Problems for HW 7

C. Gwinn

February 19, 2010

Due **Tuesday**, 23 Feb 2010, 5 pm

1 HW7 Problem 1

a) Consider a semi-infinite solenoid, idealized as a cylindrical current sheet of radius R and an azimuthal surface current $\vec{K} = \kappa\hat{\phi}$. The end of the solenoid is centered at the origin, and its axis is the negative z-axis. Use the Biot-Savart Law to show that the magnetic field on the axis is

$$\begin{aligned} B_z &= \frac{\mu_0\kappa}{2}\left[1 - \frac{z}{(R^2+z^2)^{1/2}}\right] && (1) \\ &= \frac{\mu_0\kappa}{2}\left[1-\cos\psi\right]. && (2) \end{aligned}$$

Give a geometrical interpretation of ψ.

b) Consider an infinite solenoid of radius R. Suppose that the current varies sinusoidally along its length: the surface current is $\vec{K} = \kappa_0 \sin(kz)\hat{\phi}$. Thus, the current periodically reverses direction, at distances π/k along its length (period $2\pi/k$). Show that the magnetic field along the axis is

$$\vec{B} = \mu_0\kappa_0 R^2 \frac{k}{R} K_1(kR)\sin(kz)\hat{z} \qquad (3)$$

Hint: you may need the integral:

$$\int_{-\infty}^{\infty} \frac{\cos(au)\, du}{(u^2+b^2)^{3/2}} = 2\frac{a}{b}K_1(ab) \qquad (4)$$

c) Find the magnetic field within the solenoid from part b, as a function of cylindrical coordinates (s, ϕ, z).

Hint: Note that the curl and divergence of $\vec{B}$ are both zero inside the solenoid. That means that $\vec{B}$ is the gradient of a scalar potential (or, if you prefer, minus the gradient). That scalar potential satisfies Laplace's Equation, which suggests some possible approaches.

2 HW7 Problem 2

A large region of space of volume V is filled with a constant electric field $\vec{E}_0\hat{z}$.

a) State the energy contained in electric field, within the volume.

b) The electric field is reduced to zero, and a dielectric sphere, of electric susceptibility χ_e and radius a, is placed near the center of the volume. The electric field is again increased until (far from the sphere) it is again $\vec{E}_0\hat{z}$. Find the bulk polarization of the dielectric sphere. Now find the energy contained in the volume, using Eq. 4.89 in Jackson.

c) Now suppose that the polarization is "frozen" at the value you found in the previous expression, leaving the bound surface charge fixed. Find the energy contained in the volume, using Eq. 1.59 from Jackson.

d) Suppose that the dipoles within the sphere are each composed of two charges, $\pm q$, connected together by a spring, of spring constant k. There are N such dipoles per unit volume. The bulk polarization results from the extension of these springs by some amount d, in response to the electric field. Calculate the mechanical work in the springs, corresponding to the bulk polarization $\vec{P} = \chi_e \epsilon_0 \vec{E}$, within the sphere.

3 HW7 Problem 3

Consider an infinite sheet current $\vec{K} = \kappa_0\hat{x}$ in the plane $z = 0$. Find the magnetic field above and below the sheet. Find the magnetic vector potential above and below the plane.

Compare this problem with the electrostatics problem of a uniformly charged sheet, with surface charge σ, in the plane $z = 0$.

4 HW7 Problem 4

An observer sits in a small cavity in a large volume of dielectric material. Far from the cavity, the electric field is $\vec{E} = E_0\hat{z}$, and the resulting polarization is $\vec{P} = \chi_e\epsilon_0\vec{E}$. Find the electric field within the cavity, if the cavity is:

a) needle-shaped

b) disk-shaped

c) spherical

Can you describe the electric field outside the large volume of dielectric? (Ignore the effects of the cavity, which are small so far away).