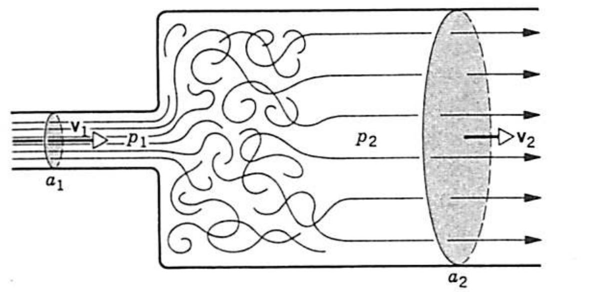

Figure 36 Problem 22.

(*b*) Show from Bernoulli's equation that in a *gradually* widening pipe we would get

$$p_2 - p_1 = \tfrac{1}{2}\rho(v_1^2 - v_2^2).$$

(*c*) Find the loss of pressure due to the abrupt enlargement of the pipe. Can you draw an analogy with elastic and inelastic collisions in particle mechanics?

23. A jug contains 15 glasses of orange juice. When you open the tap at the bottom it takes 12.0 s to fill a glass with juice. If you leave the tap open, how long will it take to fill the remaining 14 glasses and thus empty the jug?

Section 18-4 Applications of Bernoulli's Equation and the Equation of Continuity

24. A Pitot tube is mounted on an airplane wing to determine the speed of the plane relative to the air, which has a density of 1.03 kg/m³. The tube contains alcohol and indicates a level difference of 26.2 cm. What is the plane's speed relative to the air? The density of alcohol is 810 kg/m³.

25. A hollow tube has a disk *DD* attached to its end (Fig. 37). When air of density ρ is blown through the tube, the disk attracts the card *CC*. Let the area of the card be A and let v be the average air speed between the card and the disk. Calculate the resultant upward force on *CC*. Neglect the card's weight; assume that $v_0 \ll v$, where v_0 is the air speed in the hollow tube.

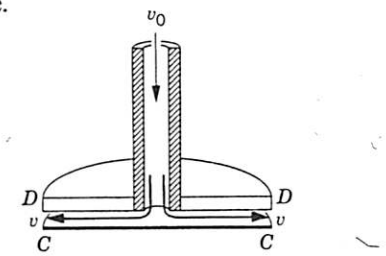

Figure 37 Problem 25.

26. A square plate with edge length 9.10 cm and mass 488 g is hinged along one side. If air is blown over the upper surface only, what speed must the air have to hold the plate horizontal? The air has density 1.21 kg/m³.

27. Air flows over the top of an airplane wing, area A, with speed v_t and past the underside of the wing with speed v_u. Show that Bernoulli's equation predicts that the upward lift force L on the wing will be

$$L = \tfrac{1}{2}\rho A(v_t^2 - v_u^2),$$

where ρ is the density of the air. (*Hint:* Apply Bernoulli's equation to a streamline passing just over the upper wing surface and to a streamline passing just beneath the lower wing surface. Can you justify setting the constants for the two streamlines equal?)

28. An airplane has a wing area (each wing) of 12.5 m². At a certain air speed, air flows over the upper wing surface at 49.8 m/s and over the lower wing surface at 38.2 m/s. (*a*) Find the mass of the plane. Assume that the plane travels with constant velocity and that lift effects associated with the fuselage and tail assembly are small. Discuss the lift if the airplane, flying at the same air speed, is (*b*) in level flight, (*c*) climbing at 15°, and (*d*) descending at 15°. The air density is 1.17 kg/m³. See Problem 27.

29. Consider the stagnant air at the front edge of a wing and the air rushing over the wing surface at a speed v. Assume pressure at the leading edge to be approximately atmospheric and find the greatest value possible for v in streamline flow; assume air is incompressible and use Bernoulli's equation. Take the density of air to be 1.2 kg/m³. How does this compare with the speed of sound under these conditions (340 m/s)? Can you explain the difference? Why should there be any connection between these quantities?

30. A Venturi tube has a pipe diameter of 25.4 cm and a throat diameter of 11.3 cm. The water pressure in the pipe is 57.1 kPa and in the throat is 32.6 kPa. Calculate the volume flux of water through the tube.

31. Consider the Venturi meter of Fig. 9. By applying Bernoulli's equation to points 1 and 2, and the equation of continuity (Eq. 3), verify Eq. 11 for the speed of flow at point 1.

32. Consider the Venturi meter of Fig. 9, containing water, without the manometer. Let $A = 4.75a$. Suppose that the pressure at point 1 is 2.12 atm. (*a*) Compute the values of v at point 1 and v' at point 2 that would make the pressure p' at point 2 equal to zero. (*b*) Compute the corresponding volume flow rate if the diameter at point 1 is 5.20 cm. The phenomenon at point 2 when p' falls to nearly zero is known as *cavitation*. The water vaporizes into small bubbles.

Section 18-5 Fields of Flow

33. Show that the constant in Bernoulli's equation is the same for *all* streamlines in the case of the steady, irrotational flow of Fig. 14.

34. A force field is conservative if $\oint \mathbf{F} \cdot d\mathbf{s} = 0$. The circle on the integration sign means that the integration is to be taken along a closed curve (a round trip) in the field. A flow is a potential flow (hence irrotational) if $\oint \mathbf{v} \cdot d\mathbf{s} = 0$ for every closed path in the field. Using this criterion, show that the fields of (*a*) Fig. 14 and (*b*) Fig. 17 are fields of potential flow.

35. In flows that are sharply curved, centrifugal effects are appreciable. Consider an element of fluid that is moving with speed v along a streamline of a curved flow in a horizontal plane (Fig. 38). (*a*) Show that $dp/dr = \rho v^2/r$, so that the pressure increases by an amount $\rho v^2/r$ per unit distance perpendicular to the streamline as we go from the concave to the convex side of the streamline. (*b*) Then use Bernoulli's equation and this result to show that vr equals a constant, so that speeds increase toward the center of curvature. Hence streamlines that are uniformly spaced in a straight pipe will

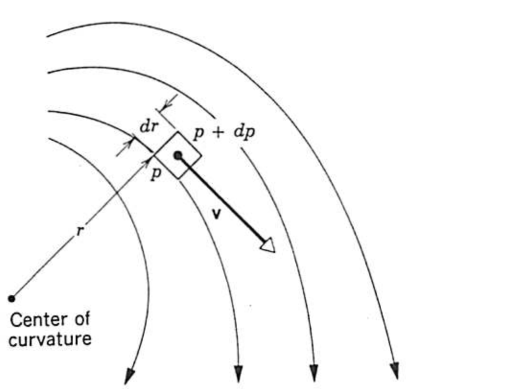

Figure 38 Problem 35.

be crowded toward the inner wall of a curved passage and widely spaced toward the outer wall. This problem should be compared to Problem 29 of Chapter 17 in which the curved motion is produced by rotating a container. There the speed varied directly with r, but here it varies inversely. (*c*) Show that this flow is irrotational.

36. Before Newton proposed his theory of gravitation, a model of planetary motion proposed by René Descartes was widely accepted. In Descartes' model the planets were caught in and dragged along by a whirlpool of ether particles centered around the Sun. Newton showed that this vortex scheme contradicted observations because: (*a*) the speed of an ether particle in the vortex varies inversely as its distance from the Sun; (*b*) the period of revolution of such a particle varies directly as the square of its distance from the Sun; and (*c*) this result contradicts Kepler's third law. Prove (*a*), (*b*), and (*c*).

Section 18-6 Viscosity, Turbulence, and Chaotic Flow

37. Figure 39 shows a cross section of the upper layers of the Earth. The surface of the Earth is broken into several rigid blocks, called plates, that slide (slowly!) over a "slushy" lower layer called the asthenosphere. See the figure for typical dimensions. Suppose that the speed of the rigid plate shown is $v_0 = 48$ mm/y, and that the base of the asthenosphere does not move. Calculate the shear stress on the base of the plate. The viscosity of the asthenosphere material is 4.0×10^{19} Pa·s. Ignore the curvature of the Earth.

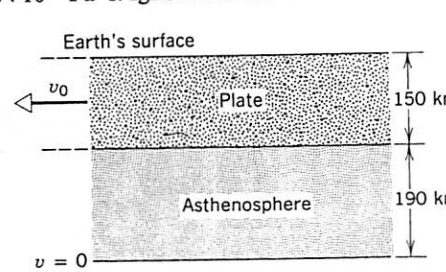

Figure 39 Problem 37.

38. Calculate the greatest speed at which blood, at 37°C, can flow through an artery of diameter 3.8 mm if the flow is to remain laminar.

39. Liquid mercury (viscosity $= 1.55 \times 10^{-3}$ N·s/m^2) flows through a horizontal pipe of internal radius 1.88 cm and length 1.26 m. The volume flux is 5.35×10^{-2} L/min. (*a*) Show that the flow is laminar. (*b*) Calculate the difference in pressure between the two ends of the pipe.

40. The streamlines of the Poiseuille field of flow are shown in Fig. 40. The spacing of the streamlines indicates that although the motion is rectilinear, there is a velocity gradient in the transverse direction. Show that the Poiseuille flow is rotational.

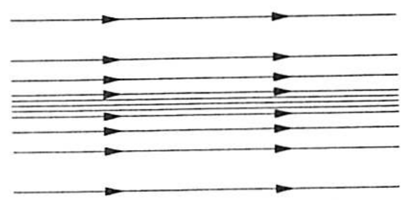

Figure 40 Problem 40.

41. A fluid of viscosity η flows steadily through a horizontal cylindrical pipe of radius R and length L, as shown in Fig. 41. (*a*) Consider an arbitrary cylinder of fluid of radius r. Show that the viscous force F due to the neighboring layer is $F = -\eta(2\pi rL)dv/dr$. (*b*) Show that the force F' pushing that cylinder of fluid through the pipe is $F' = (\pi r^2)\Delta p$. (*c*) Use the equilibrium condition to obtain an expression for dv in terms of dr. Integrate the expression to obtain Eq. 18.

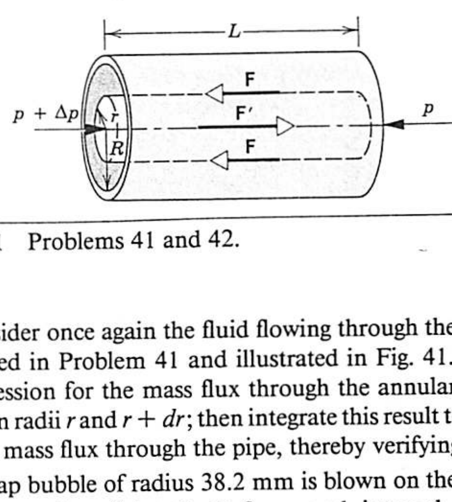

Figure 41 Problems 41 and 42.

42. Consider once again the fluid flowing through the pipe described in Problem 41 and illustrated in Fig. 41. Find an expression for the mass flux through the annular ring between radii r and $r + dr$; then integrate this result to find the total mass flux through the pipe, thereby verifying Eq. 20.

43. A soap bubble of radius 38.2 mm is blown on the end of a narrow tube of length 11.2 cm and internal diameter 1.08 mm. The other end of the tube is exposed to the atmosphere. Find the time taken for the bubble radius to fall to 21.6 mm. Assume Poiseuille flow in the tube. (For the surface tension of the soap solution use 2.50×10^{-2} N/m; the viscosity of air is 1.80×10^{-5} N·s/m^2.)