Phys 210A: Problems for HW 8

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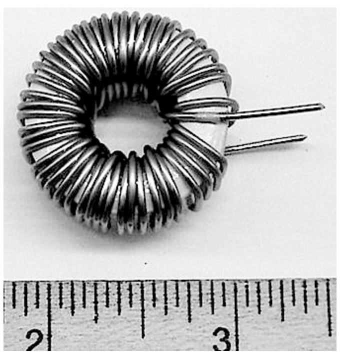
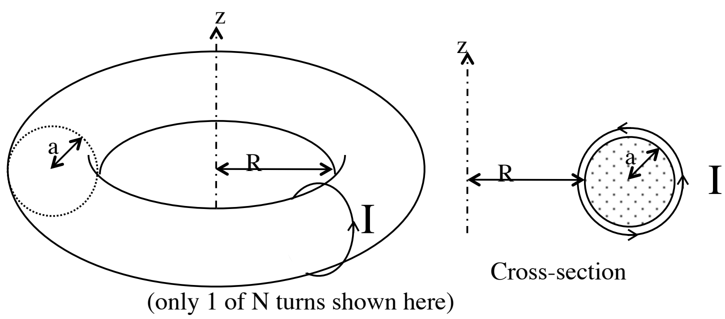
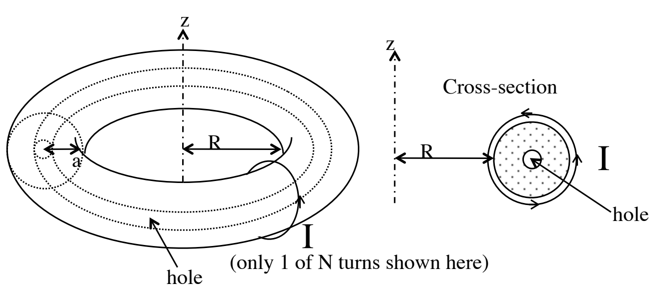
1 HW8 Problem 1

A torus is made of material with magnetic susceptibility χ_m. The outer surface of the torus is covered with a toroidal coil, with N turns. The coil carries a current I. (Note: Assume that the coil windings are even, unlike the figure; and that they have negligible thickness, and are in contact with the torus.) The torus has a circular cross-section, with minor radius a and inner radius R. (In math terminology, the major radius of the torus is $R + a$.) Do not assume that $a << R$; try to be as general as possible.

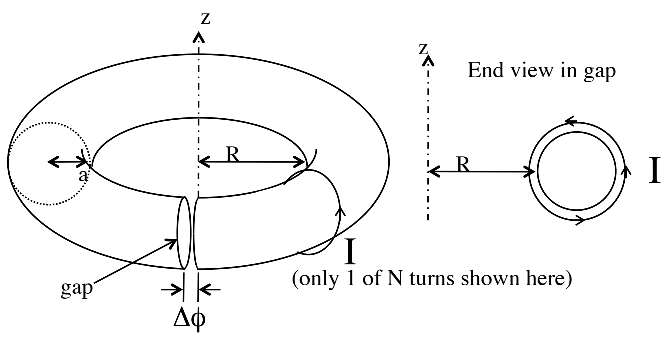




a) Find the magnetic field $\vec{B}$, and the auxiliary field $\vec{H}$, inside the torus. Find the magnetization $\vec{M}$.



b) A second torus is manufactured, but this one has a thin, toroidal hole along the axis of the original toroid. The torus is still wound with N turns, carrying current I. Find the magnetic field $\vec{B}$, and the auxiliary field $\vec{H}$, within the toroidal hole. Explain your reasoning.



c) Now a very thin wedge-shaped gap is cut from the torus, as shown. The gap has angular width about the axis of $\Delta\phi$, where $\Delta\phi << 1$ rad. The current through the N turns surrounding the torus is still I, and the susceptibility is still χ_m. Find the magnetic field $\vec{B}$, and the auxiliary field $\vec{H}$, within the gap. Explain your reasoning.

2 HW8 Problem 2

Monopoles Note: Dirac originally worked out this problem. The existence and role of monopoles remains controversial among theorists, and a goal for experimentalists.

A simple model for a magnetic monopole is a very thin, half-infinite solenoid. Suppose that the solenoid extends from $-\infty$ to 0 along the z-axis. The monopole lies at the origin: at the end of the solenoid. Except inside the solenoid (which we choose to ignore), the magnetic field is:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{g}{r^2}\hat{r}, \tag{1}$$

where g is the magnetic charge of the monopole.

a) Suppose that an electron (with charge $-e$) approaches the monopole from far away, at high speed v, with impact parameter b. Specifically, far away the electron is at position $(x, y, z) = (-\infty, 0, b)$ and it velocity is $(v_x, v_y, v_z) = (+v_0, 0, 0)$. Estimate the effect of the interaction by estimating the change in momentum $\Delta\vec{p}$, as given by the impulse $\vec{F}\Delta t$. For your estimate, take $\vec{F}$ as constant, equal to the force at the closest approach b (if the electron were undeflected), and Δt as the time for the electron to travel twice a distance b. (This crude approximation is good in the limit of large b and/or large v_0; it's also good in these limits, with the correct force law, for Rutherford scattering.) What is the final speed and direction of the electron?

b) Find the initial angular momentum of the electron about the origin $\vec{L}_0$, and its final angular momentum $\vec{L}_1$. Find the change in angular momentum $\Delta\vec{L}$. Show that this change is independent of b.

c) Angular momentum is quantized, in units of $\bar{h}$. Thus, $|\Delta\vec{L}| = N\bar{h}$, where N is an integer. Find e in terms of g. Therefore, if the universe contains one or more magnetic monopoles, electric charge must be quantized.

3 HW8 Problem 3

a) Prove the conditions on $\vec{H}$ at a surface between two media, with magnetic permeability μ_1 and μ_2:

$$\mu_1\vec{H}_1 \cdot \hat{n} = \mu_2\vec{H}_2 \cdot \hat{n} \tag{2}$$

$$\vec{H}_1 \times \hat{n} = \vec{H}_2 \times \hat{n} \tag{3}$$

$$\tag{4}$$

b) Consider a hollow cylinder of permeability μ, with inner radius a and outer radius b. The cylinder is very long – you may regard it as infinitely long. The axis of the cylinder is the X-axis. The cylinder is placed in vacuum, in a large region of previously-uniform magnetic field $\vec{B} = B_0\hat{z}$. Find the magnetic field inside the cylinder $s < a$. Also find the perturbed field outside the cylinder $s > b$, and the field in the material $a < s < b$. (Hint: Make use of the magnetic scalar potential in the 3 regions).