cisco and event B be its arrival in New York. Is it possible to find two observers who disagree about the time order of these events? Explain.

16. Two observers, one at rest in S and one at rest in S', each carry a meter stick oriented parallel to their relative motion. *Each* observer finds upon measurement that the *other* observer's meter stick is the shorter of the two sticks. Does this seem like a paradox to you? Explain. (*Hint:* Compare with the following situation. Harry waves goodbye to Walter who is in the rear of a station wagon driving away from Harry. Harry says that Walter gets smaller. Walter says that Harry gets smaller. Are they measuring the same thing?)
17. How does the concept of simultaneity enter into the measurement of the length of an object?
18. In relativity the time and space coordinates are intertwined and treated on a more or less equivalent basis. Are time and space fundamentally of the same nature, or is there some essential difference between them that is preserved even in relativity?
19. In the "twin paradox," explain (in terms of heartbeats, physical and mental activities, and so on) why the younger returning twin has not lived any longer than her own proper time even though her stay-at-home brother may say that she has. Hence explain the remark: "You age according to your own proper time."
20. Can we simply substitute γm for m in classical equations to obtain the correct relativistic equations? Give examples.
21. If zero-mass particles have a speed c in one reference frame, can they be found at rest in any other frame? Can such particles have any speed other than c?
22. A particle with zero mass (a neutrino, possibly) can transport momentum. But, by Eq. 23, $\mathbf{p} = m\mathbf{v}/\sqrt{1 - v^2/c^2}$, the momentum is directly proportional to the mass and therefore should be zero if the mass is zero. Explain.
23. How many relativistic expressions can you think of in which the Lorentz factor γ enters as a simple multiplier?
24. Is the mass of a stable, composite particle (a gold nucleus, for example) greater than, equal to, or less than the sum of the masses of its constituents? Explain.
25. "The mass of the electron is 0.511 MeV." Exactly what does this statement mean?
26. "The relation $E_0 = mc^2$ is essential to the operation of a power plant based on nuclear fission but has only a negligible relevance for a fossil-fuel plant." Is this a true statement? Explain why or why not.
27. A hydroelectric plant generates electricity because water falls under gravity through a turbine, thereby turning the shaft of a generator. According to the mass–energy concept, must the appearance of energy (the electricity) be identified with a mass decrease somewhere? If so, where?
28. Some say that relativity complicates things. Give examples to the contrary, wherein relativity simplifies matters.

PROBLEMS

Section 21-3 Consequences of Einstein's Postulates

1. Quite apart from effects due to the Earth's rotational and orbital motions, a laboratory frame is not strictly an inertial frame because a particle placed at rest there will not, in general, remain at rest; it will fall under gravity. Often, however, events happen so quickly that we can ignore free fall and treat the frame as inertial. Consider, for example, a 1.0-MeV electron (for which $v = 0.941c$) projected horizontally into a laboratory test chamber and moving through a distance of 20 cm. (*a*) How long would it take, and (*b*) how far would the electron fall during this interval? What can you conclude about the suitability of the laboratory as an inertial frame in this case?
2. A 100-MeV electron, for which $v = 0.999987c$, moves along the axis of an evacuated tube that has a length of 2.86 m as measured by a laboratory observer S with respect to whom the tube is at rest. An observer S' moving with the electron, however, would see this tube moving past with speed v. What length would this observer measure for the tube?
3. A rod lies parallel to the x axis of reference frame S, moving along this axis at a speed of $0.632c$. Its rest length is 1.68 m. What will be its measured length in frame S?
4. The mean lifetime of muons stopped in a lead block in the laboratory is measured to be 2.20 μs. The mean lifetime of high-speed muons in a burst of cosmic rays observed from the Earth is measured to be 16.0 μs. Find the speed of these cosmic ray muons.
5. An unstable high-energy particle enters a detector and leaves a track 1.05 mm long before it decays. Its speed relative to the detector was $0.992c$. What is its proper lifetime? That is, how long would it have lasted before decay had it been at rest with respect to the detector?
6. The length of a spaceship is measured to be exactly half its rest length. (*a*) What is the speed of the spaceship relative to the observer's frame? (*b*) By what factor do the spaceship's clocks run slow, compared to clocks in the observer's frame?
7. A particle moves along the x' axis of frame S' with a speed of $0.413c$. Frame S' moves with a speed of $0.587c$ with respect to frame S. What is the measured speed of the particle in frame S?
8. Frame S' moves relative to frame S at $0.620c$ in the direction of increasing x. In frame S' a particle is measured to have a velocity of $0.470c$ in the direction of increasing x'. (*a*) What is the velocity of the particle with respect to frame S? (*b*) What would be the velocity of the particle with respect to S if it moved (at $0.470c$) in the direction of *decreasing* x' in

the S' frame? In each case, compare your answers with the predictions of the classical velocity transformation equation.

9. A spaceship of rest length 130 m drifts past a timing station at a speed of $0.740c$. (*a*) What is the length of the spaceship as measured by the timing station? (*b*) What time interval between the passage of the front and back end of the ship will the station monitor record?

10. A pion is created in the higher reaches of the Earth's atmosphere when an incoming high-energy cosmic-ray particle collides with an atomic nucleus. A pion so formed descends toward Earth with a speed of $0.99c$. In a reference frame in which they are at rest, pions have a lifetime of 26 ns. As measured in a frame fixed with respect to the Earth, how far will such a typical pion move through the atmosphere before it decays?

11. To circle the Earth in low orbit a satellite must have a speed of about 7.91 km/s. Suppose that two such satellites orbit the Earth in opposite directions. (*a*) What is their relative speed as they pass? Evaluate using the classical Galilean velocity transformation equation. (*b*) What fractional error was made because the (correct) relativistic transformation equation was not used?

Section 21-4 The Lorentz Transformation

12. What must be the value of the speed parameter β if the Lorentz factor γ is to be (*a*) 1.01? (*b*) 10.0? (*c*) 100? (*d*) 1000?

13. Find the speed parameter of a particle that takes 2 years longer than light to travel a distance of 6.0 ly.

14. Observer S assigns to an event the coordinates $x = 100$ km, $t = 200\ \mu$s. Find the coordinates of this event in frame S', which moves in the direction of increasing x with speed $0.950c$. Assume $x = x'$ at $t = t' = 0$.

15. Observer S reports that an event occurred on the x axis at $x = 3.20 \times 10^8$ m at a time $t = 2.50$ s. (*a*) Observer S' is moving in the direction of increasing x at a speed of $0.380c$. What coordinates would S' report for the event? (*b*) What coordinates would S'' report if S'' were moving in the direction of *decreasing* x at this same speed?

16. Inertial frame S' moves at a speed of $0.60c$ with respect to frame S in the direction of increasing x. In frame S, event 1 occurs at the origin at $t = 0$ and event 2 occurs on the x axis at $x = 3.0$ km and at $t = 4.0\ \mu$s. What times of occurrence does observer S' record for these same events? Explain the reversal of the time order.

17. An experimenter arranges to trigger two flashbulbs simultaneously, a blue flash located at the origin of his reference frame and a red flash at $x = 30.4$ km. A second observer, moving at a speed of $0.247c$ in the direction of increasing x, also views the flashes. (*a*) What time interval between them does she find? (*b*) Which flash does she say occurs first?

18. Derive Eqs. 17 for the inverse Lorentz transformation by algebraically inverting the equations for the Lorentz transformation, Eqs. 14.

Section 21-6 The Transformation of Velocities

19. Suppose observer S fires a light beam in the y direction ($v_x = 0$, $v_y = c$). Observer S' is moving at speed u in the x direction. (*a*) Find the components v'_x and v'_y of the velocity of the light beam according to S', and (*b*) show that S' measures a speed of c for the light beam.

20. One cosmic-ray proton approaches the Earth along its axis with a velocity of $0.787c$ toward the North Pole and another, with velocity $0.612c$, toward the South Pole. See Fig. 23. Find the relative speed of approach of one particle with respect to the other. (*Hint:* It is useful to consider the Earth and one of the particles as the two inertial reference frames.)

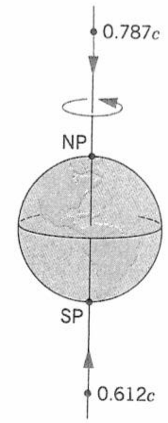

Figure 23 Problem 20.

21. Galaxy A is reported to be receding from us with a speed of $0.347c$. Galaxy B, located in precisely the opposite direction, is also found to be receding from us at this same speed. What recessional speed would an observer on galaxy A find (*a*) for our galaxy and (*b*) for galaxy B?

22. It is concluded from measurements of the red shift of the emitted light that quasar Q_1 is moving away from us at a speed of $0.788c$. Quasar Q_2, which lies in the same direction in space but is closer to us, is moving away from us at speed $0.413c$. What velocity for Q_2 would be measured by an observer on Q_1?

23. A spaceship, at rest in a certain reference frame S, is given a speed increment of $0.500c$. It is then given a further $0.500c$ increment in this new frame, and this process is continued until its speed with respect to its original frame S exceeds $0.999c$. How many increments does it require?

24. A radioactive nucleus moves with a constant speed of $0.240c$ along the x axis of a reference frame S fixed with respect to the laboratory. It decays by emitting an electron whose speed, measured in a reference frame S' moving with the nucleus, is $0.780c$. Consider first the cases in which the emitted electron travels (*a*) along the common xx' axis and (*b*) along the y' axis and find, for each case, its velocity (magnitude and direction) as measured in frame S. (*c*) Suppose, however, that the emitted electron, viewed now from frame S, travels along the y axis of that frame with a speed of $0.780c$. What is its velocity (magnitude and direction) as measured in frame S'?

25. In Fig. 24, A and B are trains on perpendicular tracks, shown radiating from station S. The velocities are in the station frame (S frame). (*a*) Find $\mathbf{v}_{AB}$, the velocity of train B with respect to train A. (*b*) Find $\mathbf{v}_{BA}$, the velocity of train A with

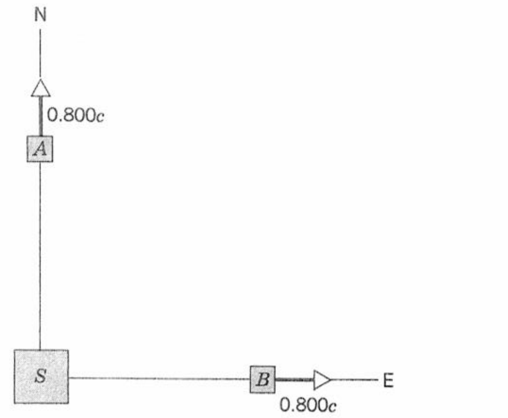

Figure 24 Problem 25.

respect to train B. (c) Comment on the fact that these two relative velocities do not point in opposite directions.

Section 21-7 Consequences of the Lorentz Transformation

26. An electron is moving at a speed such that it could circumnavigate the Earth at the equator in 1 s. (a) What is its speed, in terms of the speed of light? (b) Its kinetic energy K? (c) What percent error do you make if you use the classical formula to calculate K?

27. The rest radius of the Earth is 6370 km and its orbital speed about the Sun is 29.8 km/s. By how much would the Earth's diameter appear to be shortened to an observer stationed so as to be able to watch the Earth move past at this speed?

28. An airplane whose rest length is 42.4 m is moving with respect to the Earth at a constant speed of 522 m/s. (a) By what fraction of its rest length will it appear to be shortened to an observer on Earth? (b) How long would it take by Earth clocks for the airplane's clock to fall behind by 1 μs? (Assume that only special relativity applies.)

29. A spaceship whose rest length is 358 m has a speed of $0.728c$ with respect to a certain reference frame. A micrometeorite, with a speed of $0.817c$ in this frame, passes the spaceship on an antiparallel track. How long does it take this micrometeorite to pass the spaceship?

30. A clock moves along the x axis at a speed of $0.622c$ and reads zero as it passes the origin. (a) Calculate the Lorentz factor. (b) What time does the clock read as it passes $x = 183$ m?

31. An observer S sees a flash of red light 1210 m away and a flash of blue light 730 m closer and on the same straight line. S measures the time interval between the occurrence of the flashes to be 4.96 μs, the red flash occurring first. (a) Find the relative velocity, magnitude and direction, of a second observer S' who would record these flashes as occurring at the same place. (b) From the point of view of S', which flash occurs first and what is the measured time interval between the flashes?

32. Consider the previous problem. Suppose now that observer S sees the two flashes in the same positions as in that problem but occurring closer together in time. How close together in time can they be and still have it possible to find a frame S' in which they occur at the same place?

33. A space traveler takes off from Earth and moves at speed $0.988c$ toward the star Vega, which is 26.0 ly distant. How much time will have elapsed by Earth clocks (a) when the traveler reaches Vega and (b) when the Earth observers receive word from him that he has arrived? (c) How much older will the Earth observers calculate the traveler to be when he reaches Vega than he was when he started the trip?

34. You wish to make a round trip from Earth in a spaceship, traveling at constant speed in a straight line for 6 months and then returning at the same constant speed. You wish further, on your return, to find the Earth as it will be 1000 years in the future. (a) How fast must you travel? (b) Does it matter whether or not you travel in a straight line on your journey? If, for example, you traveled in a circle for 1 year, would you still find that 1000 years had elapsed by Earth clocks when you returned?

35. Observers S and S' stand at the origins of their respective frames, which are moving relative to each other with a speed $0.600c$. Each has a standard clock, which, as usual, they set to zero when the two origins coincide. Observer S keeps the S' clock visually in sight. (a) What time will the S' clock record when the S clock records 5.00 μs? (b) What time will observer S *actually read* on the S' clock when the S clock reads 5.00 μs?

36. (a) Can a person, in principle, travel from Earth to the galactic center (which is about 23,000 ly distant) in a normal lifetime? Explain, using either time-dilation or length-contraction arguments. (b) What constant speed would be needed to make the trip in 30 y (proper time)?

Section 21-8 Relativistic Momentum

37. Show that 1 kg·m/s $= 1.875 \times 10^{21}$ MeV/c.

38. A particle has a momentum equal to mc. Calculate its speed.

39. Calculate the speed parameter of a particle with a momentum of 12.5 MeV/c if the particle is (a) an electron and (b) a proton.

Section 21-9 Relativistic Energy

40. Find the speed parameter β and the Lorentz factor γ for an electron whose kinetic energy is (a) 1.0 keV, (b) 1.0 MeV, and (c) 1.0 GeV.

41. Find the speed parameter β and the Lorentz factor γ for a particle whose kinetic energy is 10 MeV if the particle is (a) an electron, (b) a proton, and (c) an alpha particle.

42. A particle has a speed of $0.990c$ in a laboratory reference frame. What are its kinetic energy, its total energy, and its momentum if the particle is (a) a proton or (b) an electron?

43. Quasars are thought to be the nuclei of active galaxies in the early stages of their formation. A typical quasar radiates energy at the rate of 1.20×10^{41} W. At what rate is the mass of this quasar being reduced to supply this energy? Express your answer in solar mass units per year, where one solar mass unit (smu) is the mass of our Sun.

44. Calculate the speed of a particle (a) whose kinetic energy is equal to twice its rest energy and (b) whose total energy is equal to twice its rest energy.

45. Find the momentum of a particle of mass m in order that its total energy be three times its rest energy.

46. Use the velocities given in Fig. 19 in the S' frame and show that, according to S', the kinetic energies before and after the collision, computed classically, are given by Eqs. 26.

47. Reconsider the collision shown in Fig. 19. Using Eq. 27 for the relativistic kinetic energy, calculate the initial and final kinetic energies in frame S' and thereby show that kinetic energy is conserved in this frame as in frame S.

48. Consider the following, all moving in free space: a 2.0-eV photon, a 0.40-MeV electron, and a 10-MeV proton. (*a*) Which is moving the fastest? (*b*) The slowest? (*c*) Which has the greatest momentum? (*d*) The least? (*Note:* A photon is a light-particle of zero mass.)

49. (*a*) If the kinetic energy K and the momentum p of a particle can be measured, it should be possible to find its mass m and thus identify the particle. Show that

$$m = \frac{(pc)^2 - K^2}{2Kc^2}.$$

(*b*) What does this expression reduce to as $v/c \to 0$, in which v is the speed of the particle? (*c*) Find the mass of a particle whose kinetic energy is 55.0 MeV and whose momentum is 121 MeV/c; express your answer in terms of the mass m_e of the electron.

50. In a high-energy collision of a primary cosmic-ray particle near the top of the Earth's atmosphere, 120 km above sea level, a pion is created with a total energy of 135 GeV, traveling vertically downward. In its proper frame this pion decays 35.0 ns after its creation. At what altitude above sea level does the decay occur? The rest energy of a pion is 139.6 MeV.

51. How much work must be done to increase the speed of an electron from (*a*) $0.18c$ to $0.19c$ and (*b*) $0.98c$ to $0.99c$? Note that the speed increase ($= 0.01c$) is the same in each case.

52. Two identical particles, each of mass 1.30 mg, moving with equal but opposite velocities of $0.580c$ in the laboratory reference frame, collide and stick together. Find the mass of the resulting particle.

53. A particle of mass m traveling at a relativistic speed makes a completely inelastic collision with an identical particle that is initially at rest. Find (*a*) the speed of the resulting single particle and (*b*) its mass. Express your answers in terms of the Lorentz factor γ of the incident particle.

54. (*a*) Suppose we have a particle accelerated from rest by the action of a force F. Assuming that Newton's second law for a particle, $F = dp/dt$, is valid in relativity, show that the final kinetic energy K can be written, using the work–energy theorem, as $K = \int v\, dp$. (*b*) Using Eq. 23 for the relativistic momentum, show that carrying out the integration in (*a*) leads to Eq. 27 for the relativistic kinetic energy.

55. (*a*) In modern experimental high-energy physics, energetic particles are made to circulate in opposite directions in so-called storage rings and permitted to collide head-on. In this arrangement each particle has the same kinetic energy K in the laboratory. The collisions may be viewed as totally inelastic, in that the rest energy of the two colliding particles, plus all available kinetic energy, can be used to generate new particles and to endow them with kinetic energy. Show that the available energy in this arrangement can be written in the form

$$E_{\text{new}} = 2mc^2\left(1 + \frac{K}{mc^2}\right)$$

where m is the mass of the colliding particles. (*b*) How much energy is made available when 100-GeV protons are used in this fashion? (*c*) What proton energy would be required to make 100 GeV available? (*Note:* Compare your answers with those in Problem 56, which describes another—less energy-effective—bombarding arrangement.)

56. (*a*) A proton, mass m, accelerated in a proton synchrotron to a kinetic energy K strikes a second (target) proton at rest in the laboratory. The collision is entirely inelastic in that the rest energy of the two protons, plus all the kinetic energy consistent with the law of conservation of momentum, is available to generate new particles and to endow them with kinetic energy. Show that the energy available for this purpose is given by

$$E_{\text{new}} = 2mc^2\sqrt{1 + \left(\frac{K}{2mc^2}\right)}.$$

(*b*) How much energy is made available when 100-GeV protons are used in this fashion? (*c*) What proton energy would be required to make 100 GeV available? (*Note:* Compare with Problem 55.)

57. (*a*) Consider the decay of the kaon described in Sample Problem 8, but use a frame of reference (the center-of-mass frame) in which the kaons are initially at rest. Show that the two pions emitted in the decay travel in opposite directions with equal speeds of $0.827c$. (*b*) What is the velocity of the original kaons as observed in the laboratory frame? (*c*) Assume the two pions are emitted in the center-of-mass frame with velocities of $v'_x = +0.827c$ and $v'_x = -0.827c$. By calculating the corresponding velocities in the laboratory frame, show that the kinetic energies in the laboratory frame are identical with those found in the solution to Sample Problem 8.

58. An alpha particle with kinetic energy 7.70 MeV strikes a ^{14}N nucleus at rest. An ^{17}O nucleus and a proton are produced, the proton emitted at 90° to the direction of the incident alpha particle and carrying kinetic energy 4.44 MeV. The rest energies of the various particles are: alpha particle, 3730.4 MeV; ^{14}N, 13,051 MeV; proton, 939.29 MeV; ^{17}O, 15,843 MeV. (*a*) Find the kinetic energy of the ^{17}O nucleus. (*b*) At what angle with respect to the direction of the incident alpha particle does the ^{17}O nucleus move?