Practice Final Solutions

1 a) The immersed vol is the same
The vol. of the sphere is the same (given), and the mass is the same since it is sealed. Hence upward buoyancy force and downward gravitational forces remain the same.

b) In both cases it is better to land and take off into the wind, to increase lift, and reduce the speed necessary to take off / land.

c) Air in the pen remains at atmospheric pressure, whereas cabin pressure is lower. This difference in pressure pushes ink out. It get worse if the pen is used and there is more air in the chamber. ~~You can~~ If you ~~g~~ ~~it~~ put the pen tip up, then air can escape without pushing ink out.

d) In the absence of external torques, the total angular momentum of the plane/propeller system has to be zero. ~~If the pro~~ In a single engine plane, if the propeller turns in one direction, the plane would turn in the opposite direction. Trim counteracts this by applying ~~a~~ an opposite torque.
~~Trim is not necessary in a twin engine plane~~.

e) The center of mass will fall straight down, since the ground can only exert an upward force without friction. So the center of mass cannot move sideways.

f) The period i) first increases, then decreases. The center of mass moves downward as the water drains out. Since $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$ it initially has a longer period. This shortens once again as the ~~wa~~ mass of water becomes negligible and the center of mass is once again the center of the sphere.

g) The system would be wildly unstable. In fact,

$$F = m\ddot{x} = kx$$

$$\Rightarrow x = x_0 e^{\omega t}, \text{ where } \omega^2 = \frac{k}{m}$$

↳ the amplitude increases exponentially

2 a) If stick is rotated through θ, spring is stretched

$$\Delta x = \left(\frac{L}{2}\right)\theta$$

$$F = -k\Delta x$$
$$= -\frac{kL\theta}{2}.$$

Torque

$$\tau = I\ddot{\theta} = \vec{r} \times \vec{F}$$
$$= -\left(\frac{L}{2}\,\frac{kL\theta}{2}\right) \times 2$$

← since there are 2 springs

$$\Rightarrow \ddot{\theta} + 2\left(\frac{L}{2}\right)^2 \frac{k}{I}\theta = 0$$

$$I = \frac{ML^2}{12}$$

$$\Rightarrow \boxed{\ddot{\theta} + \frac{6k}{m}\theta = 0}$$ equation for simple harmonic oscillator

(b) $$\omega^2 = \frac{6k}{m}$$

$$\Rightarrow \boxed{\omega = \left(\frac{6k}{m}\right)^{1/2}}$$

(c) $\theta = \theta_0 \cos \omega t$ (so $\theta = \theta_0$ at $t = 0$)

Hence

$$\dot{\theta} = -\omega\theta_0 \sin\omega t$$

When stick passes through horizontal, $\theta = 0$

$$\Rightarrow \cos\omega t = 0 \Rightarrow \sin\omega t = 1$$

$$\Rightarrow \quad \dot{\theta} = -\omega\theta_0$$

$$\Rightarrow |v| = \left|\tfrac{L}{2}\dot{\theta}\right| = \frac{L}{2}\omega\theta_0 = \left(\frac{3k}{2m}\right)^{1/2} L\theta_0$$

3. (a) Let's apply Bernoulli's equation ~~from to~~ the top of the tank to just before hitting the ground

$$\underbrace{p_0}_{\text{atmospheric pressure}} + \frac{1}{2}\rho \overset{0}{\cancel{v_1^2}} + \rho g h = \underbrace{p_0}_{\text{atmospheric pressure}} + \frac{1}{2}\rho v_2^2 + \rho g(0) \qquad v_1 = 0$$

$$\Rightarrow v_2 = \sqrt{2gh} = \sqrt{2(9.8)(5)}$$

$$\simeq 10 \text{ ms}^{-1}$$

~~&~~ It acts as though it's in free fall the whole way.

(b) Continuity $A v = \text{const}$

$$\Rightarrow A_1 \frac{dh}{dt} = A_2 v_2$$

speed at which height of column changes

$$= A_2\sqrt{2gh} \qquad \text{(fr. Torricelli's theorem)}$$

$$\Rightarrow t = \frac{A_1}{A_2\sqrt{2g}} \int_0^{h_0} \frac{dh}{\sqrt{h}}$$

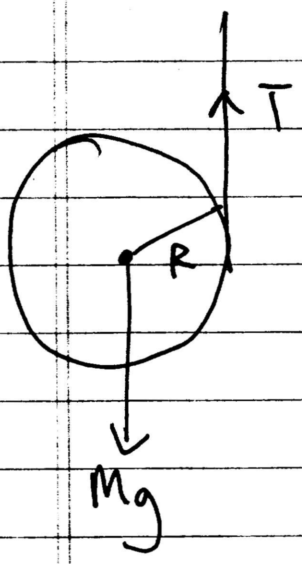
$$= \frac{A_1}{A_2}\sqrt{\frac{2h_0}{g}}$$

$$= \left(\frac{1}{10^{-2}}\right)^2 \left(\frac{2(3)}{10}\right)^{1/2} \simeq 7.75 \times 10^3 \text{ s}$$

$$= 2 \text{ hr } 9 \text{ mins}$$

4 (a) Since the center of mass is not accelerating,

$$\boxed{T = Mg}$$



(b) Take torque about central axis:

$$\tau = I\alpha = TR = MgR$$

$$\Rightarrow \quad \alpha = \frac{MgR}{I} = \frac{MgR}{\frac{1}{2}MR^2}$$

$$= \frac{2g}{R}$$

I for cylinder and uniform disk are the same

(c) $a = R\alpha = R\left(\frac{2g}{R}\right) = 2g$

5. (a) For a damped harmonic oscillator,

$$x = x_0 e^{-\frac{\gamma t}{2}} \sin(\omega_0 t)$$

Over 1 period, $t_2 - t_1 = \frac{2\pi}{\omega_0}$

At maximum displacement, $\sin(\omega_0 t_1) = \sin(\omega_0 t_2) = 1$

$$\Rightarrow \delta = \ln\left(\frac{x_1}{x_2}\right)$$

$$= \ln\left(e^{-\frac{\gamma(t_1 - t_2)}{2}}\right) = \ln e^{+\frac{\gamma}{2}\frac{2\pi}{\omega_0}}$$

$$= \ln e^{\pi/Q} = \pi/Q$$

(b) $\delta = \pi/Q = 0.02$

$$Q = \pi/\delta = \frac{\omega_0}{\gamma}$$

$$\Rightarrow \gamma = \frac{b}{m} = \frac{\delta\omega_0}{\pi} = \frac{\delta 2\pi\nu}{\pi}$$

$$\Rightarrow b = 2\delta\nu m$$

$$\Rightarrow b = 2(0.02)(0.5)(5)$$

$$= 0.1 \ N.s/m.$$

$$k = m\omega_0^2 = 5(2\pi(0.5))^2 = \cancel{1.97 \ N/m}$$

$$= 49.3 \ N/m$$