PHYS 21: Assignment 4 Solutions

Due on Feb 4 2013

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H + R 12.43

We see immediately that there is friction in this problem, so we *can't* use conservation of energy. We know that for the hoop:

$$v_h = \omega R \text{Rolling condition}$$

And for the box:

$$a_b = g\cos\theta - \mu_k g\sin\theta$$

The fact that the hoop keeps pace with the box means that

$$v_h = v_b$$

Which automatically gives us

$$a_h = a_b$$

We see that:

$$\begin{aligned} a_h &= \frac{d}{dt}(\omega R) \\ &= \dot{\omega} R \\ &= \frac{\tau}{I_h} R \\ &= \frac{\tau}{m_h R^2} R \\ &= \frac{\tau}{m_h R} \end{aligned}$$

We realise that gravity exerts *no torque* because it is exerted at the centre of mass of the hoop. The only force which exerts a torque is friction.

$$\begin{aligned} \tau &= fR \\ &= \mu_k m_h g \sin\theta R \\ a_h &= \frac{\mu_k m_h g \sin\theta R}{m_h R} \\ &= \mu_k g \sin\theta \\ \mu_k g \sin\theta &= g\cos\theta - \mu_k g \sin\theta \\ 2\mu_k \sin\theta &= \cos\theta \\ \mu_k &= \frac{1}{2}\tan\theta \end{aligned}$$

H + R 12.49

We see that there is no friction in this problem, so we can use conservation of energy. Let's call the speed of the ball as it exits the track v_f:

$$\begin{aligned}
PE &= mgh \\
KE &= \frac{1}{2}mv^2 + \frac{I}{2}\omega^2 \\
E &= PE + KE \\
\frac{dE}{dt} &= 0 \\
mgH &= mgh + \frac{1}{2}mv_f^2 + \frac{mR^2}{5}\omega_f^2
\end{aligned}$$

From the rolling condition, we see that

$$\omega_f^2 = \frac{v_f^2}{R^2}$$

So:

$$\begin{aligned}
mgH &= mgh + \frac{1}{2}mv_f^2 + \frac{m}{5}v_f^2 \\
g(H-h) &= \frac{7}{10}v_f^2 \\
v_f &= \sqrt{\frac{10g(H-h)}{7}}
\end{aligned}$$

Then, using our kinematics knowledge, we see that:

$$\begin{aligned}
h &= \frac{gt^2}{2} \\
t &= \sqrt{\frac{2h}{g}} \\
x &= v_f t \\
&= \sqrt{\frac{20(H-h)h}{7}} \\
&= \sqrt{\frac{20(60-20)(20)}{7}} \\
&= 20m\sqrt{\frac{40}{7}} \\
&= 47.8m
\end{aligned}$$

H + R 12.50

Again there is no friction, so we can use conservation of energy. In order for the marble to just stay on the track, the normal force exerted on it by the track at the top of the loop should be exactly 0. Since the marble is on a circular section of track, this is the same as saying that gravity provides all the centripetal acceleration. This condition can be written:

$$g = \frac{v_{top}^2}{R}$$

So, from conservation of energy:

$$\begin{aligned}
mgh =& mgR + \frac{1}{2}mv_{top}^2 + \frac{I}{2}\omega^2 \\
=& mgR + \frac{1}{2}mgR + \frac{2mR^2}{2\cdot 5}\frac{v_{top}^2}{R^2} \\
=& \frac{3}{2}mgR + \frac{1}{5}mgR \\
=& \frac{17}{10}mgR \\
h =& \frac{17}{10}R
\end{aligned}$$

H + R 12.51

To solve this, we can just write down the torques and use Newton's third law. We know that:

$$\begin{aligned}
ma =& W - T \\
\tau =& TR \\
=& I\alpha \\
=& \frac{mR^2}{2}\frac{a}{R} \\
TR =& \frac{mRa}{2} \\
=& \frac{RW}{2} - \frac{RT}{2} \\
\frac{3}{2}TR =& \frac{RW}{2} \\
T =& \frac{W}{3}
\end{aligned}$$

To find the angular acceleration α:

$$\begin{aligned}
\alpha =& \frac{TR}{I} \\
=& \frac{2TR}{mR^2} \\
=& \frac{2W}{3mR} \\
=& \frac{2g}{3R}
\end{aligned}$$

H + R 12.52

We use Newton's third law again:

$$
\begin{aligned}
ma =& mg\sin\theta - T \text{ where } T \text{ is the tension} \\
TR =& I\alpha \\
I =& \frac{m}{2}R^2 \\
T =& \frac{m}{2}R\alpha \\
=& \frac{m}{2}R\frac{a}{R} \\
=& \frac{m}{2}a \\
ma =& mg\sin\theta - \frac{1}{2}ma \\
\frac{3}{2}ma =& mg\sin\theta \\
a =& \frac{2}{3}g\sin\theta
\end{aligned}
$$

So the tape unwinds with constant acceleration. Now we can use a kinematics equation:

$$
\begin{aligned}
\ell =& v_0 t + \frac{1}{2}at^2 \\
L =& \frac{1}{2}aT^2 \text{ where } T \text{ is the unwinding time} \\
T^2 =& \frac{2L}{a} \\
T =& \sqrt{\frac{2L \cdot 3}{2g\sin\theta}} \\
=& \sqrt{\frac{3L}{g\sin\theta}}
\end{aligned}
$$

H + R 12.57

We can use energy methods again:

$$
\begin{aligned}
mgh =& \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \\
mgh =& \frac{2}{5}\frac{1}{2}mR^2\frac{v^2}{R^2} + \frac{1}{2}mv^2 \\
=& \frac{7}{10}mv^2 \\
v =& \sqrt{\frac{10}{7}gh} \\
x =& v\cos\theta t \\
y =& -v\sin\theta t - \frac{1}{2}gt^2 \\
y =& -x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta} \\
y =& -x\tan\theta - \frac{7x^2}{20h\cos^2\theta}
\end{aligned}
$$

Plugging in the numbers the question gives us, we find:

$$x = 5.34 \text{ m}$$

H + R 12.59

Let's call the velocity of the centre of mass v_c, and the velocity of the far end (the one her hand isn't on) v_f. We see that she throws the stick with ω_0. Let's call the angular velocity about the centre of mass ω.

$$\begin{aligned}
v_c &= \frac{\omega_0 L}{2} \\
v_f &= \omega_0 L = \frac{\omega L}{2} \\
\Rightarrow \omega &= 2\omega_0 \\
2\pi N &= \theta(T) \\
&= \omega t \\
v_c &= gt \\
t &= \frac{v_c}{g} \\
2\pi N &= \omega \frac{v_c}{g} \\
2g\pi N &= 2\omega_0 \frac{\omega_0 L}{2} \\
2g\pi N &= \omega_0^2 L \\
\omega_0 &= \sqrt{\frac{2g\pi N}{L}} \\
2ay &= (v_c^f)^2 - (v_c^i)^2 \\
2gh &= v_c^2 - 0 \text{ look at the falling behavior} \\
h &= \frac{v_c^2}{2g} \\
&= \frac{\omega_0^2 L^2}{4} \frac{1}{2g} \\
&= \frac{2g\pi N L^2}{8Lg} \\
&= \frac{\pi N L}{4}
\end{aligned}$$

H + R 12.60

We realize that the ball stops slipping when the rolling condition is met, when

$$\omega R = v$$

We see that the only torque on the ball comes from friction:

$$\begin{aligned}
\tau =& I\alpha \\
fR =& \frac{2mR^2}{5}\alpha \\
\alpha =& \frac{5\mu mgR}{2mR^2} \\
=& \frac{5\mu g}{2R} \\
\omega =& \alpha t \\
=& \frac{5\mu g}{2R}
\end{aligned}$$

And the ball is decelerating linearly due to friction:

$$\begin{aligned}
v =& v_0 - \mu mgt \\
\omega R =& v_0 - \mu gT \\
\frac{5\mu gT}{2} =& v_0 - \mu gT \\
\frac{7\mu gT}{2} =& v_0 \\
T =& \frac{2v_0}{7\mu g}
\end{aligned}$$

We can check our units - $ms^{-1} \cdot m^{-1}s^2 = s$ so our units make sense.

K + K 6.35

As the block rocks sideways without slipping, the point of contact between it and the sphere is $R\theta$. As long as the centre of mass is "inside" the $R\theta$ "envelope" (closer to the vertical than $R\theta$) then the block won't fall off. If the centre of mass obeys this condition, a restoring torque exists. If the centre of mass doesn't obey the condition, a restorting torque is impossible. Assuming that $R \gg L$, the block will be unstable if:

$$\begin{aligned}
R\theta <& \frac{L}{2}\tan\theta \\
<& \frac{L}{2}\theta \text{ for small angles} \\
2R <& L
\end{aligned}$$