PHYS 21: Assignment 8 Solutions

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HRK P18.6

We'll use the fact that volumetric flow rate is conserved.

$$v_{bay}A_{bay} = v_{channel}A_{channel}$$

We need to find the average speed of water in the bay. We see that the water level is described by

$$h(t) = 1m \cdot \cos \frac{2\pi}{12.5\text{h}} t$$

So:

$$|v|(t) = 1m \cdot \frac{2\pi}{12.5\text{h}} |\sin \frac{2\pi}{12.5h} t|$$

The average velocity is then $\frac{2}{\pi}\frac{2\pi}{12.5} = \frac{4}{12.5}\text{m/h} = 8.889 \times 10^{-5}\text{m/s}$. Now we use the volumetric flow condition:

$$8.889 \times 10^{-5} \cdot 5.2 \times 10^3 \cdot 6.1 \times 10^3 = v_{channel} \cdot 190 \cdot 6.5$$
$$v_{channel} = 2.28\text{m/s}$$

HRK P18.15

a:
We write Bernoulli's equation as:

$$\frac{v_1^2}{2} + gh + \frac{p}{\rho} = \frac{v_3^2}{2} + \frac{p}{\rho}$$

Because the density has not changed at points 1, 2, or 3. Neither has the pressure. We see that at point 1, $v_1 = 0$ (this is a bit of an idealization - obviously as the water drains out of the spout, the water level drops and so there is some downward velocity, but we claim that this is negligible for a big enough bucket).

$$gh = \frac{v_3^2}{2}$$
$$2gh = v_3^2$$
$$v_3 = \sqrt{2gh}$$

b:
Again we use Bernoulli's equation - at the apex of the liquid's arc, its velocity is zero. So:

$$gh' = \frac{2gh}{2}$$
$$\Rightarrow h' = h$$

This makes sense - energy is conserved!
c:
If we had some sort of viscosity or turbulence, we might not conserve energy as the fluid streamline moved through the spigot. In that case, we'd need to figure out how much kinetic energy we would have lost to the walls of the spigot and to the viscous effects in the fluid. In particular, Bernoulli's equation assumes inviscid flow so we could not use it.

HRK P18.21

a:
At points A and C, the liquid is under atmospheric pressure. Again we assume that the velocity at point A is negligible.

$$\frac{v_A^2}{2} + gh_2 + \frac{p_{atm}}{\rho} = \frac{v_C^2}{2} + g \cdot 0 + \frac{p_{atm}}{\rho}$$
$$gh_2 = \frac{v_C^2}{2}$$
$$v_C = \sqrt{2gh_2}$$

b:
We assume that the velocity must be the same throughout the siphon pipe - this makes sense to do because otherwise we'd form vacuum gaps. Then:

$$\frac{p_{atm}}{\rho} = \frac{v_B^2}{2} + g(h_1 + d) + \frac{p_B}{\rho}$$
$$\frac{p_{atm}}{\rho} = gh_2 + g(h_1 + d) + \frac{p_B}{\rho}$$
$$p_{atm} = g\rho(h_1 + h_2 + d) + p_B$$
$$p_B = p_{atm} - g\rho(h_1 + h_2 + d)$$

c:
In this case, $p_B = 0$ - we can't have a negative pressure in the pipe!

$$0 = p_{atm} - g\rho(h_1 + h_2 + d)$$
$$g\rho h_2 = p_{atm} - g\rho(h_1 + d)$$
$$h_2 = \frac{p_{atm}}{g\rho} - h_1 - d$$

HRK P18.22

Because there's an abrupt transition, there will probably be some sort of turbulent flow at the transtion (this is what the diagram is trying to tell us). However, as we can see from the diagram, eventually the flow regularizes and becomes laminar again.

a:

We're supposed to use "momentum arguments" - what this means is that considering the impulse on a cross section of fluid is an easy way to look at this problem. We know that:

$$v_1 a_1 = v_2 a_2$$

So:

$$\begin{aligned}
I =& \Delta P \\
=& \Delta F \Delta t \\
=& \Delta(pA) \cdot \Delta t \\
=& (p_2 a_2 - p_1 a_1) \cdot \Delta t \\
\Delta P =& \Delta m \Delta v \\
=& \rho \Delta(At)\Delta(v^2) \\
=& \rho \Delta t (a_1 v_1^2 - a_2 v_2^2) \\
(p_2 a_2 - p_1 a_1) \cdot \Delta t =& \rho \Delta t (a_1 v_1^2 - a_2 v_2^2) \\
(p_2 a_2 - p_1 a_1) =& \rho (a_1 v_1^2 - a_2 v_2^2) \\
(p_2 a_2 - p_1 a_1) =& \rho (a_2 v_2 v_1 - a_2 v_2^2) \\
(p_2 a_2 - p_1 a_1) =& \rho a_2 v_2 (v_1 - v_2)
\end{aligned}$$

At the interface, we can say that $p_2 a_2 - p_1 a_1 \approx p_2 a_2 - p_1 a_2$. Then:

$$\begin{aligned}
(p_2 a_2 - p_1 a_1) \approx& (p_2 a_2 - p_1 a_2) = \rho a_2 v_2 (v_1 - v_2) \\
(p_2 a_2 - p_1 a_2) =& \rho a_2 v_2 (v_1 - v_2) \\
p_2 - p_1 =& \rho v_2 (v_1 - v_2)
\end{aligned}$$

b:

Now we use Bernoulli's equation:

$$\begin{aligned}
\rho \frac{v_1^2}{2} + g\rho h + p_1 =& \rho \frac{v_2^2}{2} + g\rho h + p_2 \\
p_2 - p_1 =& \frac{\rho}{2}\left(v_1^2 - v_2^2\right)
\end{aligned}$$

c:

We can combine our results from the previous two parts - the system loses energy in the turbulent flow when the pipe widens, so the "constant" in Bernoulli's equation changes. The total change in pressure is the decrease we'd get from Bernoulli's equation, minus the increase we get from the momentum considerations in part (a). Then:

$$\begin{aligned}
p_2 - p_1 =& \frac{\rho}{2}\left(v_1^2 - v_2^2\right) - \rho v_2 (v_1 - v_2) \\
=& \frac{\rho}{2}\left(v_1^2 - v_2^2\right) - \rho v_2 v_1 + \rho v_2^2 \\
=& \frac{\rho}{2}\left(v_1^2 - 2 v_1 v_2 + v_2^2\right) \\
=& \frac{\rho}{2}\left(v_1 - v_2\right)^2
\end{aligned}$$

HRK P18.25

We want to figure out the difference in pressure on either side of the card. Obviously the pressure on the bottom is atmospheric pressure. We can use Bernoulli's equation (again!), realising that we don't need to know the height of the pipe because the airspeed is constant within the pipe:

$$\begin{aligned}
\frac{\rho v_0^2}{2} + g\rho h + p_{atm} =& \frac{\rho v^2}{2} + g\rho h + p_{new} \\
p_{atm} - p_{new} \approx& \frac{\rho v^2}{2} \\
\Delta p =& \frac{\rho v^2}{2} \\
F_{above} =& A p_{above} \\
F_{below} =& A p_{below} \\
F_{net} =& A\Delta p \\
=& \frac{A\rho v^2}{2}
\end{aligned}$$

HRK P18.38

We need to calculate the Reynolds number for blood. We can look up the viscosity of blood and find out that its

$$\mu = 4 \times 10^{-3} \mathrm{Pa} \cdot \mathrm{s}$$

So:

$$\begin{aligned}
\mathrm{Re} =& \frac{\rho v L}{\mu} \\
\rho =& 1060 \mathrm{kg/m}^3 \\
L =& 3.8 \times 10^{-3} \mathrm{m}
\end{aligned}$$

The point where the flow becomes nonlaminar is $\mathrm{Re} = 5 \times 10^5$. Plugging in numbers:

$$\begin{aligned}
5 \times 10^5 =& \frac{1060 \cdot 3.8 \times 10^{-3}}{4 \times 10^{-3}} v \\
=& 1007 v \\
v =& 496.5 m/s
\end{aligned}$$