Physics 21: Practice Final (Winter 2013)

Exam is closed book, closed notes, no cell phones. Calculators are OK.

For question 1 (conceptual questions), do **6** of the 7 questions. For questions 2-5, do **3** of the 4 computational questions. **Do only the required number of questions. If you do more than that, only the first 6 conceptual and the first 3 computational problems will be graded.**

Rotational Motion

$v = \omega r$; $a = \alpha r$
$I_1 = Mh^2 + I_0$, where h is perpendicular distance between 2 parallel axes I_0, I_1
$I(\text{rod about center}) = \frac{1}{12}ML^2$; $I(\text{disk}) = \frac{2}{5}MR^2$
$L_z = I_0\omega + (\mathbf{r} \times m\mathbf{v})_z$
$\tau = \mathbf{r} \times \mathbf{F} = \frac{d\mathbf{L}}{dt} = I\alpha$
$K = \frac{1}{2}I_0\omega^2 + \frac{1}{2}Mv^2$; $U = Mgh$

Harmonic Motion

Simple Harmonic Oscillator: $m\ddot{x} + kx = 0$
has solution $x = A\cos(\omega t + \phi)$; where $\omega = \sqrt{\frac{k}{m}}$

Pendulum $\omega = \sqrt{g/l}$; $\omega = \frac{2\pi}{T}$

Damped Harmonic Oscilllator: $m\ddot{x} + b\dot{x} + kx = 0$. has solution $x = Ae^{-\gamma t/2}cos(\omega t + \phi)$ where $\omega = \sqrt{\frac{k}{m}}$, and energy $E = E_0e^{-\gamma t}$; $\gamma = b/m$.

Quality factor $Q = E/(\Delta E \text{ in } 1 \text{ radian}) \approx \omega/\gamma$

Fluids

Pressure=Force/Area
Continuity $R = \rho v A$ =const
Bernoulli $p + \frac{1}{2}\rho v^2 + \rho g h$ =const

1. [**28 points**] Conceptual questions: **short** answers, not more than 3 lines. Do **6** of the 7 questions below.

a) [**4 points**] Consider a light hollow glass sphere filled with air and sealed; it floats in water with 2/3 of its volume immersed. Suppose you now heat the sphere so that the air inside doubles in temperature. Ignore thermal expansion; the sphere does not expand when heated. If you put it back in water, is the immersed volume more, less, or the same? Why?

b)[**4 points**] On takeoff, would it be better for an airplace to move with the wind or against the wind? On landing? Why?

c)[**4 points**] Your Pilot V-5 pen (a liquid ink pen) tends to leak when you take it on an airplane. Why? Does it matter if it is new or closed to used up? Can you orient your pen in such a way as to prevent leakage?

d)[**4 points**] A single-engine propeller plane must be "trimmed" to fly level (trimming consists of raising one aileron and lower the opposite one). Why is this necessary?

e)[**4 points**] Consider a straight stick standing on end on frictionless ice. What would be the path of its center of mass if it falls? Why?

f)[**4 points**] A hollow sphere is filled with water through a small hole. It is hung by a long thread, and as the water flows out of the hole at the bottom. Does the period of oscilllation i) first increase, then decrease, ii) first decrease, then increase, iii) stay the same? Explain.

g)[**4 points**] What would happen to the motion of an oscillating system if instead of $F = -kx$, the force law obeyed $F = kx$? Explain.

Do **3** of the 4 problems below.

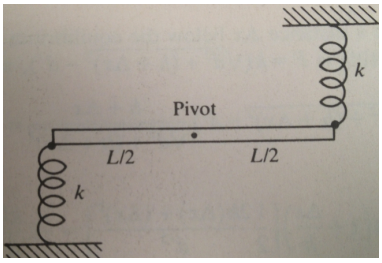

Fig. 1.— Question 2.

2. [**10 points**] The uniform stick in Fig 1 has mass m and length L, and is pivoted at its center. In the equilibrium position shown, the identical light springs have their natural length.
(a) [**5 pts**] Show that the stick will undergo simple harmonic motion when turned through a small angle θ_0 from the position shown and released.
(b) [**2 pts**] What is the frequency of the motion?
(c) [**3 pts**] How fast will the tip of the stick be moving when the stick passes through the horizontal?

3. [**10 points**] A tank of radius r=1 m rests on a platform 2 m high. The tank is filled with water to a depth $h_0 = 3\,m$. A plug in hole with radius 1 cm is removed from the bottom of the tank.
(a) [**5 points**] What is the speed of the stream as it strikes the ground?
(b) [**5 points**] How long does it take to empty the tank?

4. [**10 points**] A string is wound around an otherwise unsupported, homogeneous, horizontal cylinder of mass M and radius R. As the string unwinds and the cylinder spins, the end of the string is continually pulled vertically upward with a force just sufficient to keep the cylinder from descending relative to the ground.
(a)[**3 points**] What is the tension in the vertical portion of the string?
(b)[**4 points**] What is the angular acceleration of the cylinder?
(c) [**3 points**] What is the upward acceleration of any given point along the vertical portion of the string?

5. [**10 points**] The logarithmic decrement δ is defined to be the natural logarithm of the ratio of successive maximum displacements (in the same direction) of a free damped oscillator.
(a)[**6 points**] Show that $\delta = \pi/Q$.
(b)[**4 points**] Find the spring constant k and damping constant b of a damped oscillator having a mass of 5 kg, frequency of oscillation 0.5 Hz, and logarithmic decrement $\delta = 0.02$.