QUESTIONS

1. How could you prove experimentally that energy is associated with a wave?
2. Energy can be transferred by particles as well as by waves. How can we experimentally distinguish between these methods of energy transfer?
3. Can a wave motion be generated in which the particles of the medium vibrate with angular simple harmonic motion? If so, explain how and describe the wave.
4. In analyzing the motion of an elastic wave through a material medium, we often ignore the molecular structure of matter. When is this justified and when isn't it?
5. How do the amplitude and the intensity of surface water waves vary with the distance from the source?
6. How can one create plane waves? Spherical waves?
7. A passing motor boat creates a wake that causes waves to wash ashore. As time goes on, the period of the arriving waves grows shorter and shorter. Why?
8. The following functions in which A is a constant are of the form $y = f(x \pm vt)$:

$$y = A(x - vt), \qquad y = A(x + vt)^2,$$
$$y = A\sqrt{x - vt}, \qquad y = A\ln(x + vt).$$

   Explain why these functions are not useful in wave motion.
9. Can one produce on a string a waveform that has a discontinuity in slope at a point, that is, a sharp corner? Explain.
10. The inverse-square law does not apply exactly to the decrease in intensity of sounds with distance. Why not?
11. When two waves interfere, does one alter the progress of the other?
12. When waves interfere, is there a loss of energy? Explain your answer.
13. Why don't we observe interference effects between the light beams emitted from two flashlights or between the sound waves emitted by two violins?
14. As Fig. 20 shows, twice during the cycle the configuration of standing waves in a stretched string is a straight line, exactly what it would be if the string were not vibrating at all. Discuss from the point of view of energy conservation.
15. Two waves of the same amplitude and frequency are traveling on the same string. At a certain instant the string looks like a straight line. Are the two waves necessarily traveling in the same direction? What is the phase relationship between the two waves?
16. If two waves differ only in amplitude and are propagated in opposite directions through a medium, will they produce standing waves? Is energy transported? Are there any nodes?
17. The partial reflection of wave energy by discontinuities in the path of transmission is usually wasteful and can be minimized by insertion of "impedance matching" devices between sections of the path bordering on the discontinuity. For example, a megaphone helps match the air column of mouth and throat to the air outside the mouth. Give other examples and explain qualitatively how such devices minimize reflection losses.
18. Consider the standing waves in a string to be a superposition of traveling waves and explain, using superposition ideas, why there are no true nodes in the resonating string of Fig. 25, even at the "fixed" end. (*Hint:* Consider damping effects.)
19. Standing waves in a string are demonstrated by an arrangement such as that of Fig. 25. The string is illuminated by a fluorescent light and the vibrator is driven by the same electric outlet that powers the light. The string exhibits a curious color variation in the transverse direction. Explain.
20. In the discussion of transverse waves on a string, we have dealt only with displacements in a single plane, the xy plane. If all displacements lie in one plane, the wave is said to be *plane polarized.* Can there be displacements in a plane other than the plane dealt with? If so, can two different plane polarized waves be combined? What appearance would such a combined wave have?
21. A wave transmits energy. Does it transfer momentum? Can it transfer angular momentum? (See "Energy and Momentum Transport in String Waves," by D. W. Juenker, *American Journal of Physics,* January 1976, p. 94.)
22. In the Mexico City earthquake of September 19, 1985, areas with high damage alternated with areas of low damage. Also, buildings between 5 and 15 stories high sustained the most damage. Discuss these effects in terms of standing waves and resonance.

PROBLEMS

Section 19-3 Traveling Waves

1. A wave has a wave speed of 243 m/s and a wavelength of 3.27 cm. Calculate (*a*) the frequency and (*b*) the period of the wave.
2. By rocking a boat, a child produces surface water waves on a previously quiet lake. It is observed that the boat performs 12 oscillations in 30 s and also that a given wave crest reaches shore 15 m away in 5.0 s. Find (*a*) the frequency, (*b*) the speed, and (*c*) the wavelength of the waves.
3. A sinusoidal wave travels along a string. The time for a particular point to move from maximum displacement to zero displacement is 178 ms. The wavelength of the wave is 1.38 m. Find (*a*) the period, (*b*) the frequency, and (*c*) the speed of the wave.
4. Write an expression describing a transverse wave traveling along a cord in the $+x$ direction with wavelength 11.4 cm, frequency 385 Hz, and amplitude 2.13 cm.

5. Write the equation for a wave traveling in the negative direction along the x axis and having an amplitude of 1.12 cm, a frequency of 548 Hz, and a speed of 326 m/s.

6. A wave of frequency 493 Hz has a speed of 353 m/s. (*a*) How far apart are two points differing in phase by 55.0°? (*b*) Find the difference in phase between two displacements at the same point but at times differing by 1.12 ms.

Section 19-4 Wave Speed

7. Show (*a*) that the maximum transverse speed of a particle in a string owing to a traveling wave is given by $u_{max} = \omega y_m$, and (*b*) that the maximum transverse acceleration is $a_{max} = \omega^2 y_m$.

8. The equation of a transverse wave traveling along a string is given by

$$y = (2.30 \times 10^{-3}) \sin (18.2x - 588t),$$

where x and y are in meters and t is in seconds. Find (*a*) the amplitude, (*b*) the frequency, (*c*) the velocity, (*d*) the wavelength of the wave, and (*e*) the maximum transverse speed of a particle in the string.

9. The equation of a transverse wave traveling along a very long string is given by $y = 6.0 \sin (0.020\pi x + 4.0\pi t)$, where x and y are expressed in centimeters and t in seconds. Calculate (*a*) the amplitude, (*b*) the wavelength, (*c*) the frequency, (*d*) the speed, (*e*) the direction of propagation of the wave, and (*f*) the maximum transverse speed of a particle in the string.

10. Calculate the speed of a transverse wave in a cord of length 2.15 m and mass 62.5 g under a tension of 487 N.

11. The speed of a wave on a string is 172 m/s when the tension is 123 N. To what value must the tension be increased in order to raise the wave speed to 180 m/s?

12. Show that, in terms of the tensile stress S and mass density ρ, the speed v of transverse waves in a wire is given by $v = (S/\rho)^{1/2}$.

13. The equation of a particular transverse wave on a string is $y = 1.8 \sin (23.8x + 317t)$, where x is in meters, y is in millimeters, and t is in seconds. The string is under a tension of 16.3 N. Find the linear mass density of the string.

14. A continuous sinusoidal wave is traveling on a string with speed 82.6 cm/s. The displacement of the particles of the string at $x = 9.60$ cm is found to vary with time according to the equation $y = 5.12 \sin (1.16 - 4.08t)$, where y is in centimeters and t is in seconds. The linear mass density of the string is 3.86 g/cm. (*a*) Find the frequency of the wave. (*b*) Find the wavelength of the wave. (*c*) Write the general equation giving the transverse displacement of the particles of the string as a function of position and time. (*d*) Calculate the tension in the string.

15. A simple harmonic transverse wave is propagating along a string toward the left (or $-x$) direction. Figure 26 shows a plot of the displacement as a function of position at time $t = 0$. The string tension is 3.6 N and its linear density is 25 g/m. Calculate (*a*) the amplitude, (*b*) the wavelength, (*c*) the wave speed, (*d*) the period, and (*e*) the maximum speed of a particle in the string. (*f*) Write an equation describing the traveling wave.

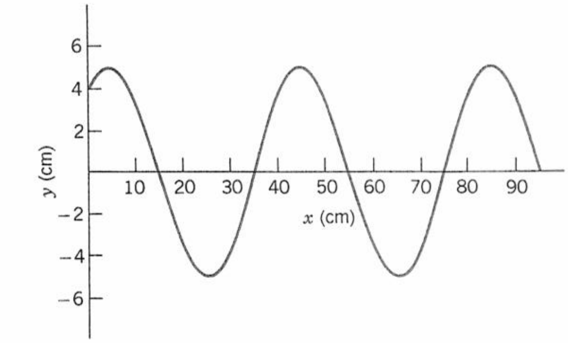

Figure 26 Problem 15.

16. Prove that the slope of a string at any point is numerically equal to the ratio of the particle speed to the wave speed at that point.

17. For a wave on a stretched cord, find the ratio of the maximum particle speed (the maximum speed with which a single particle in the cord moves transverse to the wave) to the wave speed. If a wave having a certain frequency and amplitude is imposed on a cord, would this speed ratio depend on the material of which the cord is made, such as wire or nylon?

18. In Fig. 27*a*, string #1 has a linear mass density of 3.31 g/m, and string #2 has a linear mass density of 4.87 g/m. They are under tension owing to the hanging block of mass $M = 511$ g. (*a*) Calculate the wave speed in each string. (*b*) The block is now divided into two blocks (with $M_1 + M_2 = M$) and the apparatus is rearranged as shown in Fig. 27*b*. Find M_1 and M_2 such that the wave speeds in the two strings are equal.

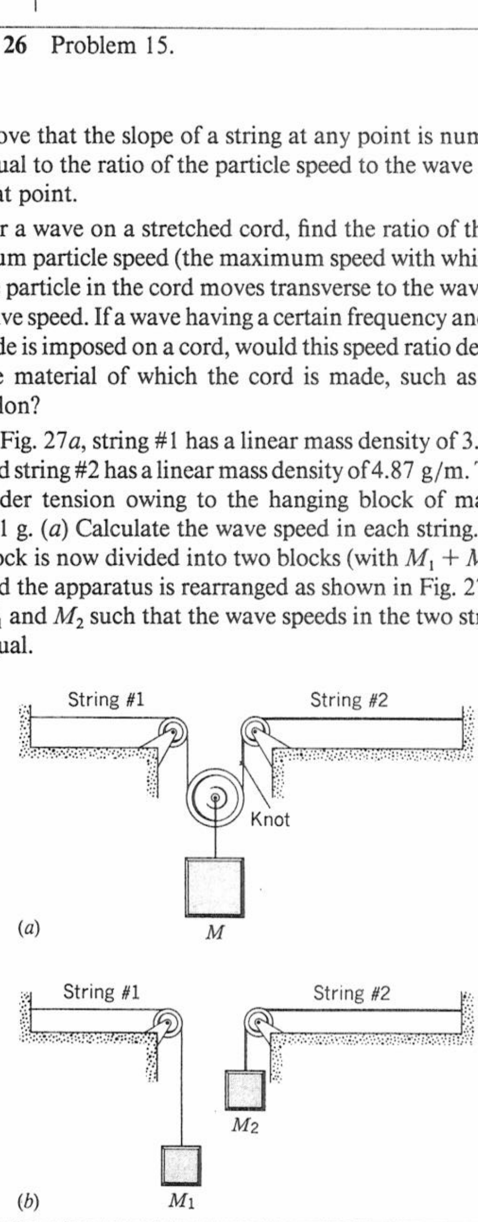

Figure 27 Problem 18.

19. A wire 10.3 m long and having a mass of 97.8 g is stretched under a tension of 248 N. If two pulses, separated in time by 29.6 ms, are generated one at each end of the wire, where will the pulses meet?

20. Find the speed of the fastest transverse wave that can be sent

along a steel wire. Allowing for a reasonable safety factor, the maximum tensile stress to which steel wires should be subject is 720 MPa. The density of steel is 7.80 g/cm³. Show that your answer does not depend on the diameter of the wire.

21. The type of rubber band used inside some baseballs and golfballs obeys Hooke's law over a wide range of elongation of the band. A segment of this material has an unstretched length L and a mass m. When a force F is applied, the band stretches an additional length ΔL. (*a*) What is the speed (in terms of m, ΔL, and the force constant k) of transverse waves on this rubber band? (*b*) Using your answer to (*a*), show that the time required for a transverse pulse to travel the length of the rubber band is proportional to $1/\sqrt{\Delta L}$ if $\Delta L \ll L$ and is constant if $\Delta L \gg L$.

22. A uniform rope of mass m and length L hangs from a ceiling. (*a*) Show that the speed of a transverse wave in the rope is a function of y, the distance from the lower end, and is given by $v = \sqrt{gy}$. (*b*) Show that the time it takes a transverse wave to travel the length of the rope is given by $t = 2\sqrt{L/g}$. (*c*) Does the actual mass of the rope affect the results of (*a*) and (*b*)?

23. A nonuniform wire of length L and mass M has a variable linear mass density given by $\mu = kx$, where x is the distance from one end of the wire and k is a constant. (*a*) Show that $M = kL^2/2$. (*b*) Show that the time t required for a pulse generated at one end of the wire to travel to the other end is given by $t = \sqrt{8ML/9F}$, where F is the tension in the wire.

24. A uniform circular hoop of string is rotating clockwise in the absence of gravity (see Fig. 28). The tangential speed is v_0. Find the speed of waves on this string. (*Remark:* The answer is independent of the radius of the hoop and the linear mass density of the string!)

Figure 28 Problem 24.

Section 19-6 Power and Intensity in Wave Motion

25. A string 2.72 m long has a mass of 263 g. The tension in the string is 36.1 N. What must be the frequency of traveling waves of amplitude 7.70 mm in order that the average transmitted power be 85.5 W?

26. A line source emits a cylindrical expanding wave. Assuming the medium absorbs no energy, find how (*a*) the intensity and (*b*) the amplitude of the wave depend on the distance from the source.

27. A wave travels out uniformly in all directions from a point source. (*a*) Justify the following expression for the displacement y of the medium at any distance r from the source:

$$y = \frac{Y}{r} \sin k(r - vt).$$

Consider the speed, direction of propagation, periodicity, and intensity of the wave. (*b*) What are the dimensions of the constant Y?

28. An observer measures an intensity of 1.13 W/m² at an unknown distance from a source of spherical waves whose power output is also unknown. The observer walks 5.30 m closer to the source and measures an intensity of 2.41 W/m² at this new location. Calculate the power output of the source.

29. (*a*) Show that the intensity I is the product of the energy density u (energy per unit volume) and the speed of propagation v of a wave disturbance; that is, show that $I = uv$. (*b*) Calculate the energy density in a sound wave 4.82 km from a 47.5-kW siren, assuming the waves to be spherical, the propagation isotropic with no atmospheric absorption, and the speed of sound to be 343 m/s.

30. A transverse sinusoidal wave is generated at one end of a long, horizontal string by a bar that moves up and down through a distance of 1.12 cm. The motion is continuous and is repeated regularly 120 times per second. The string has linear density 117 g/m and is kept under a tension of 91.4 N. Find (*a*) the maximum value of the transverse speed u and (*b*) the maximum value of the transverse component of the tension. (*c*) Show that the two maximum values calculated above occur at the same phase values for the wave. What is the transverse displacement y of the string at these phases? (*d*) What is the maximum power transferred along the string? (*e*) What is the transverse displacement y for conditions under which this maximum power transfer occurs? (*f*) What is the minimum power transfer along the string? (*g*) What is the transverse displacement y for conditions under which this minimum power transfer occurs?

Section 19-8 Interference of Waves

31. What phase difference between two otherwise identical traveling waves, moving in the same direction along a stretched string, will result in the combined wave having an amplitude 1.65 times that of the common amplitude of the two combining waves? Express your answer in both degrees and radians.

32. Determine the amplitude of the resultant wave when two sinusoidal waves having the same frequency and traveling in the same direction are combined, if their amplitudes are 3.20 cm and 4.19 cm and they differ in phase by $\pi/2$ rad.

33. Two pulses are traveling along a string in opposite directions, as shown in Fig. 29. (*a*) If the wave speed is 2.0 m/s and the pulses are 6.0 cm apart, sketch the patterns after 5.0, 10, 15, 20, and 25 ms. (*b*) What has happened to the energy at $t = 15$ ms?

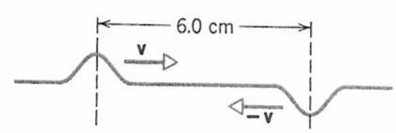

Figure 29 Problem 33.

34. Three sinusoidal waves travel in the positive x direction along the same string. All three waves have the same frequency. Their amplitudes are in the ratio $1:\frac{1}{2}:\frac{1}{3}$ and their phase angles are 0, $\pi/2$, and π, respectively. Plot the resultant waveform and discuss its behavior as t increases.

35. Four sinusoidal waves travel in the positive x direction

along the same string. Their frequencies are in the ratio 1:2:3:4 and their amplitudes are in the ratio $1:\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$, respectively. When $t = 0$, at $x = 0$, the first and third waves are 180° out of phase with the second and fourth. Plot the resultant waveform when $t = 0$ and discuss its behavior as t increases.

36. Consider two point sources S_1 and S_2 in Fig. 30, which emit waves of the same frequency and amplitude. The waves start in the same phase, and this phase relation at the sources is maintained throughout time. Consider points P at which r_1 is nearly equal to r_2. (*a*) Show that the superposition of these two waves gives a wave whose amplitude y_m varies with the position P approximately according to

$$y_m = \frac{2Y}{r} \cos \frac{k}{2} (r_1 - r_2),$$

in which $r = (r_1 + r_2)/2$. (*b*) Then show that total cancellation occurs when $r_1 - r_2 = (n + \frac{1}{2})\lambda$, n being any integer, and that total re-enforcement occurs when $r_1 - r_2 = n\lambda$. The locus of points whose difference in distance from two fixed points is a constant is a hyperbola, the fixed points being the foci. Hence each value of n gives a hyperbolic line of constructive interference and a hyperbolic line of destructive interference. At points at which r_1 and r_2 are not approximately equal (as near the sources), the amplitudes of the waves from S_1 and S_2 differ and the cancellations are only partial. (This is the basis of the OMEGA navigation system.)

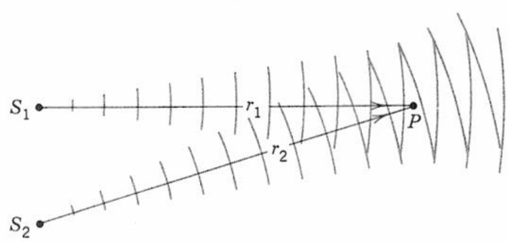

Figure 30 Problem 36.

37. A source S and a detector D of high-frequency waves are a distance d apart on the ground. The direct wave from S is found to be in phase at D with the wave from S that is reflected from a horizontal layer at an altitude H (Fig. 31). The incident and reflected rays make the same angle with the reflecting layer. When the layer rises a distance h, no signal is detected at D. Neglect absorption in the atmosphere and find the relation between d, h, H, and the wavelength λ of the waves.

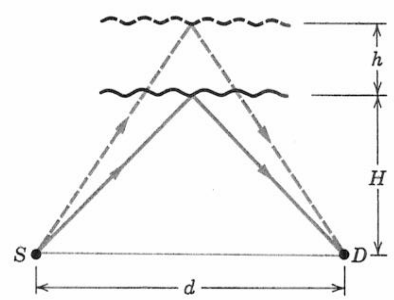

Figure 31 Problems 37 and 38.

38. Refer to Problem 37 and Fig. 31. Suppose that $d = 230$ km and $H = 510$ km. The waves are 13.0 MHz radio waves ($v = 3.00 \times 10^8$ m/s). At the detector D the combined signal strength varies from a maximum to zero and back to a maximum again six times in 1 min. At what vertical speed is the reflecting layer moving? (The layer is moving slowly, so that the vertical distance moved in 1 min is small compared to H and d.)

Section 19-9 Standing Waves

39. A string fixed at both ends is 8.36 m long and has a mass of 122 g. It is subjected to a tension of 96.7 N and set vibrating. (*a*) What is the speed of the waves in the string? (*b*) What is the wavelength of the longest possible standing wave? (*c*) Give the frequency of that wave.

40. A nylon guitar string has a linear mass density of 7.16 g/m and is under a tension of 152 N. The fixed supports are 89.4 cm apart. The string is vibrating in the standing wave pattern shown in Fig. 32. Calculate the (*a*) speed, (*b*) wavelength, and (*c*) frequency of the component waves whose superposition gives rise to this vibration.

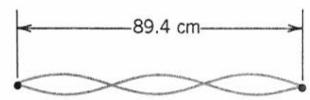

Figure 32 Problem 40.

41. The equation of a transverse wave traveling in a string is given by

$$y = 0.15 \sin (0.79x - 13t),$$

in which x and y are expressed in meters and t is in seconds. (*a*) What is the displacement at $x = 2.3$ m, $t = 0.16$ s? (*b*) Write down the equation of a wave that, when added to the given one, would produce standing waves on the string. (*c*) What is the displacement of the resultant standing wave at $x = 2.3$ m, $t = 0.16$ s?

42. A string vibrates according to the equation

$$y = 0.520 \sin (1.14x) \cos (137t),$$

where x and y are in centimeters and t is in seconds. (*a*) What are the amplitude and speed of the component waves whose superposition can give rise to this vibration? (*b*) Find the distance between nodes. (*c*) What is the velocity of a particle of the string at the position $x = 1.47$ cm at time $t = 1.36$ s?

43. Vibrations from a 622-Hz tuning fork set up standing waves in a string clamped at both ends. The wave speed for the string is 388 m/s. The standing wave has four loops and an amplitude of 1.90 mm. (*a*) What is the length of the string? (*b*) Write an equation for the displacement of the string as a function of position and time.

44. Consider a standing wave that is the sum of two waves traveling in opposite directions but otherwise identical. Show that the maximum kinetic energy in each loop of the standing wave is $2\pi^2\mu y_m^2 v v$.

45. An incident traveling wave, amplitude A_i, is only partially reflected from a boundary, with the amplitude of the reflected wave being A_r. The resulting superposition of two waves with different amplitudes and traveling in opposite

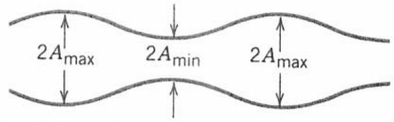

Figure 33 Problems 45 and 46.

directions gives a standing wave pattern of waves whose envelope is shown in Fig. 33. The *standing wave ratio* (SWR) is defined as $(A_i + A_r)/(A_i - A_r) = A_{max}/A_{min}$, and the percent reflection is defined as the ratio of the average power in the reflected wave to the average power in the incident wave, times 100. (*a*) Show that for 100% reflection SWR $= \infty$ and that for no reflection SWR $= 1$. (*b*) Show that a measurement of the SWR just before the boundary reveals the percent reflection occurring at the boundary according to the formula

$$\% \text{ reflection} = [(\text{SWR} - 1)^2/(\text{SWR} + 1)^2](100).$$

46. Estimate (*a*) the SWR (standing wave ratio) and (*b*) the percent reflection at the boundary for the envelope of the standing wave pattern shown in Fig. 33.

47. Two strings of linear mass density μ_1 and μ_2 are knotted together at $x = 0$ and stretched to a tension F. A wave $y = A \sin k_1(x - v_1 t)$ in the string of density μ_1 reaches the junction between the two strings, at which it is partly transmitted into the string of density μ_2 and partly reflected. Call these waves $B \sin k_2(x - v_2 t)$ and $C \sin k_1(x + v_1 t)$, respectively. (*a*) Assuming that $k_2 v_2 = k_1 v_1 = \omega$ and that the displacement of the knot arising from the incident and reflected waves is the same as that arising from the transmitted wave, show that $A = B + C$. (*b*) If it is assumed that both strings near the knot have the same slope (why?)—that is, dy/dx in string 1 $= dy/dx$ in string 2—show that

$$C = A\frac{k_2 - k_1}{k_2 + k_1} = A\frac{v_1 - v_2}{v_1 + v_2}.$$

Under what conditions is C negative?

Section 19-10 Resonance

48. A 15.0-cm violin string, fixed at both ends, is vibrating in its $n = 1$ mode. The speed of waves in this wire is 250 m/s, and the speed of sound in air is 348 m/s. What are (*a*) the frequency and (*b*) the wavelength of the emitted sound wave?

49. What are the three lowest frequencies for standing waves on a wire 9.88 m long having a mass of 0.107 kg, which is stretched under a tension of 236 N?

50. A 1.48-m-long wire has a mass of 8.62 g and is held under a tension of 122 N. The wire is held rigidly at both ends and set into vibration. Calculate (*a*) the speed of waves on the wire, (*b*) the wavelengths of the waves that produce one- and two-loop standing waves on the wire, and (*c*) the frequencies of the waves in (*b*).

51. One end of a 120-cm string is held fixed. The other end is attached to a weightless ring that can slide along a frictionless rod as shown in Fig. 34. What are the three longest possible wavelengths for standing waves in this string? Sketch the corresponding standing waves.

52. A 75.6-cm string is stretched between fixed supports. It is observed to have resonant frequencies of 420 and 315 Hz, and no other resonant frequencies between these two. (*a*) What is the lowest resonant frequency for this string? (*b*) What is the wave speed for this string?

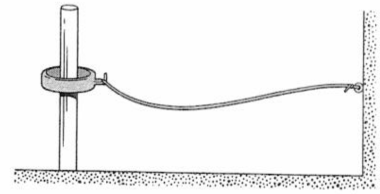

Figure 34 Problem 51.

53. In an experiment on standing waves, a string 92.4 cm long is attached to the prong of an electrically driven tuning fork, which vibrates perpendicular to the length of the string at a frequency of 60.0 Hz. The mass of the string is 44.2 g. How much tension must the string be under (weights are attached to the other end) if it is to vibrate with four loops?

54. An aluminum wire of length $L_1 = 60.0$ cm and of cross-sectional area 1.00×10^{-2} cm^2 is connected to a steel wire of the same cross-sectional area. The compound wire, loaded with a block m of mass 10.0 kg, is arranged as shown in Fig. 35 so that the distance L_2 from the joint to the supporting pulley is 86.6 cm. Transverse waves are set up in the wire by using an external source of variable frequency. (*a*) Find the lowest frequency of excitation for which standing waves are observed such that the joint in the wire is a node. (*b*) What is the total number of nodes observed at this frequency, excluding the two at the ends of the wire? The density of aluminum is 2.60 g/cm^3 and that of steel is 7.80 g/cm^3.

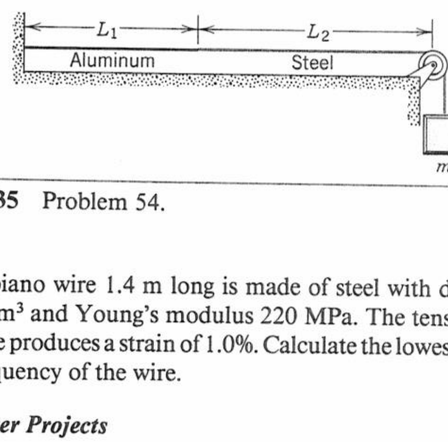

Figure 35 Problem 54.

55. A piano wire 1.4 m long is made of steel with density 7.8 g/cm^3 and Young's modulus 220 MPa. The tension in the wire produces a strain of 1.0%. Calculate the lowest resonant frequency of the wire.

Computer Projects

56. (*a*) Initially a taut string has a shape given by $f_1(x) = 0.02e^{-(x-5)^2/9}$, where f and x are in meters. Suppose the pulse moves with speed $v = 25$ m/s in the positive x direction, so the displacement of the string at coordinate x and time t is given by $y_1(x, t) = f(x - vt) = 0.02e^{-(x-vt-5)^2/9}$. Use a computer program or spreadsheet to plot $y_1(x, t)$ as a function of x from $x = 0$ to $x = 50$ m for $t = 0$, 0.5, 1.0, and 1.5 s. Preferably plot the graphs on a monitor screen and design the program so you can easily change the value of t and replot. Note the position of the pulse maximum on each graph and verify that the graphs depict a pulse that travels in the positive x direction, has a speed of 25 m/s, and moves without change in shape. (*b*) A second pulse has the form $f_2(x) = 0.02e^{-(x-45)^2/9}$ at $t = 0$ and moves in the negative x

direction with a speed of 25 m/s. Use your program to plot $y_2(x, t) = f_2(x + vt)$ from $x = 0$ to $x = 50$ m for $t = 0$, 0.5, 0.8, 1.0, and 1.5 s. Verify that the graphs depict a pulse moving in the negative x direction. (*c*) Suppose both pulses are on the string at the same time. Use your program to plot $y_1(x, t) + y_2(x, t)$ from $x = 0$ to $x = 50$ m for $t = 0, 0.5, 1.0$, and 1.5 s. Verify that the graphs depict the pulses moving toward each other and that when they meet the string displacement is large in the region of overlap. The pulses then move away from each other without change in shape. (*d*) Suppose the second wave has the form $f_2(x) = -0.02e^{-(x-45)^2/9}$ at $t = 0$ and travels in the negative x direction with a speed of 25 m/s. Use your program to plot $y_1(x, t) + y_2(x, t)$ from $x = 0$ to $x = 50$ m for $t = 0, 0.5, 0.8$, 1.0, and 1.5 s. When the two pulses meet, the action of one tends to nullify the action of the other. For one value of the time, the displacement of the string is zero everywhere. The pulses then continue on their ways without change in shape.

57. Waves can be generated on a taut string by moving one end. Suppose the string is extremely long and let $g(t)$ be the displacement of the end being moved, presumed to be at $x = 0$. If the string stretches along the positive x axis, at time t the displacement at the point at x is the same as the displacement at the end but at the earlier time $t - x/v$, where v is the wave speed. Thus the displacement at x is given by $y(x, t) = g(t - x/v)$. (*a*) Suppose that, starting at $t = 0$ and continuing for 0.20 s, the string at $x = 0$ is pulled upward in the positive y direction with a constant speed of 0.15 m/s. It is then held at its final displacement. Thus $g(t) = 0$ for $t < 0$, $g(t) = 0.15t$ for $0 < t < 0.20$ s, and $g(t) = 0.15 \times 0.20 = 0.030$ m for $t > 0.20$ s. Take the wave speed to be 5.0 m/s and use a computer program to make separate graphs of $y(x, t)$ from $x = 0$ to $x = 20$ m for $t = 0$, 0.1, 0.2, 1.0, 2.0, and 3.0 s. To do this, have the computer calculate $u = x - vt$ for each selected value of x, then set $y = 0$ if $u < 0$, set $y = 0.15u$ if $0 < u < 0.20$, and set $y = 0.03$ if $u > 0.20$. (*b*) Take the wave speed to be 15 m/s and plot $y(x, t)$ from $x = 0$ to $x = 20$ m for $t = 0$, 0.1, 0.2, 0.5, 0.75, 1.0, and 1.25 s. (*c*) What determines the slope of the string as the pulse moves along? If the string end is raised at a greater rate does the string slope increase or decrease? If the wave speed is increased does the slope increase or decrease?

58. Starting at time $t = 0$ and continuing for 0.40 s, the end of a taut string is jiggled up and down in simple harmonic motion. Its displacement is given by $g(t) = 0.020 \sin(31.4t)$, where g is in meters and t is in seconds. Use a computer to make separate graphs of the string displacement $y(x, t)$ from $x = 0$ to $x = 20$ m for each of the times $t = 0$, 0.1, 0.2, 0.3, 0.4, 0.5, 1.0, 1.5, 2.0, and 2.5 s. See the previous computer project for some hints.

59. A taut string is initially distorted into the shape given by $f(x) = 0.02e^{-(x-5)^2/9}$, where f and x are in meters. The pulse travels at 5.0 m/s in the positive x direction along the string until it gets to the fixed end at $x = 20$ m, where it is reflected. The displacement of the string is given by $y(x, t) = y_1(x, t) + y_2(x, t)$, where y_1 is the incident pulse and y_2 is the reflected pulse. The incident pulse, of course, is given by $y_1(x, t) = f(x - vt) = 0.02e^{-(x-vt-5)^2/9}$. Show that the reflected pulse is given by $y_2(x, t) = -f(2L - x - vt) = -0.02e^{-(2L-x-vt-5)^2/9}$, where L is the coordinate of the fixed point. This is the only function of $x + vt$ such that $y_1(L, t) + y_2(L, t) = 0$. Use a computer program or spreadsheet to make separate graphs of the string displacement from $x = 0$ to $x = 20$ m for $t = 0$, 1.0, 2.0, 2.5, 2.75, 3.0, 3.25, 3.5, 4.0, and 5.0 s. The function to plot is $y(x, t) = 0.020e^{-(x-vt-5)^2/9} - 0.020e^{-(2L-x-vt-5)^2/9}$.

60. A taut string carrying a wave has energy: kinetic energy because it is moving and potential energy because it is distorted. If μ is the linear mass density, then the kinetic energy in an infinitesimal length dx is given by $\frac{1}{2}\mu(\partial y/\partial t)^2\, dx$. If F is the tension in the string, then the potential energy in an infinitesimal length is given by $\frac{1}{2}F(\partial y/\partial x)^2\, dx$. Since $y(x, t) = f(x \pm vt)$ and $v = \sqrt{F/\mu}$, these two quantities are exactly equal for the same string length. Thus the total mechanical energy in the string from x to $x + \Delta x$ is given by

$$E = \mu \int_x^{x+\Delta x} (\partial y/\partial t)^2\, dx.$$

You can use the numerical integration program described in the computer projects of Chapter 8 to evaluate integrals of this form.

(*a*) The tension in a string with a linear mass density of 0.080 kg/m is 2.0 N. At time $t = 0$ the string is distorted so it has the shape given by $f(x) = 0.02e^{-(x-5)^2/9}$, where f and x are in meters. Assume the pulse moves in the positive x direction. Show that

$$E = (0.04/9)^2 \mu v^2 \int_x^{x+\Delta x} (x - vt - 5)^2 e^{-(x-vt-5)^2/4.5}\, dx.$$

(*b*) Use numerical integration to calculate the total energy in the string segment from $x = 0$ to $x = 20$ m at $t = 1$ s. This segment includes all of the pulse except for the very small tails. Using 200 intervals should produce 4 significant figure accuracy. (*c*) Use numerical integration to calculate the total energy in the string segment from $x = 30$ m to $x = 50$ m at $t = 7$ s. The result should be the same as that of part (*b*) and should indicate to you that the energy has moved from the region around $x = 10$ m to the region around $x = 40$ m. This makes sense because the wave speed is 5.0 m/s and the wave traveled 30 m in the intervening 6 s. (*d*) The rate at which energy passes the point at x is given by $P = -F(\partial y/\partial x)(\partial y/\partial t)$, so in the time interval from t to $t + \Delta t$ the energy passing x is given by

$$E = \int_t^{t+\Delta t} P\, dt = -F \int_t^{t+\Delta t} (\partial y/\partial x)(\partial y/\partial t)\, dt.$$

For the pulse described above show that

$$E = (0.04/9)^2 F v \int_t^{t+\Delta t} (x - vt - 5)^2 e^{-(x-vt-5)^2/4.5}\, dt.$$

Use numerical integration to calculate the energy that passed the point at $x = 25$ m from $t = 1$ s to $t = 7$ s. The result is again the same as before, indicating that all of the energy around $x = 10$ m at $t = 1$ s passed $x = 25$ m on its way to the region around $x = 40$ m.