Phys 21 HW 1 Solutions

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Note: see last page for diagrams

I. KK 6.1

a) The total linear momentum vanishes: $\sum_i \vec{p}_i = 0$. The angular momentum about a particular origin is $L = \sum_i \vec{r}_i \times \vec{p}_i$. The angular momentum about a different origin is $\tilde{L} = \sum_i \vec{\tilde{r}}_i \times \vec{p}_i$. Notice that the radial coordinate changes, but the linear momentum is unaffected. The new and old radial coordinates are related via $\vec{\tilde{r}} = \vec{r} + \vec{a}$, for some constant vector $\vec{a}$. So then

$$\tilde{L} = \sum_i \vec{\tilde{r}}_i \times \vec{p}_i = \sum_i (\vec{r}_i + \vec{a}) \times \vec{p}_i = \vec{L} + \vec{a} \times \sum_i \vec{p}_i = \vec{L}.$$

b) The torque on the system about a particular origin is $\vec{\tau} = \sum_i \vec{r}_i \times \vec{F}_i$. This is related to the torque about a different origin via

$$\vec{\tilde{\tau}} = \sum_i \vec{\tilde{r}}_i \times \vec{F}_i = \sum_i (\vec{r}_i + \vec{a}) \times \vec{F}_i = \vec{\tau} + \vec{a} \times \sum_i \vec{F}_i = \vec{\tau}.$$

II. KK 6.2

The key concept is that the total angular momentum is constant. Using that the moment of inertia of a drum is $I = MR^2$, and that the initial angular frequency of the B drum is zero, we have

$$L_i = I_A a^2 \omega_A(0) = (M_A + M_s) a^2 \omega_A(0),$$
$$L_f = I_A a^2 \omega_A(t) + I_B b^2 \omega_B(t) = (M_A + M_s - \lambda t) a^2 \omega_A(t) + (M_B + \lambda t) b^2 \omega_B(t).$$

Notice that the mass of the drums is changing in time, and that the change in B is equal to minus the change in A. Next, we must note that $\omega_A(t) = \omega_A(0)$–the A drum is not changing it's rotational velocity. This is because the sand particles are not exerting any torque on the A drum. Solving for $L_i = L_f$ yields

$$\omega_B(t) = \left(\frac{\lambda t}{M_B + \lambda t}\right) \frac{a^2}{b^2} \omega_A(0)$$

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III. KK 6.3

The key concept here is that the total angular momentum $L_{\text{bug}} + L_{\text{ring}}$ vanishes.

a) bug opposite pivot

The ring is rotating about pivot with angular velocity ω. The bug is moving *relative to the ring* with velocity v. So the bug is moving about the pivot with velocity

$$v - (\text{distance from bug to pivot})\omega = v - 2R\omega.$$

The angular momentum of the bug is then

$$L_{\text{bug}} = m(\text{velocity of bug about pivot})(\text{distance from bug to pivot}) = m(v - 2R\omega)2R.$$

The moment of inertia of the ring about an axis through it's center is $I_0 = MR^2$, so the moment of inertia through the pivot can be found using the parallel axis theorem: $I = I_0 + MR^2 = 2MR^2$. The angular momentum of the ring is then $L_{\text{ring}} = -I\omega = -2MR^2\omega$. The minus sign comes about because it's rotating in the opposite direction of the bug. So then we have

$$L_{\text{bug}} + L_{\text{ring}} = 0 \qquad \Rightarrow \qquad \omega = \frac{mv}{(M + 2m)R}$$

b) bug at pivot

Since the bug is at the pivot, it's angular momentum is zero. Since angular momentum is conserved, the angular momentum of the ring is also zero. So $L_{\text{bug}} = L_{\text{ring}} = \omega = 0$.

IV. KK 6.4

The package is falling through a gravitational field, and therefore feels at all times a force directed to the planet's center. Consider the angluar momentum of the packgage with respect to the center of the planet. Because the gravitational force is always acting radially, it exerts no torque; therefore this angular momentum is conserved. Also energy is conserved. With these two conserved quantities we can find the critical angle at which the package will graze the planet.

Initially, the radius of the package is $r = 5R$. When the package grazes the planet, $r = R$. The initial and final energies are:

$$E_i = \frac{1}{2}mv_0^2 - \frac{GMm}{5R} = E_f = \frac{1}{2}mv^2 - \frac{GMm}{R}.$$

The initial and final angular momenta are:

$$L_i = mv_0 \sin\theta(5R) = L_f = mvR.$$

We can now solve for θ: from the angular momentum equation we have $\frac{1}{2}mv_0^2 - \frac{GMm}{5R}$, and then using this in the energy equation we have

$$\frac{1}{2}mv_0^2 - \frac{GMm}{5R} = \frac{25}{2}mv_0^2\sin^2\theta - \frac{5GMm}{5R},$$
$$\theta = \frac{1}{5}\sqrt{1 + \frac{8}{5}\frac{GM}{Rv_0^2}}.$$

V. KK 6.5

Consider first the center of mass of the car. The gravitational force acting downwards is mg, which can be decomposed into $mg\cos\theta$ acting normal to the ramp, and $mg\sin\theta$ acting tangentially down the ramp. Therefore, the normal force is $N = mg\cos\theta$. Similarly, the frictional force must be $f = mg\sin\theta$ in order to prevent the car from sliding down the ramp. Additionally, the only contact the car has with the ramp are the two sets of wheels, so $N = N_1 + N_2$, and $f = f_1 + f_2$.

Both the normal forces and the frictions will exert a torque on the car. But the car should be stationary, so these must sum to zero. If l_1 is the height of the center of mass above the ramp, and l_2 is the distance between the axles and the center of mass (see diagram), then

$$\begin{aligned} 0 &= (N_1 - N_2)l_2 + (f_1 + f_2)l_1 \\ &= (N_1 - N_2)l_2 + (mg\sin\theta)l_1. \end{aligned}$$

Solving for N_1, N_2 yields

$$N_{1,2} = \frac{Mg}{2}\Big(\cos\theta \mp \frac{l_1}{l_2}\sin\theta\Big).$$

For the given numerical values, $N_1 = 924$lbs, $N_2 = 1674$ lbs.

VI. KK 6.6

In a similar vein as the previous problem, before considering the individual feet, consider the forces acting on the center of mass. The gravitational force is acting downwards, and the man feels a centripetal force pushing him radially outwards.

Since the man is not accelerating vertically, $N = mg$. Since he's not radially accelerating, $f = mv^2/R$. And since his only point of contact with the railroad car are his feet, $N = N_1 + N_2$ and $f = f_1 + f_2$.

Both of these forces will produce torques about his center of mass. Since his angular velocity is constant, these torques must cancel:

$$0 = (N_o - N_i)\frac{d}{2} = (f_o + f_i)L.$$

Solving these equations yields

$$N_{i,o} = \frac{1}{2}\Big(Mg \mp \frac{Mv^2L}{R(d/2)}\Big).$$

VII. KK 6.13

a) Since the force is acting parallel to the axis, it produces no torque, so angular momentum is conserved but energy is not. Therefore

$$L = mv_ir_i = mv_fr_f \qquad \Rightarrow \qquad v_f = v_i\frac{r_i}{r_f}.$$

b) The force is no longer central, so angular momentum is not conserved. But energy is conserved because the work is zero, $W = \int \vec{T} \cdot d\vec{r} = \int (\vec{T} \cdot \vec{v})dt = 0$ (since T and v are perpendicular). So

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 \qquad \Rightarrow \qquad v_f = v_i.$$

VIII. KK 6.14

a) $\tau = (Mg)(l/2)$

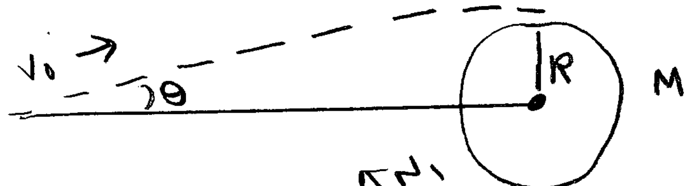
b) $\tau = I\alpha, \qquad \alpha = \frac{\tau}{I} = 3g/2l$

c) $a = \alpha(l/2) = 3g/4.$

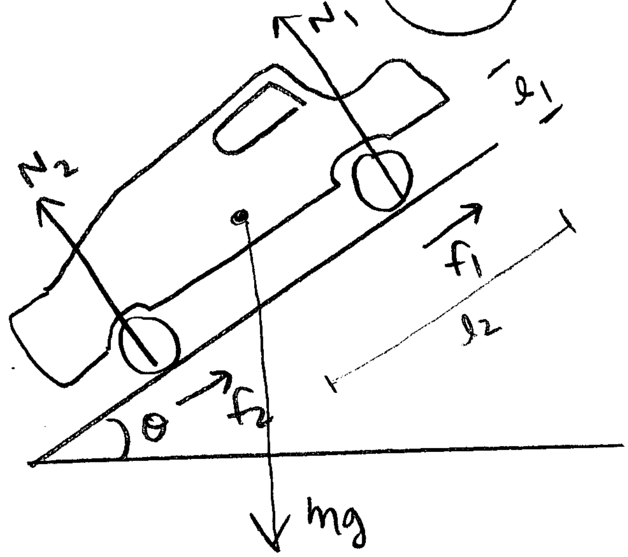
d) $Ma = Mg - F_{\text{vert}}, \qquad F_{\text{vert}} = Mg/4.$

Diagrams

6.4



6.5



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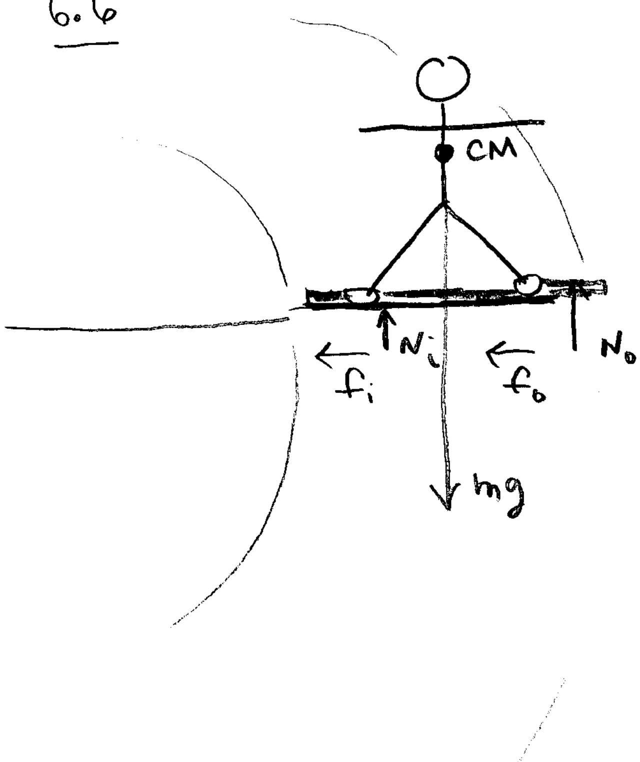

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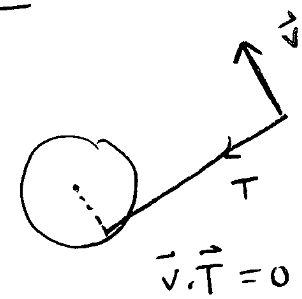

6.14