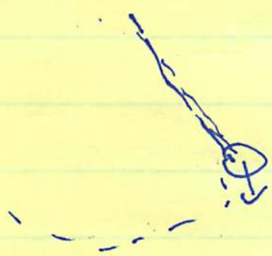
Phys 21 Midterm Solutions

1 a) The solid sphere reaches the bottom first. 2

3 $\frac{I}{MR^2}$ is smaller for a solid object compared to a shell, since there is less mass at large radius

b) The amplitude of the swing increases 2



As pendulum descends, gravity does work in direction of motion, increasing amplitude of oscillations

3

[This is actually a subtle problem. If the pendulum is lowered slowly, then $\frac{E}{\omega} = I$ is ~~an~~ adiabatic invariant.

Of course, I don't expect them to know that. ~~Some~~ variant of "potential energy is converted to kinetic energy" is acceptable, even though total energy isn't really conserved.]

c) ~~The velocity at~~ Velocity at point of contact has to be equal

$\cancel{\omega_A \omega_B}$ $V_A = V_B$

3

$$\Rightarrow \quad \omega_A R_A = \omega_B R_B$$

$$\Rightarrow \quad \frac{\omega_A}{\omega_B} = \frac{R_B}{R_A} = \frac{R}{2R} = \frac{1}{2}.$$

2

d) This increases the length of day 2

Transferring mass to the equator increases the momentum of inertia.

By conservation of $L = I\omega$, $I\uparrow \Rightarrow \omega\downarrow$

$\Rightarrow T\uparrow$

3

e) 1 (a) Toward the front of the ship will not stabilize the ship fr. side to side rolling: in this case, $\vec{\tau}$ is parallel to $\vec{L}$

1

1 (b) Parallel to the ship deck will ~~interfere~~ interfere with steering: if the ship rolls from side to side, the gyroscope will precess to the left or right.

1

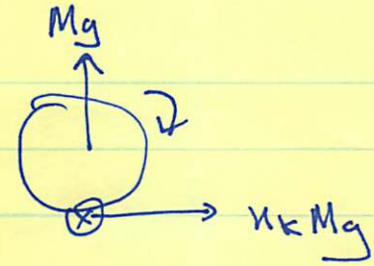
Hence, best stabilizing direction for gyroscope is (c) toward the sky 1

1 f) Linear 1 & angular 1 momentum ~~are both~~ about the pivot are both not conserved. 2

2 One easy way to see this is to realize that the sign of L and $\vec{p}$ change when the pendulum swings fr. left to right or right to left.

1 The torque & forces the pendulum exerts on the pivot are transmitted to the earth. The $\vec{L}, \vec{p}$ of the combined earth/pendulum system is conserved.

2 (a)



Torque ~~has magnitu~~ about c.m.

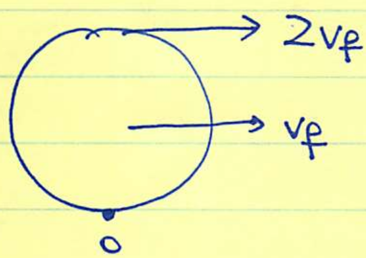
$$\tau = R \times F = \underline{R \mu_k M g}, \text{ directed } \underline{\text{out of page}}$$

2 2

Torque about point of contact : $\underline{\text{zero}}$

1

(b)



Bottom : $v = 0$ 1.5

Top : $v = 2v_f$ 1.5

(c) (i) $K = \frac{1}{2} M v_f^2 + \frac{1}{2} I \omega^2$

$$= \frac{1}{2} M v_f^2 + \frac{1}{2} \underset{\left(\frac{2}{5} M R^2\right)}{\cancel{I}} \left(\frac{v_f}{R}\right)^2$$

3

$$= \frac{7}{10} M v_f^2$$

(ii) $L_z = I_0 \omega + (r \times m v)_z$

$$= \frac{2}{5} M R^2 \left(\frac{v_f}{R}\right) + R M v_f$$

$$\Rightarrow L_z = \tfrac{7}{5} M R v_f$$ 3

(d) Kinetic energy is <u>not</u> conserved, since friction does work when the ball is sliding. 2

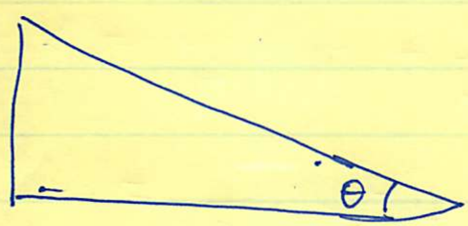
L_z <u>is</u> conserved, since there is no torque about the point of contact. 2

Hence

$$L_f = \tfrac{7}{5} M R v_f = L_i = M R v_i$$

$$\Rightarrow \boxed{v_f = \tfrac{5}{7} v_i}$$ 2

(e)



$$\frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = mgh$$

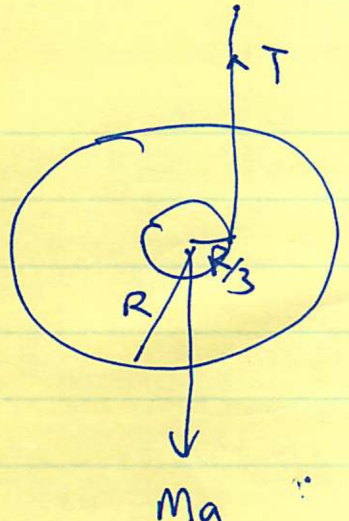
$$\frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{2}{5} m R^2\right)\left(\frac{v}{R}\right)^2 = mgh$$

5

$$\Rightarrow \frac{1}{2}\left(v^2 + \frac{2}{5} v^2\right) = gh$$

$$\Rightarrow \frac{7}{10} v^2 = gh \Rightarrow v = \sqrt{\frac{10}{7} gh}$$

3.



Forces

$Mg - T = Ma$ (2)

Torques

$T\left(\frac{R}{3}\right) = I\alpha = I\left(\frac{a}{R/3}\right)$ (2)

Algebra = 2.

$\Rightarrow T = I\,\frac{9a}{R^2}$

$= \frac{1}{2}MR^2\left(\frac{9a}{R^2}\right) = \frac{9}{2}Ma.$

$\Rightarrow Mg - \frac{9}{2}Ma = Ma$

$\Rightarrow \frac{11}{2}Ma = Mg \Rightarrow \boxed{a = \frac{2}{11}g}$ (2)

$T = \frac{9}{2}Ma = \frac{9}{2}M\left(\frac{2}{11}g\right) = \boxed{\frac{9}{11}g}$ (2)