Problems for HW 5

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Due **Tuesday**, 9 Feb 2010, 5 pm

1 HW5 Problem 1

Recall the image-charge solution for the problem of a point charge q at height z', above a grounded conducting plane at $z = 0$. Suppose that the point charge is a distance $s' = \sqrt{(x')^2 + (y')^2}$ from the z-axis. Use this solution to write down $G_D(\vec{r}, \vec{r}')$, the Green's function for Dirichlet boundary conditions, in the upper half-space.

Calculate the normal derivative at the bounding plane, $\frac{\partial G_D}{\partial n}|_{z'=0}$. Explain in a couple of sentences how this function relates to the discussion of "dipole layers" in the introductory chapter of Jackson.

Suppose that an annulus of inner radius a and outer radius b is cut out of the grounded plane, and replaced with an annulus of the same dimensions but at fixed potential V_0. Choose the center of the annulus as the origin. No charge resides in the upper half-space. Use your expression for $\frac{\partial G_D}{\partial n}|_{z'=0}$, and the appropriate expression for Dirichlet boundary conditions, to find the potential as a function of position along the z-axis.

Find the electric field at the origin, and the surface charge on the boundary there. Note that the potential is zero at this point, the Green's function is zero, and the contribution of the surface integral in Jackson's Eq. 1.44 is zero. Argue that the image charge is zero at that point, as well. How then can the surface charge be nonzero?

2 HW Problems from Jackson

Jackson 3.9, 3.10, 4.4