Physics 21: Midterm (Winter 2013)

Exam is closed book, closed notes, no calculators or cell phones.

Potentially useful formulae: (subscript 0 refers to center of mass; i.e. I_0 is moment of inertia about center of mass):

$v = \omega r$; $a = \alpha r$
$I_1 = Mh^2 + I_0$, where h is perpendicular distance between 2 parallel axes I_0, I_1
$L_z = I_0\omega + (\mathbf{r} \times m\mathbf{v})_z$
$\tau = \mathbf{r} \times \mathbf{F} = \frac{d\mathbf{L}}{dt} = I\alpha$
$K = \frac{1}{2}I_0\omega^2 + \frac{1}{2}Mv^2$
$F_{\text{friction}} = \mu_k F_{\text{normal}}$

1. [**30 points**] Conceptual questions: **short** answers, not more than 3 lines. For each of these, the correct answer is worth **2 points**, and your explanation is worth **3 points**.

a) [**5 points**] A small **solid** steel sphere (radius R, mass 4M) and a large **hollow** wooden shell (radius 2R, mass M) roll down an inclined plane at the same time. Which one reaches the bottom first? Explain.

b) [**5 points**] In Edgar Allan Poe's story, 'The Pit and the Pendulum', a prisoner is tied to the floor, and a swinging pendulum with "a crescent of glittering steel...the under edge as keen as that of a razor" is lowered onto the unfortunate prisoner. Does the amplitude of the swing increase, decrease or stay the same as the pendulum is lowered? Why?

c) [**5 points**] Consider a pair of coupled gears; the larger gear A has radius $R_A = 2R$, the smaller gear B has radius $R_B = R$. What is ω_A/ω_B? Explain.

d) [**5 points**] Many great rivers flow toward the equator, carrying sediment to the sea with them. What effect does this have on the length of the day? Why?

e) [**5 points**] Massive gyroscopes are often used to stabilize ships. There are 3 possible directions for the axis of rotation: a) toward the front of the ship; b) parallel to the ship deck; c) toward the sky. Of these 3, one will *not* stabilize the ship against rolling from side to side: which is it? Explain. Of the two which do stabilize the ship from rolling from side to side, one will interfere with steering. Explain which one. Hence, what is the best orientation for a gyroscope to stabilize a ship without interfering with steering?

f) [**5 points**] Consider a swinging pendulum; assume the pivot to be frictionless. Is the a) linear momentum of the bob; b) angular momentum about the pivot, conserved? Explain (if your answer is 'no' for either, explain what happens to the missing momentum).

2. [**20 points**] Just after it has landed, a bowling ball of mass M and radius R slides on a bowling alley with some initial velocity v_i in the x-direction. The ball's moment of inertia is

$I = \frac{2}{5}MR^2$.

(a) [**5 points**] Let the coefficient of kinetic friction between the ball and the alley be μ_k. What is the *magnitude* and *direction* (draw a picture) of the torque about the center of the bowling ball? What is the torque about the point of contact?

(b) [**3 points**] Due to this torque, the ball eventually rolls without slipping, with a center of mass velocity v_f. What is the instantaneous speed, relative to the ground, of the bottom of the ball? Of the top of the ball? Express in terms of v_f.

(c) [**6 points**] When the ball is rolling without slipping, express the (a) kinetic energy, (b) angular momentum L_z about the point of contact, in terms of M, R, v_f.

(d) [**6 points**] Is i) kinetic energy; ii) angular momentum L_z about the point of contact) conserved in the transition from sliding to no slip roll? Justify your answers. Hence, solve for v_f in terms of v_i.

(e) [**5 points**] Suppose we now release this bowling ball from rest down an inclined plane of height h at an angle θ to the horizontal. The plane has coefficient of friction $\mu_{\rm k}$. If the ball does not slip, what is the final velocity of the ball?

3. [**10 points**] Consider a yo-yo of mass M and radius R. The radius of the inner axel is $R/3$. Assume the yo-yo is approximately a solid disk with moment of inertia $I = \frac{1}{2}MR^2$. Someone releases the yo-yo from rest, and it falls down the string without slipping. Find: a) the acceleration a of the yo-yo; b) the tension T in the string. *Hint: conservation laws are not going to work if you need to find forces.*