Problems for HW 1

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1 HW1 Problem 1 (Review)

a) Consider a slab of charge density ρ_0 perpendicular to the z-axis. The slab is parallel to the $x - y$ plane, and extends over $-h < z < h$. The problem contains no other charge.

This system possesses a great deal of symmetry, and consequently the electric field has a simple form. For example, the charge distribution is invariant under translation by x or y, and so the electric field cannot depend on these: $\vec{E}(x_1, y_1, z_1) = \vec{E}(x_2, y_2, z_1) = \vec{E}(z_1)$.

List the symmetries of the charge distribution and the resulting consequences for $\vec{E}$.

Give an appropriate Gaussian box and find the electric field as a function of position.

b) State the symmetries and find the electric field for a charged shell of surface charge σ_0 and radius d.

2 HW1 Problem 2

Prove that the average of the electric potential over the surface of a sphere containing zero charge equals the potential at the center of the sphere.
Is the preceding statement true if the sphere contains zero *net* charge? Prove that it is, or present a counterexample.

3 HW1 Problem 3

A 1D electrostatics problem provides a simple example of Green functions. Specifically, suppose that charge density depends only on z: $\rho = \rho(z)$. Thus, all charge is in sheets perpendicular to $\hat{z}$. Given local boundary conditions, we can find a solution to the Poisson Equation for a single sheet, and superpose copies of that solution to find a solution for any charge density.

a) Write down Laplace's Equation, and show that in 1D, its solution is a linear function:

$$V(z) \quad = \quad A_0 + A_1 z, \tag{1}$$

where $\{A_0, A_1\}$ are constants. Note that in these expressions, z is the field point.

b) Consider a thin sheet of surface charge density σ_0, at the "source point" z'. Outside the sheet, the potential is given by Eq. 1. If only one sheet is present, you can assume that the potential is symmetric across the sheet: $V(d + z') = V((-d) + z')$.

Find the relations among the constants $\{A_0, A_1\}$ for field points above and below the sheet: $z > z'$ and for $z < z'$. (You might call these constants $\{A_{0>}, A_{1>}\}$ and $\{A_{0<}, A_{1<}\}$).

c) In any integral, the charge density of a single sheet, at z_s, can be expressed as a delta-function: $\sigma_0\, \delta(z' - z_s) dz'$. Furthermore, any given 1D charge distribution $\rho(z')dz'$ can be expressed as a superposition of sheets. In this case $V(z)$ is just the superposition of V's. Give an expression for $V(z)$, expressed as an integral of ρ over z'.

d) As a simple example, consider the charge distribution

$$\rho(z') \quad = \quad \begin{cases} \rho_0 \sin(\pi z'/a), & -a \le z' \le a \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Find the potential $V(z)$. How could you include the effects of an unknown charge distribution outside $-a \le z \le a$?

e) Suppose that the boundary conditions are: $V = 0$ at $z = \pm h$. What is the potential resulting from a single sheet charge σ_0 at z', where $-h < z' < h$? What is the general expression for the potential due to an arbitrary charge distribution $\rho(z')$?