HRK Chap 17

Fluid Statics

Division between fluids & solids is not always clear
→ e.g., glass is a fluid
Church windows are thicker on bottom than on top.

Plastic is also intermediate
→ e.g. ~~ø~~ clay.

Can change state by changing temperature or **pressure** → e.g., ~~so lava~~ rock layers
→ rock flows at high pressure
→ aluminium can be made into wires.

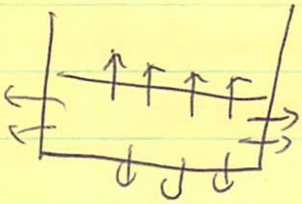
Plasmas : ionized gas w. very different behaviour fr. ordinary gas.
Fluorescent light.
Sun.

Solids
relatively incompressible → Liquids
Gases ← compressible.

↑ Stronger intermolecular forces (intermolecular forces become smaller)

We now develop force laws for a fluid.

Pressure



Fluid can flow - unable to sustain shear stress.
All forces between interior & exterior ate at right angles to fluid boundary.

Pressure is a scalar, no direction. $P = F/A$

Result of collisions of molecules with surface, imparts momentum ~~& vice v~~

$$\Delta\vec{F} = p\,\Delta\vec{A}$$

$$\Rightarrow p = \lim_{\Delta\vec{A}\to 0}\frac{\Delta\vec{F}}{\Delta\vec{A}}$$ ← so that independent of size of element

Pressure can vary fr. point to point on surface.
↓
But ∇P is a vector (direction of force exerted by pressure gradient).

~~Pa~~

1 Pa = 1 Nm^{-2}

1atm = 10^5 Pa.

normal Blood pressure = 1.6×10^4 Pa

Center of Earth ≃ 4×10^{11} Pa

" " Sun ≃ 2×10^{16} Pa.

Density $\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}$ a scalar.

[If density is uniform, then $\rho = \frac{m}{V}$]

Some examples ρ(interstellar space) $\simeq 10^{-20}$ kg m^{-3}

ρ(water) $\simeq 10^{3}$ kg m^{-3}

ρ(air, at 1 atm) $\simeq 1$ kg m^{-3}.

ρ(black hole) $\simeq 10^{19}$ kg m^{-3}.

Density can depend on pressure & T.
(varies little for solids + liquids, lots for gas]

$$B = -\frac{\overbrace{\Delta p}^{\text{stress}}}{\underbrace{\Delta V / V}_{\text{strain}}}$$

Bulk modulus [has same units as p, since $\frac{\Delta V}{V}$ is dimensionless].

makes B +ve (since Δp & ΔV have opposite sign).

$$\Delta p > 0 \longrightarrow \Delta V < 0$$

B is <u>large</u>.

Then large $\Delta p \Rightarrow$ small $\frac{\Delta V}{V}$

resists compression.

Water has $B \simeq 2.2 \times 10^{9}$ N m^{-2}

(nearly incompressible).

At bottom of Pacific ocean, $p = 400$ atm
$\simeq 10^{7}$ Pa

$$\Rightarrow \frac{\Delta V}{V} = -\frac{\Delta p}{\Delta B} \simeq \frac{10^{7}}{2.2 \times 10^{9}} \simeq 0.5\%$$

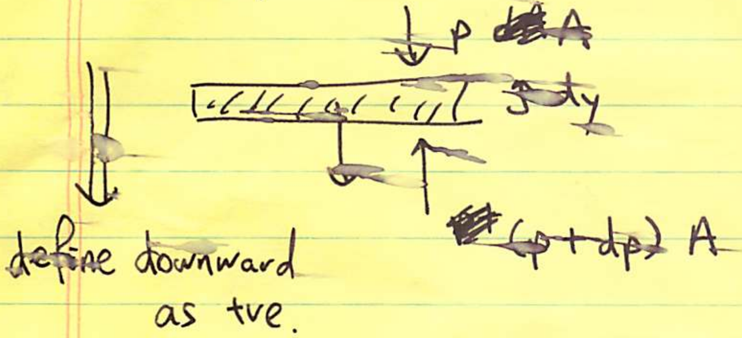
B large incompressible
B small compressible
Gas has $B \simeq 10^5 \ \mathrm{N\,m^{-2}}$

$$\Delta p \simeq 0.1 \text{atm} \simeq 10^4 \ \mathrm{Pa}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta p}{B} \simeq 0.1 = 10\%.$$

Variation of Pressure in Fluid at Rest

For fluid at rest, net force + net torque on every fluid element = 0.



~~All~~ Force balance:

$F_{gravity}$

$= mg$

$= \rho dV g$

$= \rho A g dy$

$$F_{pressure} = pA - (p + dp) A$$

$$F_{gravity} = F_{pressure}$$

$$\Rightarrow \quad \rho A g dy = pA - (p+dp)A = -dp\,A$$

$$\Rightarrow \boxed{\frac{dp}{dy} = -\rho g}$$ equation of hydrostatic equilibrium.

Can integrate this:

can take this out if $\rho = \text{const}$ e.g. water

$$\int_{p_1}^{p_2} dp = -\int_{y_1}^{y_2} \rho g \, dy$$

$$\Rightarrow (p_2 - p_1) = -\rho g (y_2 - y_1)$$

atmospheric pressure

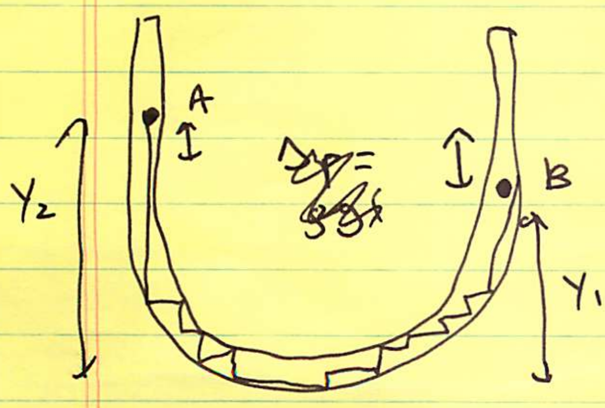
If fluid has free surface, then let that be the zero point

$$(p - p_0) = -\rho g (-h - 0)$$

$$\Rightarrow \quad p = p_0 + \rho g h.$$

Pressure increases with depth, but is the same for all points at the same depth.

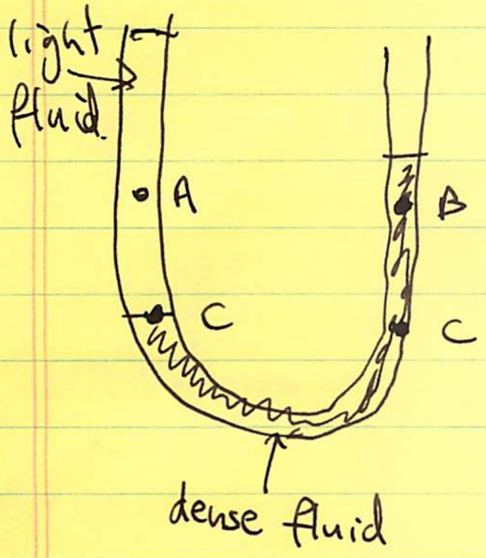
<u>True regardless of the shape of container!</u>



$$\Delta p = \rho g (y_2 - y_1)$$

Any 2 points can be connected by mix of horizontal & vertical steps

pressure doesn't change on horizontal steps, pressure changes by $\Delta p = -\rho g \Delta y$ on vertical steps.



If 2 immiscible fluids have different densities, pressure can be different at same level. on different sides.

$\boxed{\text{Ask class}}$

Where is the pressure greater, A or B?

Liquid between C on left & C on right in equilibrium ⇒ forces at C are same. But pressure falls less between C & A than C & B, since A is less dense

$\Rightarrow$ $\boxed{A}$ is at higher pressure.

Variation of Pressure in Atmosphere

$\Delta p = -\rho g \Delta y$ assumed that density ρ was constant.

This is no longer true if Δy is large.

Ideal gas law $P = \frac{\rho}{\mu m_p} k_B T$

$\propto \rho T$

$\propto \rho$ if T is const (not exactly true, only approximate).

Then write $P = P_0 \left(\frac{\rho}{\rho_0}\right)$

$$\frac{dP}{dy} = -\rho g$$

$$\Rightarrow \quad \frac{P_0}{\rho_0}\frac{d\rho}{dy} = -\rho g$$

$$\Rightarrow \quad \frac{d\rho}{\rho} = -\frac{\rho_0}{P_0} g \, dy$$

$$\Rightarrow \quad \ln \rho = -\frac{\rho_0}{P_0} g y + C$$

At $y = 0$, $\rho = \rho_0$

$$\Rightarrow \quad \rho = \rho_0 e^{-\frac{\rho_0}{P_0} g y}$$

$$P \propto \rho \Rightarrow P = P_0 e^{-\frac{\rho_0}{P_0} g y}$$

$$\frac{g\rho_0}{P_0} = \frac{10 \cdot 1}{10^5} \simeq 10^{-4}\ \mathrm{m}^{-1}$$ (must have units of L^{-1}))

$\simeq$ ~~10~~ $0.1\ \mathrm{km}^{-1}$.

or $a \simeq \left(\frac{g\rho_0}{P_0}\right)^{-1} \simeq 10$ km.

(more careful $a = 8.55$ km)

$$P = P_0 e^{-y/a}$$

⑧

a is the ~~e-folding~~ pressure scale height : pressure drops by e^{-1} everytime go 1 a.

Hence drop by a factor 10 when

$$e^{-y/a} = \cancel{10}\ 0.1$$

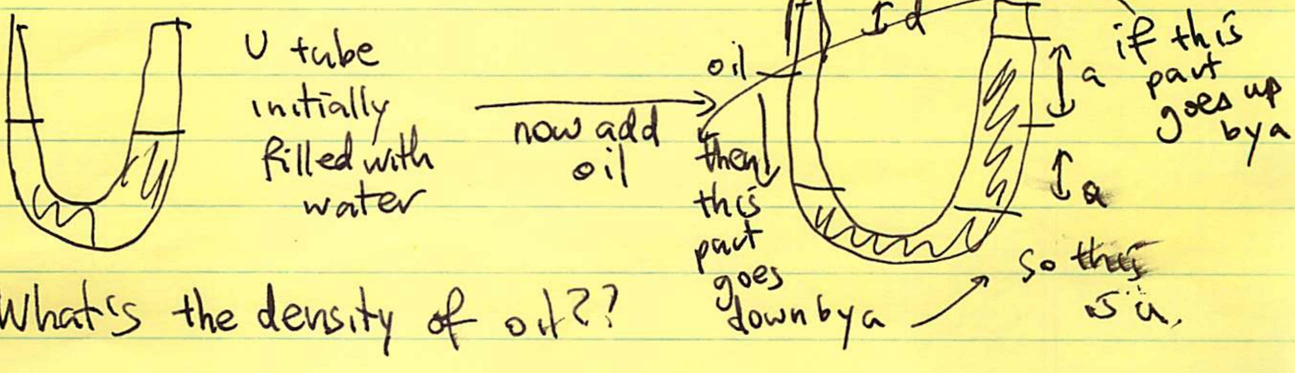
$$\Rightarrow \quad -\frac{y}{a} = \ln 0.1 \Rightarrow y = a \ln 10$$
$$= 2.3a$$
$$= \underline{20\,\text{km}}$$

So drop by factor ~10 every 20 km; at 40 km, P = 0.01 atm.

[Actual pressure drops a little faster than this, bcos temperature falls with height as well].

Note: pressure changes linearly with height for liquid but exponentially for gas. Liquids are incompressible, but gas is compressible → density & hence weight of fluid above changes much faster w/ height.

Example



What's the density of oil??

For U-shaped portion to be static, pressure has to be same on both sides.

Pressure on exposed ends has to be the same (atmospheric pressure).

So $\rho_{oil}(2a+d) = \rho_{water} 2a$

$$\Rightarrow \rho_{oil} = \left(\frac{2a}{2a+d}\right)\rho_{water}.$$

✱ This device is used to find <u>relative densities</u> (or <u>specific gravity</u>).

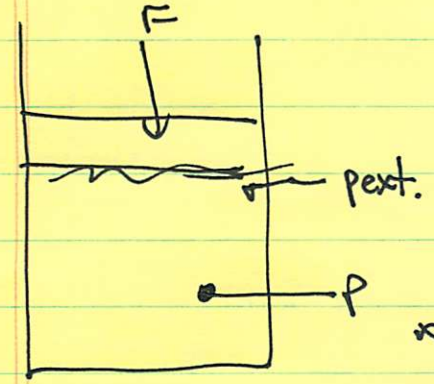
<u>Pascal's Principle & Archimedes Principle</u>

Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.

↳ so if increase pressure at one location by Δp, that increase is felt everywhere!

↳ <u>Examples</u>

- squeeze toothpaste
- hydralics [brakes, flaps on aircraft, dentist chair
  --- enables one to amplify a small force, as well as transmit forces over large distances.



Consider an incompressible fluid

$$p = p_{ext} + \rho g h.$$

← consider pressure here.

If add pressure Δp_{ext}

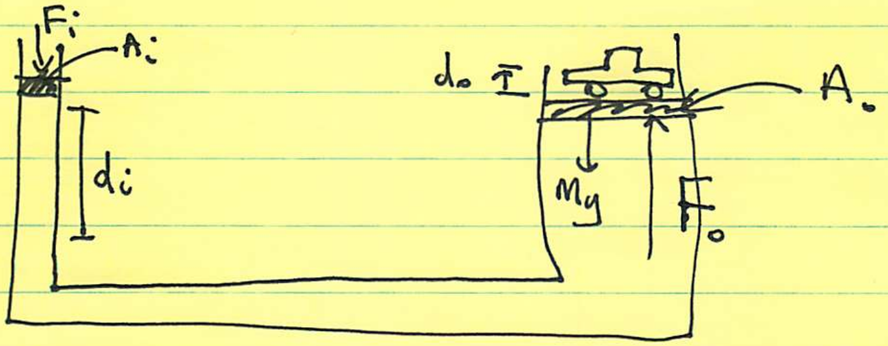
$$\Delta p = \Delta p_{ext} + \Delta(\cancel{\rho g h}) \quad \to 0 \text{ since incompressible}$$

(actually, is also true for compressible fluids → get initial sound waves due to change in density which die out.

E.g. when inflate a balloon w/ air, it expands evenly in all directions.

Example

Hydralic Lever



Using hydralic level, can apply a small force to lift a heavy object like a car!

$$p_i = p_o$$

$$\Rightarrow \frac{F_i}{A_i} = \frac{F_o}{A_o} \Rightarrow F_i = F_o \frac{A_i}{A_o} = Mg \frac{A_i}{A_o}$$

So if $\left(\frac{A_i}{A_o}\right) \ll 1$, can use small force to lift a heavy car.

~~a Are we~~ getting something for nothing??

For incompressible fluids, ~~volume of liquid dis~~

Vol of liquid displaced downwards

= Vol of liq moved upwards

$$\Rightarrow A_i d_i = A_o d_o$$

$$\Rightarrow d_o = d_i\left(\frac{A_i}{A_o}\right) \ll d_i \text{ if } \left(\frac{A_i}{A_o}\right) \ll 1$$

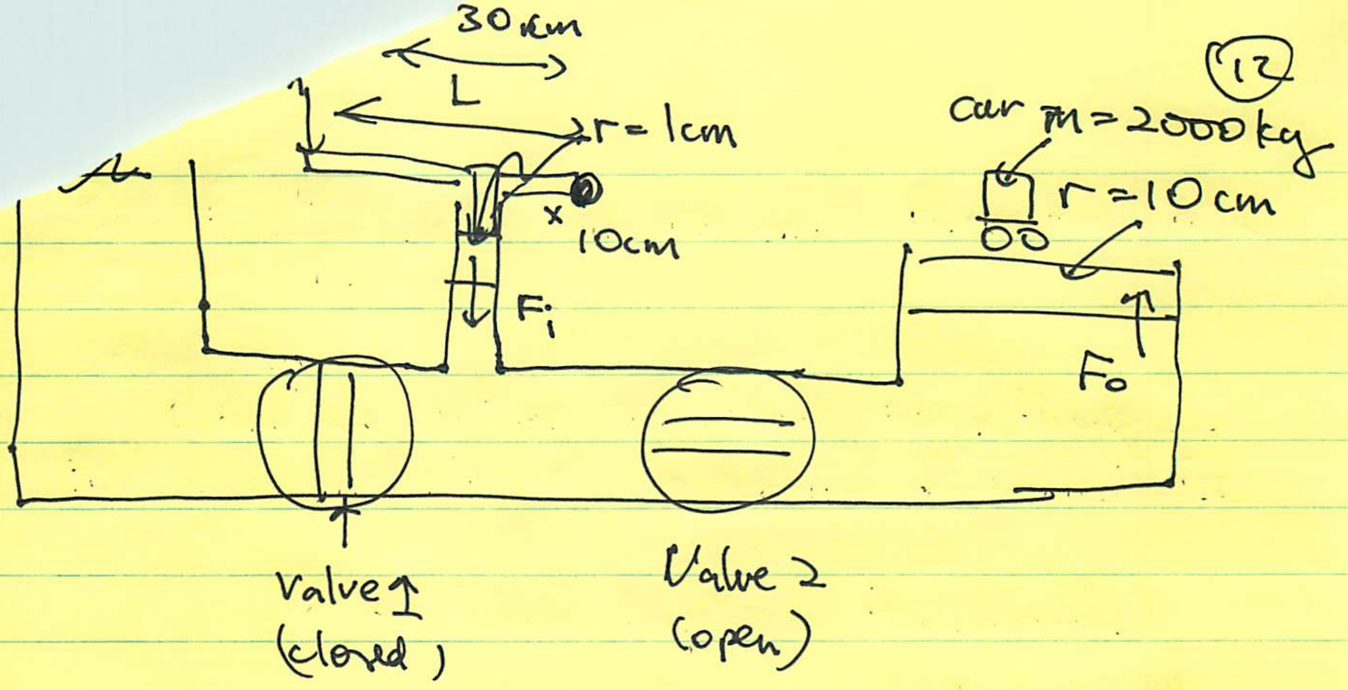
Work done

$$F_i d_i = F_o\left(\frac{A_i}{A_o}\right) d_o\left(\frac{A_o}{A_i}\right) = F_o d_o$$

→ energy is conserved (ignoring frictional losses) → not getting a free lunch!

Example 2

Let's consider a hydralic jack used to raise an automobile. How is it possible that our hands can raise such a heavy object?



F ① What force do we need to apply?
② How far will the car move?

$$F_i = Mg \frac{A_i}{A_o} = 2000(10) \frac{\pi (1)^2}{\pi (10)^2}$$

$$= 200 \text{ N}$$

There's also a lever on the pump.

Assuming angular acceleration of pump handle is negligible,

$$\sum \tau = F_h L - F_i x = 0$$

$$\Rightarrow F_h = F_i \left(\frac{x}{L}\right) = F_i \left(\frac{10\text{cm}}{30\text{cm}}\right)$$

$$\simeq 67 \text{ N}$$

This is small! Like lifting a 6.7 kg weight.

~~di~~

Suppose hand moves through distance $h \simeq 30\,\text{cm}$
Small piston moves through distance

$$d_i \simeq h\left(\frac{x}{L}\right) \simeq \frac{h}{3} \simeq 10\,\text{cm}.$$

Large piston moves through distance:

$$d_o = d_i\left(\frac{A_i}{A_o}\right) = 10\left(\frac{1}{100}\right) = 0.1\,\text{cm}$$
$$= 1\,\text{mm}!$$

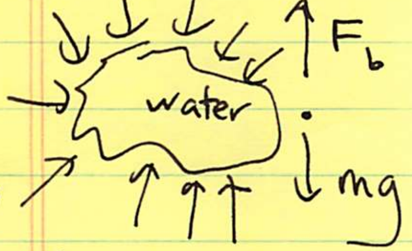
So need to apply many strokes to raise car.

Use valves to transfer fluid fr. reservoir to pistons:

downstroke	1 (closed)	2 (open)
upstroke	~~#~~1 (open)	2 (closed)

To let car down → both valves open. fluid goes back to reservoir.

Archimedes' Principle

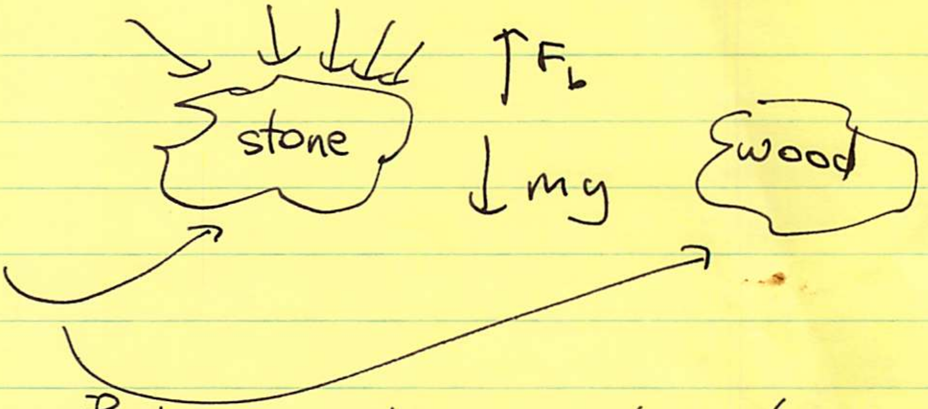


larger pressure (since pressure increases w/ depth)

Water doesn't rise or fall

$F_b = mg$ balance

(vector sum of all forces on surface)



if they are the same shape (same vol) then F_b must be the same as for the parcel of water.

Will then rise ($F_b > mg$)
or sink ($F_b < mg$)

depending on $mg \rightarrow M = \rho V$ ← since vol is fixed, depends on relative density.

Hence buoyancy force depends on pressure difference between top & bottom of fluid element.

So we have Archimedes' principle:

> A body wholly or partially immersed in a fluid is buoyed up by a force equal in magnitude to the weight of fluid displaced by the body.

Denser than water → sinks
less dense than water → rises.

Also: things weigh less in water, due to the buoyant force. So, for example : → astronauts practice in underwater tanks, to simulate weightless conditions in space.

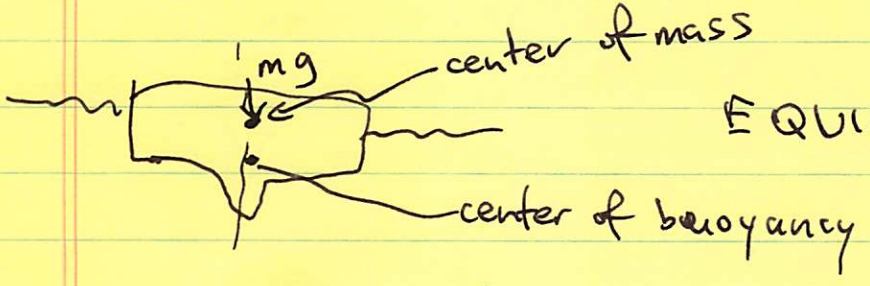
Object less dense than water floats on surface. Submerged portion has volume V
s.t.

$$\rho_{water} V = m_{obj} g \quad \leftarrow \text{equilibrium condition}$$

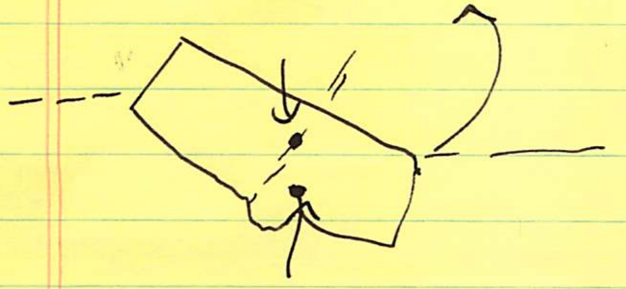
Buoyant force acts on <u>center of buoyancy</u> : center of gravity of displaced fluid.

Gravity acts on <u>center of mass</u>.

As ship tips, center of buoyancy moves → depending on relative position of c. buoyancy & c. mass, torques develop which restore balance or tip the ship over.



EQUILIBRIUM



low c.m. → restoring torque right ship

High c.m. → ship tips over!

<u>Iceberg</u> → what fraction of iceberg is exposed, given that

$\rho_{water} = 1024 \text{ kg m}^{-3}$.
$\rho_{ice} = 917 \text{ kg m}^{-3}$.

weight $W_i = \rho_i V_i g$
buoyant force $F_b = \rho_w V_w g$ ← vol. of water displaced

$$W_i = F_b$$
$$\rho_w V_w g = \rho_i V_i g$$

$$\Rightarrow \frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{917 \text{ kg m}^{-3}}{1024 \text{ kg m}^{-3}} = 89.6\%$$

So only ~ 10% of iceberg is above water!

Measurement of Pressure

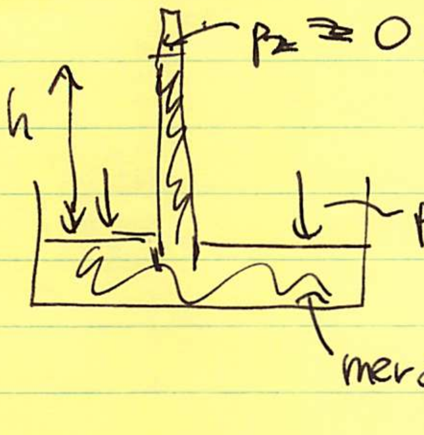
How to do it?
- static (discuss now)
- dynamic (based on speed of moving fluid, we discuss in chap 18)

Use atmospheric pressure as reference level;

measure $\underset{\text{guage pressure}}{\Delta p} = \underset{\text{absolute pressure}}{p_{actual}} - p_{atm}$

we use mercury because it is dense, so required height of fluid is less.

Mercury barometer → used to measure atmospheric pressure.



simply created when mercury ~~falls~~ flows down.

0 (since vacuum)

$$p_1 = p_2 + \rho g h$$

$$\Rightarrow p_1 = \rho g h.$$

$\rho_{mercury} = 1.36 \times 10^4 \ kg \ m^{-3}$.

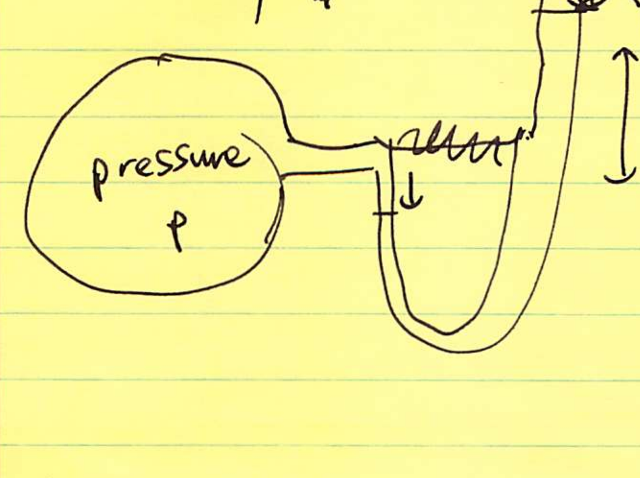
$h = 760$ mm

1 bar $= 10^5$ Pa

$$p_1 = (1.36 \times 10^4)(9.8)(0.76)$$
$$= 1.013 \times 10^5 \ N \ m^{-2} = 1.013 \text{ bar}$$

This is the weight of column density of air from bottom to top of atmosphere.

Mercury ~~ø~~ manometer can be used to measure pressure:



$$p - p_0 = \rho g h$$

(if gas is under low pressure, can use less dense liquid, like water)

Historically : Pascal ~~the~~ got brother-in law to measure height of mercury on a mountaintop → 8cm lower! Created a sensation.

Surface Tension

Leaves & insects float ~~on~~ not bcos of Archimedes' principle, but bcos of surface tension → attractive force between molecules of water.

↳ also causes droplets of water to have spherical shape, minimizing $\frac{\text{Area}}{\text{Vol.}}$

↳ can show it by floating a steel needle or razor blade, which should sink by Archimedes principle (indeed, if put it in water, will sink).

↳ If put in detergent, which reduces surface tension, will sink!