Problems for HW 4

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1 HW4 Problem 1

A semi-infinite cylinder of radius a has grounded walls. The base, at $z = 0$, is grounded as well, except for a very thin annular strip of radius $b < a$ and width $w < a$, at potential V_0. Find an expression for potential V within the cylinder. You may assume $z >> w$.

It is desired to extend the potential as far into the cylinder as possible, while keeping a, w, and V_0 fixed. For what value of b is the potential at a particular point on the z-axis, at $z_1 >> a$, as great as possible?

If a second ring is added, also with width w but at potential $-V0$, where should it be placed so as to maximize the potential at $z_1 >> a$? (Assume that $w < a/2$).

2 HW4 Problem 2

An ideal dipole resides outside a grounded, conducting sphere of radius a. Assume that the dipole lies on the z-axis at a distance d from the sphere. The dipole moment is $\vec{p} = p_0(\cos\theta\hat{z} + \sin\theta\hat{x})$. Find the potential throughout space.

3 HW4 Problem 3

a) Prove that $Y_{\ell\,\ell}(\theta, \phi) \propto \sin(\theta)^\ell e^{i\ell\phi}$, for any nonzero integer ℓ. Find the constant of proportionality.

b) Consider a conducting spherical surface, divided into $2N$ segments by N planes. The planes all intersect on the z-axis and are equally spaced in ϕ. The segments are kept at alternating potential: $+V_0$, $-V_0$, $+V_0$, $-V_0$.... . Find the potential near the center of the sphere, *to lowest order* in r.