20. List examples of the Brownian motion in physical phenomena.

21. Would Brownian motion occur in gravity-free space?

22. A golf ball is suspended from the ceiling by a long thread. Explain in detail why its Brownian motion is not readily apparent.

23. We have defined ρ_n to be the number of molecules per unit volume in a gas. If we define ρ_n for a very small volume in a gas, say, one equal to 10 times the volume of an atom, then ρ_n fluctuates with time through the range of values zero to some maximum value. How then can we justify a statement that ρ_n has a definite value at every point in a gas?

PROBLEMS

Section 24-1 Statistical Distributions and Mean Values

1. Suppose you have a collection of N objects. The number of ways you can select n of them, if the order isn't important, is $N!/n!\,(N-n)!$. Using this formula, derive Eq. 5. (*Hint*: Divide a pack of cards into 13 hearts and 39 other cards. In how many ways can you select h hearts from the 13? In how many ways can you select $13-h$ cards from the 39 non-hearts? How many ways are there to select *any* 13 cards from the deck?)

Section 24-2 Mean Free Path

2. Substitute Eq. 10 into Eq. 12 and evaluate the integrals to obtain Eq. 13.

3. The denominator of Eq. 12 is equal to the total number of particles N. Use this to show that the constant A in Eq. 14 is given by $A = N/\lambda$.

4. At 2500 km above the Earth's surface the density is about 1.0 molecule/cm^3. (*a*) What mean free path is predicted by Eq. 16 and (*b*) what is its significance under these conditions? Assume a molecular diameter of 2.0×10^{-8} cm.

5. At standard temperature and pressure (0°C and 1.00 atm) the mean free path in helium gas is 285 nm. Determine (*a*) the number of molecules per cubic meter and (*b*) the effective diameter of the helium atoms.

6. In a certain particle accelerator the protons travel around a circular path of diameter 23.5 m in a chamber of 1.10×10^{-6} mm Hg pressure and 295 K temperature. (*a*) Calculate the number of gas molecules per cubic meter at this pressure. (*b*) What is the mean free path of the gas molecules under these conditions if the molecular diameter is 2.20×10^{-8} cm?

7. The mean free path λ of the molecules of a gas may be determined from measurements (for example, from measurement of the viscosity of the gas). At 20.0°C and 75.0 cm Hg pressure such measurements yield values of λ_{Ar} (argon) $= 9.90 \times 10^{-6}$ cm and λ_{N_2} (nitrogen) $= 27.5 \times 10^{-6}$ cm. (*a*) Find the ratio of the effective cross-section diameters of argon to nitrogen. (*b*) What would the value be of the mean free path of argon at 20.0°C and 15.0 cm Hg? (*c*) What would the value be of the mean free path of argon at -40.0°C and 75.0 cm Hg?

8. Calculate the mean free path for 35 spherical jelly beans in a jar that is vigorously shaken. The volume of the jar is 1.0 L and the diameter of a jelly bean is 1.0 cm.

9. At what frequency would the wavelength of sound be on the order of the mean free path in nitrogen at 1.02 atm pressure and 18.0°C? Take the diameter of the nitrogen molecule to be 315 pm. See Problem 42 of Chapter 23.

10. (*a*) Find the probability that a molecule will travel a distance at least equal to the mean free path before its next collision. (*b*) After what distance of travel since the last collision is the probability of having suffered the next collision equal to $\frac{1}{2}$? Give the answer in terms of the mean free path.

11. In Sample Problem 3, at what temperature is the average rate of collision equal to $6.0 \times 10^9\ \text{s}^{-1}$? The pressure remains unchanged.

Section 24-3 The Distribution of Molecular Speeds

12. The speeds of a group of ten molecules are 2, 3, 4, . . . , 11 km/s. (*a*) Find the average speed of the group. (*b*) Calculate the root-mean-square speed of the group.

13. (*a*) Ten particles are moving with the following speeds: four at 200 m/s, two at 500 m/s, and four at 600 m/s. Calculate the average and root-mean-square speeds. Is $v_{rms} > \bar{v}$? (*b*) Make up your own speed distribution for the 10 particles and show that $v_{rms} \geq \bar{v}$ for your distribution. (*c*) Under what condition (if any) does $v_{rms} = \bar{v}$?

14. You are given the following group of particles (n_i represents the number of particles that have a speed v_i):

n_i	v_i (km/s)
2	1
4	2
6	3
8	4
2	5

(*a*) Compute the average speed $\bar{v}$. (*b*) Compute the root-mean-square speed v_{rms}. (*c*) Among the five speeds shown, which is the most probable speed v_p for the entire group?

15. In the apparatus of Miller and Kusch (see Fig. 12) the length L of the rotating cylinder is 20.4 cm and the angle ϕ is 0.0841 rad. What rotational speed corresponds to a selected speed v of 212 m/s?

16. It is found that the most probable speed of molecules in a gas at temperature T_2 is the same as the rms speed of the molecules in this gas when its temperature is T_1. Calculate T_2/T_1.

17. Two containers are at the same temperature. The first contains gas at pressure p_1 whose molecules have mass m_1 with a root-mean-square speed v_{rms1}. The second contains molecules of mass m_2 at pressure $2p_1$ that have an average speed $\bar{v}_2 = 2v_{rms1}$. Find the ratio m_1/m_2 of the masses of their molecules.

18. A gas, not necessarily in thermal equilibrium, consists of N particles. The speed distribution is not necessarily Maxwellian. (*a*) Show that $v_{rms} \geq \bar{v}$ regardless of the distribution of speeds. (*b*) When would the equality hold?

19. Show that, for atoms of mass m emerging as a beam from a

small opening in an oven of temperature T, the most probable speed is $v_p = \sqrt{3kT/m}$.

20. An atom of germanium (diameter = 246 pm) escapes from a furnace ($T = 4220$ K) with the root-mean-square speed into a chamber containing atoms of cold argon (diameter = 300 pm) at a density of 4.13×10^{19} atoms/cm³. (*a*) What is the speed of the germanium atom? (*b*) If the germanium atom and an argon atom collide, what is the closest distance between their centers, considering each as spherical? (*c*) Find the initial collision frequency experienced by the germanium atom.

21. For the hypothetical gas speed distribution of N particles shown in Fig. 22 [$n(v) = Cv^2$, $0 < v < v_0$; $n(v) = 0$, $v > v_0$], find (*a*) an expression for C in terms of N and v_0, (*b*) the average speed of the particles, and (*c*) the rms speed of the particles.

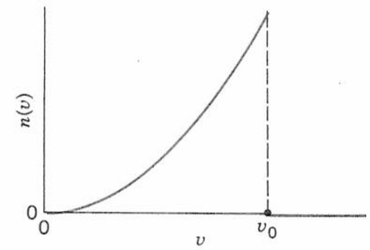

Figure 22 Problem 21.

22. A hypothetical gas of N particles has the speed distribution shown in Fig. 23 [$n(v) = 0$ for $v > 2v_0$]. (*a*) Express a in terms of N and v_0. (*b*) How many of the particles have speeds between $1.50v_0$ and $2.00v_0$? (*c*) Express the average speed of the particles in terms of v_0. (*d*) Find v_{rms}.

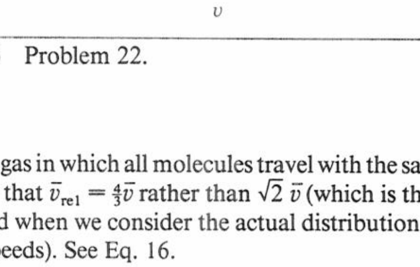

Figure 23 Problem 22.

23. For a gas in which all molecules travel with the same speed $\bar{v}$, show that $\bar{v}_{rel} = \frac{4}{3}\bar{v}$ rather than $\sqrt{2}\,\bar{v}$ (which is the result obtained when we consider the actual distribution of molecular speeds). See Eq. 16.

Section 24-4 The Distribution of Energies

24. Calculate the fraction of particles in a gas moving with translational kinetic energy between $0.01kT$ and $0.03kT$. (*Hint*: For $E \ll kT$, the term $e^{-E/kT}$ in Eq. 27 can be replaced with $1 - E/kT$. Why?)

25. (*a*) Find E_{rms} using the energy distribution of Eq. 27. (*b*) Why is $E_{rms} \neq \frac{1}{2}mv_{rms}^2$, where v_{rms} is given by Eq. 22?

26. Find the fraction of particles in a gas having translational kinetic energies within a range $0.02kT$ centered on the most probable energy E_p. (*Hint*: In this region, $n(E) \approx$ constant. Why?)

Section 24-5 Brownian Motion

27. The root-mean-square speed of hydrogen molecules at 0°C is 1840 m/s. Compute the speed of colloidal particles of "molar mass" 3.2×10^6 g/mol.

28. Particles of mass 6.2×10^{-14} g are suspended in a liquid at 26°C and are observed to have a root-mean-square speed of 1.4 cm/s. Calculate Avogadro's number from the equipartition theorem and these data.

29. Calculate the root-mean-square speed of smoke particles of mass 5.2×10^{-14} g in air at 14°C and 1.07 atm pressure.

30. A delicate spring balance has a force constant of 0.44 N/m. It vibrates randomly due to the bombardment by molecules in the air on the supported object (thermal fluctuations). (*a*) Find the rms displacement of the object from its equilibrium position at 31°C. (*Hint*: Take the average energy of the random motion to be kT. Why?) (*b*) Determine the uncertainty in a weight determination due to this effect.

31. Very small solid particles, called grains, exist in interstellar space. They are continually bombarded by hydrogen atoms of the surrounding interstellar gas. As a result of these collisions, the grains execute Brownian movement in both translation and rotation. Assume the grains are uniform spheres of diameter 4.0×10^{-6} cm and density 1.0 g/cm³, and that the temperature of the gas is 100 K. Find (*a*) the root-mean-square speed of the grains between collisions and (*b*) the approximate rate (rev/s) at which the grains are spinning.

32. Colloidal particles in solution are buoyed up by the liquid in which they are suspended. Let ρ' be the density of liquid and ρ the density of the particles. If V is the volume of a particle, show that the number of particles per unit volume in the liquid varies with height as

$$n = n_0 \exp\left[-\frac{N_A}{RT} V(\rho - \rho')gh\right],$$

where n_0 is the number density of particles at height $h = 0$. This equation was tested by Perrin in his Brownian motion studies.

Section 24-6 Quantum Statistical Distributions

33. In how many ways can three (*a*) distinguishable classical particles and (*b*) indistinguishable bosons be distributed in two states?

34. (*a*) At what energy E is $f_{FD}(E) = \frac{1}{2}$? (*b*) What is the value of $f_{BE}(E)$ at the energy found in (*a*)?

Computer Project

35. Many computers have a random number generator, which you can use to "deal" a hand of 13 cards. For example, by assigning each card a numerical value from 1 through 52 and then by generating a random number in that interval, you can "deal" a card at random from a deck of 52. Using this technique, deal a hand of 13 cards and determine the number of hearts. Repeat 1000 times and draw a histogram similar to Fig. 2. Compare your results with the predictions of Eq. 5.