HRK 18.4

Since power is defined as $\frac{dW}{dt}$ we want to know how much work the pump is doing (W_{pump}). We already know from the work-energy theorem that $W_{Total} = \Delta K$. In deriving Bernoulli's equation we saw there were two forces contributing to the total work, the pressure force and the force of gravity. In this problem the pressure force is provided by the pump so we have

$$W_{Total} = W_{pump} + W_{grav} = \Delta K \Rightarrow W_{pump} = -W_{grav} + \Delta K$$

This is just a restatement of $\Delta K + \Delta U = W_{pump}$ which appears in the textbook after the derivation of Bernoulli's equation. The units of power are $J.s^{-1}$ so we need to figure out how much energy the pump transmits to the water per second.

The water in the basement is effectively moving with speed $0m/s$ and we are told that it exits the hose at $5.30m/s$. The mass of water leaving the hose every second is

$$\Delta m = \rho A v = 10^3 \pi (.0097)^2 5.30 = 1.566kg$$

so the increase in kinetic energy per second is

$$\Delta K/s : \qquad \frac{1}{2}(\rho A v)v^2 = \frac{1}{2}1.566(5.30)^2 = 22J.s^{-1}$$

Since the water comes out of the hose $2.90m$ higher than where it entered, it gains potential energy to the tune of

$$\Delta U/s : \qquad \Delta m g(y_{out} - y_{in}) = 1.566(9.81)(2.90) = 44.55J.s^{-1}$$

Putting the two energy terms together we see

$$\text{Power} = \frac{\Delta K + \Delta U}{\Delta t} = 66.55 \text{ Watts}$$

HRK 18.10

One of the key is results of the chapter is that, when ρ is constant (incompressible fluid), the volume flow rate is constant

$$\text{Volume flow rate: } R = A_1 v_1 = A_2 v_2$$

This is all we need to know to solve part (a).

18.10 (a)

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ (4.2 \times 10^{-4})(5.18) &= (7.60 \times 10^{-4})(v_2) \\ \Rightarrow v_2 &= 2.86 \ m/s \end{aligned}$$

18.10 (b)

Here we need to use Bernoulli's equation to solve for p_B.

$$\begin{aligned} p_a + \frac{1}{2}\rho v_a^2 + \rho g y_a &= p_b + \frac{1}{2}\rho v_b^2 + \rho g y_b \\ p_a + \frac{1}{2}\rho v_a^2 &= p_b + \frac{1}{2}\rho v_b^2 + \rho g(y_b - y_a) \\ (152 \times 10^3) + \frac{1}{2}10^3(5.18)^2 &= p_b + \frac{1}{2}10^3(2.86)^2 + 10^3(9.81)(-9.66) \\ p_b &= 256.1 \times 10^3 = 256.1 \ kPa \end{aligned}$$

HRK 18.11

We will prove Torricelli's Law in problem 18.15 but for now we will just use the result which states: the speed, v of the stream of water coming out, is related to the depth below the water's surface, h, where the hole is positioned via

$$v = \sqrt{2gh}$$

18.11 (a)

The mass flux is the same for both holes. The volume δV of liquid, moving with speed v emerging from a hole of area A in a time δt is given by

$$\delta V = Av\delta t.$$

Furthermore, the mass δm emerging is related to the volume emerging via

$$\begin{aligned} \delta m &= \rho\delta V \\ \Rightarrow \delta m &= \rho Av\delta t \end{aligned}$$

Overall, we have the result that mass flux $\frac{dm}{dt} = \rho Av$ so equating this quantity for both tanks we get

$$\text{Mass flux: } \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

The set-up of the question tells us that $A_2 = 2A_1$, so let us use that piece of information and also Toricelli's Law (which is independent of density and area)

$$\begin{aligned} \rho_1 A_1 v_1 &= \rho_2 A_2 v_2 \\ \rho_1 A_1 \sqrt{2gh} &= \rho_2 2A_1 \sqrt{2gh} \\ \frac{\rho_1}{\rho_2} &= 2 \end{aligned}$$

18.11 (b)

The volume flow rate is defined as $R = Av$ so

$$\frac{R_1}{R_2} = \frac{A_1 v_1}{A_2 v_2} = \frac{A_1\sqrt{2gh}}{2A_1\sqrt{2gh}} = \frac{1}{2}$$

18.11 (c)

We want to make $R_1 = R_2$ by setting the holes at different heights in the tanks. Let the appropriate height for tank two be denoted h', then

$$\begin{aligned} R_1 &= R_2 \\ A_1\sqrt{2gh} &= 2A_1\sqrt{2gh'} \\ \sqrt{2gh} &= \sqrt{2g(4h')} \\ \Rightarrow h' &= \frac{h}{4} \end{aligned}$$

HRK 18.15

18.15 (a)

We use Bernoulli's equation for the vertical streamline $1 \rightarrow 2$ and the horizontal streamline $2 \rightarrow 3$. Intuitively you know that he water level in the tank is dropping very slowly, and using the conservation of volume flow between points 1 and 3 we have

$$\begin{aligned} v_1 A_1 &= v_3 A_3 \\ \frac{v_1}{v_3} &= \frac{A_3}{A_1}. \end{aligned}$$

We know from the setup of the question that $\frac{A_3}{A_1} \approx 0$ which tells us $v_1 \approx 0$. The same reasoning leads us to $v_2 \approx 0$ since $A_2 = A_1$.

$$\begin{aligned}
\underline{1 \to 2:} \qquad p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \\
p_0 + \frac{1}{2}\rho (0)^2 + \rho g(0) &= p_2 + \frac{1}{2}\rho (0)^2 + \rho g(-h) \\
p_2 &= p_0 + \rho g h
\end{aligned}$$

$$\begin{aligned}
\underline{2 \to 3:} \qquad p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 &= p_3 + \frac{1}{2}\rho v_3^2 + \rho g y_3 \\
p_2 + \frac{1}{2}\rho (0)^2 + \rho g y_2 &= p_0 + \frac{1}{2}\rho v_3^2 + \rho g y_2 \qquad \text{(since } y_3 = y_2\text{)} \\
p_2 &= p_0 + \frac{1}{2}\rho v_3^2
\end{aligned}$$

Now let us combine our two expressions for p_2 to find an expression for the "speed of efflux" v_3

$$\begin{aligned}
p_0 + \rho g h &= p_0 + \frac{1}{2}\rho v_3^2 \\
g h &= \frac{1}{2} v_3^2 \\
v_3 &= \sqrt{2gh}
\end{aligned}$$

This is a very useful result! If you recognize this expression from elsewhere it is because this is exactly the speed acquired by a projectile dropped from rest from a height h and accelerated by the force of gravity (ignoring friction etc.). It is not too surprising since Bernoulli's equation was derived assuming energy conservation (ignoring friction etc.).

18.15 (b)

Use energy considerations to figure out how high the water can stream. We can think of this like a 1-d projectile problem from Chapter 4; the water has zero potential energy on leaving the spout (point a) and all of its energy is kinetic. At its highest point (point b) the water will have zero kinetic energy and all potential energy.

$$\begin{aligned}
\Delta K + \Delta U &= 0 \\
\frac{1}{2}\Delta m (v_b^2 - v_a^2) + \Delta m g (y_b - y_a) &= 0 \\
\frac{1}{2}(v_b^2 - v_a^2) &= -g(y_b - y_a) \\
\frac{1}{2}(0^2 - (\sqrt{2gh})^2) &= -g(y_b - y_a) \\
\frac{1}{2}(2h) &= (y_b - y_a)
\end{aligned}$$

This tells us the maximum height the water stream can reach is h - the original height of the water surface! The connection with projectile motion appears again. This height is exactly what could be reached by a projectile hitting the ground at $v = \sqrt{2gh}$ and rebounding up without losing any energy (i.e. a perfectly elastic collision).

18.15 (c)

If we allow the fluid to be viscous or turbulent then friction-like forces enter the picture, and mechanical energy is not conserved: $\Delta K + \Delta U < 0$ so the stream would not reach as high

$$(y_b - y_a) < h$$

HRK 18.16

Again, this is a Torricelli type set-up so we will use his Law.

18.16 (a)

Similar to problem 18.15 we can treat this like a projectile question (e.g. throwing a ball off a cliff). The vertical distance the water covers in falling is $(h - H)$ and since it shoots sideways out of the tank, the initial component v_{0y} of velocity in the y direction is zero.

$$\begin{aligned} y_f - y_i &= v_{0y}t + \frac{1}{2}a_y t^2 \\ h - H &= 0t + \frac{1}{2}(-g)t^2 \\ t^2 &= \frac{2(H-h)}{g} \end{aligned}$$

See what horizontal distance the water has travelled in this time t, bearing in mind there is nothing accelerating the water in the x direction

$$\begin{aligned} x_f - x_i &= v_{0x}t + \frac{1}{2}a_x t^2 \\ x_f - x_i &= \sqrt{2gh}t + \frac{1}{2}(0)t^2 \\ x_f - x_i &= \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} \\ x_f - x_i &= 2\sqrt{h(H-h)} \end{aligned}$$

18.16 (b)

We want to see if there exist another height h' such that the analysis performed above would give the same range i.e.

$$x_f - x_i = 2\sqrt{h'(H-h')}$$

If you equate the range for both holes

$$2\sqrt{h(H-h)} = 2\sqrt{h'(H-h')}$$

then you can see that as well as the obvious solution $h' = h$ there also exists a solution

$$h' = H - h$$

By rearranging you can get a feel for what this means physically. Since $h + h' = H$ then

$$\frac{h}{H} + \frac{h'}{H} = 1$$

so that e.g. water emerging from a hole 1/3 of the way down any tank has the same horizontal range as water emerging from a hole 2/3 of the way down the same tank.

18.16 (c)

We want to optimize (maximize) $x_f - x_i$ with respect to h. Using the expression from part (a) let us take the derivative and set equal to zero

$$\begin{aligned} x_f - x_i &= 2\sqrt{h(H-h)} \\ \rightarrow \frac{d}{dh}(2\sqrt{h(H-h)}) &= 0 \\ \frac{-2h+H}{h(H-h)} &= 0 \end{aligned}$$

The final line is satisfied for $H = 2h$ or $h = \frac{H}{2}$. The range for $h = \frac{H}{2}$ is

$$\text{Max Range: } x_f - x_i = 2\sqrt{h(H-h)} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} = H$$

You can take the second derivative of the range to make sure this is a maximum not some other critical point

$$\begin{aligned}
x_f - x_i &= 2\sqrt{h(H-h)} \\
\frac{d^2}{dh^2}(2\sqrt{h(H-h)})\Big|_{h=\frac{H}{2}} &< 0? \\
\left(-\frac{(H-2h)^2}{2(h(H-h))^{\frac{3}{2}}} - \frac{2}{\sqrt{h(H-h)}}\right)\Bigg|_{h=\frac{H}{2}} &< 0? \\
-\frac{4}{H} &< 0
\end{aligned}$$

so it is a maximum.

HRK 18.20

The pressure in the water at a depth of 6.15m is $p_0 + \rho g(6.15) = 161.33kPa$. If this depth corresponds to the middle of plug, then it is very slightly less than this above the center of the plug and very slightly greater than this below the center. We could integrate over the height of the plug but that is not necessary – you can satisfy yourself that taking the pressure at the midpoint of the plug works, because pressure is linear in depth.

18.20 (a)

Anyway this pressure acts on an area $\pi r_p^2 = (3.14)(.0215)^2 = 1.452 \times 10^{-3}$ so that

$$F_{out} = (p_0 + \rho g(6.15))A = 234.29N$$

and the force pointing in, which must balance, is due to atmospheric pressure and friction

$$F_{in} = p_0 A + F_{fric} = 146.67 + F_{fric}$$

So solving $F_{out} = F_{in}$ for F_{fric} we get

$$F_{fric} = 87.6\ N$$

18.20 (b)

This could be made more complicated by assuming a small reservoir, in which case the water level would drop as water flowed out. Such an example is done for the orange juice question 18.23. Here we will assume the water level stays constant so that the pressure at a depth of $6.15m$ stays constant. We can use Torricelli's Law to find the speed of efflux of the water

$$v = \sqrt{2gh} = \sqrt{2g(6.15)} = 10.985\ m/s$$

The volume flow rate (in m^3/sec) is given by $R = Av$

$$R = Av = (1.452 \times 10^{-3}) * 10.985 = 1.595 \times 10^{-2}\ m^3/s$$

So, finally in 3 hours, the volume that comes out is

$$\text{Volume} = Av\Delta t = (1.595 \times 10^{-2})(3 \times 60 \times 60) = 172.26\ m^3$$

HRK 18.21

We will repeatedly use Bernoulli's eqn here. We will use the surface of the water in the container as a reference point S and ignore reference point A given to us in the picture. Note that since the area of the container is much greater than the area of the inside of the tube, we can say that the velocity of water on the surface is effectively zero, $v_s \approx 0$. Note also that the speed of the liquid travelling through the tube will be constant because of conservation of mass ($\rho A v$ is constant inside tube).

18.21 (a)

We want to solve for v_c in the following

$$\begin{aligned}
p_s + \frac{1}{2}\rho v_s^2 + \rho g y_s &= p_c + \frac{1}{2}\rho v_c^2 + \rho g y_c \\
p_s + \frac{1}{2}\rho (0)^2 + \rho g (y_s - y_c) &= p_c + \frac{1}{2}\rho v_c^2 \\
p_0 + \rho g (y_s - y_c) &= p_0 + \frac{1}{2}\rho v_c^2 \\
v_c^2 &= \frac{2}{\rho}(\rho g (y_s - y_c)) \\
v_c^2 &= 2g(d + h_2) \\
v_c &= \sqrt{2g(d + h_2)}
\end{aligned}$$

18.21 (b)

We want to solve for p_b in the following

$$\begin{aligned}
p_s + \frac{1}{2}\rho v_s^2 + \rho g y_s &= p_b + \frac{1}{2}\rho v_b^2 + \rho g y_b \\
p_0 + \frac{1}{2}\rho (0)^2 + \rho g (y_s - y_b) &= p_b + \frac{1}{2}\rho v_b^2 \\
p_0 + \rho g(-h_1) &= p_b + \frac{1}{2}\rho(2gd + 2gh_2) \qquad \text{since } v_b = v_c \\
p_b &= p_0 - \rho g h_1 - \rho g d - \rho g h_2
\end{aligned}$$

18.21 (c)

The siphon will stop working if the pressure at the top of the tube, p_b, falls below the vapor pressure of the liquid. When this happens the liquid at the top will become gaseous and the resulting bubbles will ruin the operation of the siphon. Assuming this liquid has a low vapor pressure $p_{vap} \approx 0$ this amounts to finding when $p_b \approx 0$. We will use the result from part (b) and find out for what h_1 we get $p_b = 0$.

$$p_b = 0 \quad = \quad p_0 - \rho g h_1 - \rho g d - \rho g h_2$$

Part (a) told us that if the siphon is to output water then we must have $(d + h_2) > 0$. Set this sum to be very small $\epsilon = (d + h_2) \gtrsim 0$ so that just a trickle is coming out at point c. Now look at the expression for p_b again.

$$p_b = 0 \quad = \quad p_0 - \rho g h_1 - \rho g \epsilon$$

Now ignore the very small ϵ term and solve for h_1

$$h_1 = \frac{p_0}{\rho g} = \frac{1.01 \times 10^5}{(10^3)(9.81)} = 10.29\ m$$

Note: the vapor pressure for water at room temperature is actually about $(.023)p_0$. If you use this value, you will get a slightly smaller (and more realistic) value for the maximum possible h_1.

HRK 18.23

18.23 (a)

We will let the jug have cross-sectional area A, with an initial juice height H (inital volume of juice= AH) . Picture the first glass of juice escaping from the tap at the bottom of the jug. The juice level in the jug falls by a distance of $\frac{H}{15}$. The volume missing from the top of the jug is equal to the volume output in 12 seconds by the tap. We know that at the instant we begin to pour the first drop of juice, the speed of the juice stream is $v = \sqrt{2gH}$ by Torricelli's Law. However as the juice level drops to $h < H$ the speed of the juice stream also drops to $v(h) = \sqrt{2gh}$. We need to set up a differential equation to capture this relationship

The volume of juice lost per unit time from the top of the jug is

$$\frac{d\text{Vol}}{dt} = \frac{d(Ah)}{dt} = A\frac{dh}{dt}$$

Let us assign a cross-sectional area a to the hole that forms the tap. The volume flux leaving through the tap is

$$\frac{d\text{Vol}}{dt} = -av = -av(h)$$

We know that these two expressions must be equal by conservation of volume (the orange juice's denity ρ does not change), so

$$A\frac{dh}{dt} = -av(h)$$

Cancelling and rearranging we get

$$\begin{aligned}
\frac{dh}{dt} &= -\frac{a}{A}v(h) \\
\frac{dh}{dt} &= -\frac{a}{A}\sqrt{2gh} \\
\frac{dh}{\sqrt{h}} &= -\frac{a\sqrt{2g}}{A}dt
\end{aligned}$$

Our ultimate goal is to get h as a function of t, i.e. $h(t)$, and then solve for $h = 0$. You could perform the indefinite integral, get a constant of integration, and find this constant of integration using the conditions given in the problem. Equivalently you can perform the definite integral over the period of draining the first glass of juice. Either way, we get around the fact that we don't know A or a.

First we will rewrite the constant term $\frac{a\sqrt{2g}}{A}$ as simply C.

$$\begin{aligned}
\int \frac{dh}{\sqrt{h}} &= -\int \frac{a\sqrt{2g}}{A}dt \\
\int \frac{dh}{\sqrt{h}} &= -C\int dt
\end{aligned}$$

Now let us use the 12-seconds-to-fill-one-glass piece of information

$$\begin{aligned}
\int_H^{\frac{14H}{15}} \frac{dh}{\sqrt{h}} &= -C\int_0^{12} dt \\
2\sqrt{h}\Big|_H^{\frac{14H}{15}} &= -Ct\Big|_0^{12} \\
2\sqrt{\frac{14H}{15}} - 2\sqrt{H} &= -12C \\
C &= \frac{\sqrt{H}}{6}\left(1 - \sqrt{\frac{14}{15}}\right)
\end{aligned}$$

Now that we know C, we can answer the question we were asked – let's call the time to drain completely T:

$$\begin{aligned}
\int_H^0 \frac{dh}{\sqrt{h}} &= -C\int_0^T dt \\
2\sqrt{h}|_H^0 &= -Ct|_0^T \\
2\sqrt{0} - 2\sqrt{H} &= -CT \\
T &= \frac{2\sqrt{H}}{\frac{\sqrt{H}}{6}\left(1-\sqrt{\frac{14}{15}}\right)} \\
T &= \frac{12}{1-\sqrt{\frac{14}{15}}} = 353.89 \text{ seconds}
\end{aligned}$$

The time to drain the jug after one glass has already been removed is $T - 12 = 341.89$ seconds.

HRK 18.41

18.41 (a)

For the set-up containing two flat planes separated by a viscous fluid, one of which is moving at v with respect to the other stationary plane, the defining equation for the force on the moving plane is

$$F = \eta A \frac{dv}{dy}$$

Here we are in a cylindrical geometry so the velocity gradient varies with r instead of y. Also note that the area of fluid in contact with the cylinder of radius r is $2\pi rL$. Putting these two observations into the equation for η we get

$$\Rightarrow F = \eta(2\pi rL)\left|\frac{dv}{dr}\right|$$

And finally $F = -\eta(2\pi rL)\frac{dv}{dr}$ since $\frac{dv}{dr}$ is negative (because velocity drops with increasing r).

18.41 (b)

The diagram accompanying the question shows that a pressure $p + \Delta p$ is pushing the cylinder from behind, and an opposing pressure of p is pushing the front of the cylinder back. Consequently the net pressure force is ΔpA where A is the area of the cylinder of fluid with radius r

$$F' = \Delta pA = \Delta p\pi r^2$$

18.41 (c)

Since the cylinder of fluid of radius r is not accelerating we must have $F = F'$.

$$-\Delta p\pi r^2 = \eta(2\pi rL)\frac{dv}{dr}$$

We want to solve this differential equation to get v in terms of r.

$$\begin{aligned}
-\frac{\Delta p\pi r^2}{\eta(2\pi rL)}dr &= dv \\
-\int \frac{\Delta pr}{\eta(2L)}dr &= \int dv
\end{aligned}$$

$$
\begin{aligned}
-\frac{\Delta p}{\eta(2L)} \int_R^r r\ dr &= \int_0^v dv \\
\frac{\Delta p}{\eta(2L)} \int_r^R r\ dr &= \int_0^v dv \\
\frac{\Delta p}{\eta(2L)} \frac{1}{2} r^2 \Big|_r^R &= v \\
\frac{\Delta p}{\eta(4L)} (R^2 - r^2) &= v \\
\frac{\Delta p R^2}{\eta(4L)} \left(1 - \frac{r^2}{R^2}\right) &= v
\end{aligned}
$$

Note how the term inside the bracket behaves for different values of r. For $r = R$ (outer walls) the velocity is zero, while the velocity is maximized for $r = 0$ (at the center of the tube).

HRK 18.42

18.42 (a)

The mass flux through an annular ring is $\frac{dm}{dt} = \rho A v$ where we have to be careful that both A and v depend on r. We need to find the area dA in a thin circular ring. One way to picture this is to imagine a circle of circumference $2\pi r$. Now, in the region between this and an infinitesimally larger circle (with radius $r + dr$), we have an area $dA = 2\pi r dr$. To get the flux through a finite region we have to integrate over r.

$$
\begin{aligned}
\text{Mass flux through tube:} \quad &= \rho \int_0^R v(r) dA \\
&= \rho \int_0^R (2\pi r) \frac{\Delta p}{\eta(4L)} (R^2 - r^2) dr \\
&= \frac{\rho \pi \Delta p}{\eta(2L)} \int_0^R (r)(R^2 - r^2) dr \\
&= \frac{\rho \pi \Delta p}{\eta(2L)} \int_0^R r R^2 - r^3 dr \\
&= \frac{\rho \pi \Delta p}{\eta(2L)} \left[\frac{r^2}{2} R^2 - \frac{r^4}{4}\right]_0^R \\
&= \frac{\rho \pi \Delta p}{\eta(2L)} \left(\frac{R^4}{2} - \frac{R^4}{4}\right) \\
&= \frac{\rho \pi \Delta p R^4}{\eta(8L)}
\end{aligned}
$$

HRK 18.43

For this problem we need to know the difference between the pressures inside and outside a soap bubble. This is problem 17.55 and I'll sketch the proof here: For a spherical soap bubble there are two surfaces which have surface tension (the inner and outer surface of the bubble). Recall that the force F parallel to the surface, divided by the length of line L over which it acts gives you the surface tension

$$\gamma = \frac{F}{L}$$

The way to approach the soap bubble is consider half of it first (i.e. a hollow hemisphere). Newton's second law says that the forces on this hemisphere must balance, in particular along the direction normal to the plane of the hemisphere (call this the $\hat{x}$ direction).

$$\begin{aligned}
(pA)_x &= (p_{in} - p_{out})\pi r^2 \\
(F^{tension})_x &= -2\gamma(2\pi r) \\
(pA)_x + (F^{tension})_x &= 0 \\
(p_{in} - p_{out})\pi r^2 &= 2\gamma(2\pi r) \\
(p_{in} - p_{out}) &= \frac{4\gamma}{r}
\end{aligned}$$

In this derivation the quantity $F_x^{tension} = -2\gamma(2\pi r)$ arises from two surfaces each providing a force $\gamma(2\pi r)$ in the opposite direction to $(pA)_x$. For a spherical droplet of water, we could do the same analysis but now there is only one surface acting as a boundary between air and water. This is the reason why $(p_{in} - p_{out})$ for a droplet is exactly half that of a soap bubble (of the same size).

Armed with this we can start problem 18.43. We need an expression for the volume of a thin spherical shell in 3 dimensions dVol (this is like the annular ring ring we saw before but one dimension higher). Imagine a sphere of radius r, which consequently has a surface area of $4\pi r^2$. The volume contained between a sphere of radius r and an infinitesimally larger sphere with radius $r + dr$ is given by $d\text{Vol} = 4\pi r^2 dr$. Recall we just showed above that $\Delta p = \frac{4\gamma}{r}$. If we write Poiseuille's law

$$\begin{aligned}
\frac{dm}{dt} &= \frac{\rho\pi R^4 \Delta p}{\eta(8L)} \\
\rho\frac{d\text{Vol}}{dt} &= \frac{\rho\pi R^4 \Delta p}{\eta(8L)} \\
\rho\frac{4\pi r^2 dr}{dt} &= \frac{\rho\pi R^4 \Delta p}{\eta(8L)} \\
\rho\frac{4\pi r^2 dr}{dt} &= \frac{\rho\pi R^4}{\eta(8L)}\frac{4\gamma}{r}
\end{aligned}$$

Cancel, rearrange and prepare to integrate

$$\begin{aligned}
\frac{\eta(8L)}{\gamma R^4} r^3 dr &= dt \\
\frac{\eta(8L)}{\gamma R^4}\int_{r_{init}}^{r_{final}} r^3 dr &= \int_0^T dt \\
\frac{\eta(8L)}{\gamma R^4}\frac{r^4}{4}\Big]_{r_{init}}^{r_{final}} &= T \\
\frac{\eta(8L)}{4\gamma R^4}\left((3.82\times10^{-2})^4 - (2.16\times10^{-2})^4\right) &= T \\
\frac{(1.8\times10^{-5})(2)(0.112)}{(2.5\times10^{-2})(\frac{1.08}{2}\times10^{-3})^4}\left((3.82\times10^{-2})^4 - (2.16\times10^{-2})^4\right) &= T \\
\Rightarrow T = 3626s &
\end{aligned}$$