Phys 21 HW 7 Solutions

Gavin Hartnett*
Physics Department, University of California, Santa Barbara
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HRK Chapter 17 Questions: $3, 7, 8, 17, 19, 22, 34, 37$

HRK Chapter 17 Problems: $3, 20, 27, 28, 45, 46$

I. HRK Q17.3

In this problem we are comparing apples and oranges. The pressure that causes pain above water is of an entirely different nature than the ambient pressure below the water. The water pressure is just the ambient pressure, while the pain-causing pressure is due to sound waves. As can be seen from the equations section 20-3, a sound wave of the same frequency under water would actually result in a much greater pain than in air (since both the speed of sound and the density are greater underwater).

II. HRK Q17.7

The paradox is resolved when one notes that we haven't accounted for all the forces in the vertical direction. The walls of the containers feel a force/area due to the pressure, and in the first two containers, this force has a verticle component.

III. HRK Q17.8

In a sense, the Archimedes principle holds even in free fall. There is no buoyancy in free fall, but this is consistent with Archimedes principle: The acceleration due to gravity is zero, so the weight of the displaced fluid is zero, and so the buoyant force is zero.

And in this same trivial sense it holds for cicular orbit, that is, there is no bouyancy force because there is no weight. If a rubber ducky is sitting in a bathtub orbiting the Earth, the rubber ducky is weightless and does not press down on the water at all.

* hartnett@physics.ucsb.edu

IV. HRK Q17.17

According to Archimedes principle, the bouyant force is the weight of the volume of water displaced by the ship. If we start with a hollow ship, and fill it with sea water, then it's weight increases, so the ship sinks a bit to displace a greater volume of water. If we keep adding water, eventually the ship sinks more and more until it is completely submerged because it's weight is exactly equal to a volume of water whose volume is that of the ship (interior+hull). Adding any more water will sink the ship, since the volume of displaed water, and hence bouyant force will not change. Here its important to remember that the hull is part of the ship, and that it's more dense than water. If it were less dense, then you'd never be able to sink the ship.

V. HRK Q17.19

Split the ice cube into a bottom and a top part. The bottom part is submerged and the top is not. The ice cube is not moving, so $\sum F_y = 0$, i.e. the bouyant force cancels the weight of the ice cube.

$$F_{\text{bouy}} = \rho_w V_b = \rho_i (V_b + V_t).$$

Then the ice cube melts. Melting changes volume but conserves mass, so that

$$\rho_i (V_t + V_b) = \rho_w V_w.$$

Now if $V_w > V_b$, there will be overflow of water, and if $V_w < V_b$, there will be room left in the container. If $V_w = V_b$, then the jar will still filled to the brim. Working it out,

$$V_w = \frac{\rho_i}{\rho_w}(V_t + V_b) = \frac{V_t + V_b}{V_b + V_t} V_b = V_b.$$

So the beaker is sill filled to the brim. Now let's add either sand or air bubbles in the ice. Which I'll model by saying that the density of ice is modified, $\rho_i \to \rho_i + \delta$. This enters into the first equation, but not the second since the sand just goes along for the ride. Then we have

$$V_w = \frac{\rho_i}{\rho_w}(V_t + V_b), \qquad \frac{\rho_w}{\rho_i + \delta} = \frac{V_b + V_t}{V_b}.$$

So then,

$$V_w = \frac{\rho_i}{\rho_w} \frac{\rho_w}{\rho_i + \delta} V_b = \frac{\rho_i}{\rho_i + \delta} V_b.$$

So if $\delta > 0$, i.e. the sand or whatever is more dense than ice, there is room left in the container, and if the stuff is less dense, then there is an overflow.

Addendum: The above answer is wrong for the air bubble case, I forgot to take into account the fact that once freed from the ice, the air bubbles will simply escape into the atmosphere. The density argument only works if the impurities remain in the container.

VI. HRK Q17.22

The density of air decreseas as you go up, and the density of water increases as you go down. So if a balloon is less heavy than it's displaced air, eventually it will rise to a height where it's weight and the displaced air are equal. The same argument might suggest that a submarine would sink until the water got so dense that the displaced water's weight balanced the submarine's weight. But remember, that the submarine is filled with *water*, whose weight is increasing along with the displaced water. So, as long as it doesn't sink to a crazy depth where the hull can be less dense than seawater, the bouyant force will never be able to balance the weight once it starts sinking.

VII. HRK Q17.34

Neither of these things is important. Looking at the derivation of the pressure gradient, neither the area nor the orientation of the tube were important. But keep in mind that you must measure the *height*, and not distance along the tube when it's not aligned vertically.

VIII. HRK Q17.37

When you try to separate the plates by diret pull, you have to work against the surface tension. When you slide them, you don't need to, until the plates begin to separate, at which point you are fighting the surface tension, but only a little bit at a time.

IX. HRK P17.3

Since pressure is force/area, so

$$\Delta F = (p_1 - p_2)A = (1 - 0.962)(3.43 \times 2.08)1.0135 \times 10^5 N = 2.75 \times 10^4 N.$$

X. HRK P17.20

We already know how the pressure and density fall-off as you go up in altitude. The same formulae apply for going down in water, simply change a sign in eq 17.13 to get

$$p(y) = p_0 e^{ya}, \qquad \rho(y) = \rho_0 e^{y/a}, \qquad \frac{1}{a} = \frac{g\rho_0}{p_0}.$$

Expanding the density near the surface ($y = 0$) yields

$$\rho(y) \approx \rho_0 \Big(1 + \frac{y}{a}\Big).$$

Now I want to write a in terms of the bulk modulus, B. To do that, use the definition

$$B = \rho \frac{dp}{d\rho}\bigg|_{y=0},$$

and the fact that $p = (p_0/\rho_0)\rho$. Then we see that $B = p_0$, and

$$\rho(y) \approx \Big(1 + \frac{g\rho_0}{B} y\Big).$$

XI. HRK P17.27

Pressure here acts like the normal force does in the problem of having a point particle on an inclined plane. So the force balance condition on an infinitesimal fluid element yields

$$ma = p \sin\theta, \qquad mg = p\cos\theta, \qquad \Rightarrow \tan\theta = \frac{a}{g}.$$

The pressure is given by $p = p_0 + \rho g h$, so the pressure is proportional to the height.

XII. HRK P17.28

For the unaccelerated case, the vertical force balance condition is

$$F_B - mg - T_0 = 0, \qquad F_B = T_0 + mg = M_w g,$$

where M_w is the mass of the displaced water. In the accelerated case, the bouyant force is going to change because the weight of the displaced water will change. We have

$$ma = F_B' - mg - T, \qquad F_B' = M_w(g + a) = M_w g(1 + a/g) = F_B(1 + a/g).$$

Solving for T then yields

$$T = T_0(1 + a/g).$$

XIII. HRK P17.45

For this problem I'll assume the density of air is zero. Originally we have

$$F_B = mg = \frac{1}{4}V\rho_{Hg}.$$

Then when we add water we have

$$F_B = mg = xV\rho_{Hg} + (1-x)V\rho_w.$$

The bouyant force doesn't change, and neither does the weight, but now the bouyant force is split between the displaced mercury and the displaced water. x is the fraction of the object underneath the mercury. Dividing these two equations yields

$$x = \frac{1-4y}{4(1-y)}, \qquad y = \frac{\rho_w}{\rho_{Hg}}$$

. Plugging y = 1/13.56 (I looked this up), I get

$$x = 0.19.$$

XIV. HRK P17.46

Let x be the vertical distance that the log sinks by, and let ρ be the density of the water. Then when $x = 0$, the force balance condition reads

$$\sum F_y = 0 = F_B^0 - mg \Rightarrow F_B^0 = mg.$$

Now sink the log by a distance x. Then we have

$$\sum F_y = ma = F_B - mg = F_B^0 + (xA\rho)g - mg = (xA\rho)g.$$

I want to solve for a, which means dividing my m. I want to write m in terms of the given variables of the problem, so use the fact that

$$F_B^0 = mg = (LA\rho)g \Rightarrow m = LA\rho.$$

Then we have that

$$a = \frac{xA\rho g}{LA\rho} = \frac{x}{L}g.$$

And this is just the equation of motion for a harmonic oscillator with period

$$T = (2\pi)\sqrt{\frac{L}{g}} = 3.2s.$$