## HRK 26.7

Summarizing the information given in the question

$$W_A = 5W_B$$
$$Q_A^{IN} = 3Q_B^{IN}$$
$$Q_A^{OUT} = 2Q_B^{OUT}$$

One way of doing this is as follows.

Use 
$$e_A = \frac{|W_A|}{|Q_A^{IN}|} = \frac{|Q_A^{IN}| - |Q_A^{OUT}|}{|Q_A^{IN}|}$$
  
and  $e_B = \frac{|W_B|}{|Q_B^{IN}|} = \frac{|Q_B^{IN}| - |Q_B^{OUT}|}{|Q_B^{IN}|}$ 

Find the ratio  $e_A/e_B$  and then, in turn,  $e_A$  and  $e_B$ .

$$\begin{array}{rcl} \displaystyle \frac{e_A}{e_B} & = & \displaystyle \frac{|W_A|}{|Q_A^{IN}|} \displaystyle \frac{|Q_B^{IN}|}{|W_B|} \\ \displaystyle \frac{e_A}{e_B} & = & \displaystyle \frac{5|W_B|}{3|Q_B^{IN}|} \displaystyle \frac{|Q_B^{IN}|}{|W_B|} \\ \displaystyle \frac{e_A}{e_B} & = & \displaystyle \frac{5}{3} \end{array}$$

$$\begin{aligned} \frac{5}{3} &= \frac{e_A}{e_B} \\ \frac{5}{3} &= \frac{|Q_A^{IN}| - |Q_A^{OUT}|}{|Q_A^{IN}|} \frac{|Q_B^{IN}|}{|Q_B^{IN}| - |Q_B^{OUT}|} \\ \frac{5}{3} &= \frac{3|Q_B^{IN}| - 2|Q_B^{OUT}|}{3|Q_B^{IN}|} \frac{|Q_B^{IN}|}{|Q_B^{IN}| - |Q_B^{OUT}|} \\ \Rightarrow 5(|Q_B^{IN}| - |Q_B^{OUT}|) &= 3|Q_B^{IN}| - 2|Q_B^{OUT}| \\ 2|Q_B^{IN}| &= 3|Q_B^{OUT}| \\ \Rightarrow e_B &= 1 - \frac{|Q_B^{OUT}|}{|Q_B^{IN}|} \\ \Rightarrow e_B &= 1 - \frac{1}{3} \end{aligned}$$

Finally

$$e_A = \frac{5}{3}e_B$$
$$e_A = \frac{5}{9}$$

### HRK 26.17

26.17(a)

An engine has efficiency

$$e = \frac{|W|}{|Q_{IN}|} = \frac{|Q_{IN}| - |Q_{OUT}|}{|Q_{IN}|}$$

and for a Carnot engine this is equal to

$$e = \frac{T_H - T_L}{T_H}$$

because

$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L}$$

for a Carnot engine regardless of the working substance. The Carnot efficiency is the best possible efficiency for any engine using a hot reservoir at temperature  $T_H$  and a cool reservoir at  $T_L$ . Since we know the temperatures we immediately have

$$e = \frac{322 - 258}{322}$$
  
$$e = 0.199$$

Since  $|Q_{IN}| - |Q_{OUT}| = W$  we can easily find the work done using our knowledge of the efficiency and the heat entering the engine from the hot reservoir

$$e = \frac{|W|}{|Q_{IN}|}$$
  

$$\Rightarrow |W| = (0.199)(568 J)$$
  

$$|W| = 112.9 J$$

#### **26.17(b)**

A refrigerator has coefficient of performance

$$K = \frac{|Q_{IN}|}{|W|} = \frac{|Q_{IN}|}{|Q_{OUT}| - |Q_{IN}|}$$

where heat is entering the system from the cold reservoir and being expelled at the hot reservoir. For a Carnot refrigerator the K value is as high as possible and equals

$$K = \frac{T_L}{T_H - T_L}$$
$$K = \frac{258}{322 - 258}$$
$$K = 4.03$$

The work supplied to a refrigerator is  $|W| = |Q_{OUT}| - |Q_{IN}|$ .

Again, we know the coefficient of performance and  $|Q_{IN}|$  so we have

$$K = \frac{|Q_{IN}|}{|W|}$$
  

$$\Rightarrow |W| = \frac{|Q_{IN}|}{K}$$
  

$$|W| = \frac{1230}{4.03}$$
  

$$|W| = 305.1 J$$

## HRK 26.22

26.22(a)



With reference to the pV diagram of this cycle we see that the intermediate step is superfluous. Heat is expelled as we traverse an isotherm in the direction of decreasing volume. Conversely, heat is absorbed as we traverse an isotherm in the direction of increasing volume. The net effect of expelling and then absorbing the same amount of heat in an isothermal process is to do nothing! (Obviously in reality it would be worse than doing nothing as this would introduce losses in a real (not Carnot) engine). We know that the efficiency of a Carnot engine is related to the temperature of the hot and cool reservoirs

$$e = \frac{T_H - T_L}{T_H}$$
$$\Rightarrow e = \frac{T_1 - T_3}{T_1}$$

#### 26.22(b)

This is just an example of the engine described in (a). The maximum efficiency, which it certainly will not reach, is

$$e = \frac{(469 + 273) - (37.8 + 273)}{(469 + 273)}$$
  
$$\Rightarrow e = 0.58$$

# HRK 26.31

**26.3**1(a)



Section 1 
$$\Delta S = \int_{a}^{b} \frac{dQ}{T}$$
  
 $\Delta S = \frac{1}{T_{H}} \int_{a}^{b} dQ$  (isothermal)  
 $\Delta S = \frac{Q_{H}}{T_{H}}$   
also  $\Delta T = 0$  (isothermal)

Section 2 
$$\Delta S = \int_{b}^{c} \frac{dQ}{T}$$
  
 $\Delta S = \int_{b}^{c} \frac{0}{T}$  (adiabatic)  
 $\Delta S = 0$   
also  $\Delta T = (T_{H} - T_{L})$  (isotherm H  $\rightsquigarrow$  isotherm L)

Section 3 
$$\Delta S = \int_{c}^{d} \frac{dQ}{T}$$
  
 $\Delta S = \frac{1}{T_{L}} \int_{c}^{d} dQ$  (isothermal)  
 $\Delta S = -\frac{|Q_{L}|}{T_{L}} = \frac{|Q_{H}|}{T_{H}}$   
also  $\Delta T = 0$  (isothermal)

Section 4 
$$\Delta S = \int_{d}^{a} \frac{dQ}{T}$$
  
 $\Delta S = \int_{d}^{a} \frac{0}{T}$  (adiabatic)  
 $\Delta S = 0$   
also  $\Delta T = -(T_H - T_L)$  (isotherm L  $\rightsquigarrow$  isotherm H)

### 26.31(b)

Using

$$|Q_H| = T_H \Delta S$$

we get

$$|Q_H| = (400 \ K)(0.5 \ J/K)$$
  
 $|Q_H| = 200 \ J$ 

## **26.31(c)**

Using

$$\begin{aligned} |Q_L| &= -T_L \Delta S \\ |Q_L| &= -(250 \ K)(-0.5 \ J/K) = 125 \ J \end{aligned}$$

and

$$|W| = |Q_H| - |Q_L|$$

we get

$$|W| = 200 - 125 = 75 J$$

and since the system does useful work on the environment. the work done on the system is

$$W = -75 \ J$$

As a check the work done by a Carnot engine (see 26.17(a)) is

$$W = \left(\frac{T_H - T_L}{T_H}\right) |Q_{IN}|$$
$$W = \left(\frac{400 - 250}{400}\right) 200$$
$$W = 75 J \text{ (on environment, so negative this on system)}$$

# HRK 26.33

Use

$$dQ = nC_v dT$$

in the normal expression for change in entropy for a reversible process

$$\Delta S = \int_{i}^{f} \frac{dQ}{T}$$

$$\Rightarrow \Delta S = \int_{T=5}^{T=10} \frac{nC_{v}dT}{T}$$

$$\Delta S = \int_{T=5}^{T=10} \frac{nAT^{3}dT}{T}$$

$$\Delta S = \int_{T=5}^{T=10} nAT^{2}dT$$

$$\Delta S = \frac{nAT^{3}}{3}\Big]_{5}^{10}$$

$$\Delta S = \frac{(4.8 \text{ mol}) (3.15 \times 10^{-5} \text{ J/(mol } K^{4}))}{3} (10^{3}K^{3} - 5^{3}K^{3})$$

$$\Delta S = .0441 \text{ J/K}$$

### HRK 26.40

#### 26.40(a)

Using the ideal gas law

$$p_1V_1 = nRT_1$$
  

$$p_2(3V_1) = nRT_2 \qquad (T_2 = T_1 \text{ since isothermal})$$
  

$$\Rightarrow p_2 = \frac{p_1}{3}$$

Find  $p_3$  using the adiabatic process connecting point 1 and 3.

$$p_1 V_1^{\gamma} = p_3 V_3^{\gamma} \quad \text{(where } \gamma = \frac{7}{5} \text{ for diatomic gas)}$$
  

$$\Rightarrow p_3 = p_1 \left(\frac{V_1}{V_3}\right)^{\frac{7}{5}}$$
  

$$p_3 = p_1 \left(\frac{1}{3}\right)^{\frac{7}{5}}$$
  

$$p_3 = (0.215)p_1$$

We can do something similar to relate  ${\cal T}_3$  to  ${\cal T}_1$ 

$$\begin{aligned} T_1 V_1^{\frac{2}{5}} &= T_3 V_3^{\frac{2}{5}} & \text{(using } pV = nRT \text{ and } p_1 V_1^{\gamma} = p_3 V_3^{\gamma} \text{ to get } T_1 V_1^{\gamma-1} = T_3 V_3^{\gamma-1} \text{)} \\ \Rightarrow T_3 &= T_1 \left(\frac{V_1}{V_3}\right)^{\frac{2}{5}} \\ T_3 &= (0.644)T_1 \end{aligned}$$

**26.40**(b)

 $\textbf{Isothermal}\ 1 \rightsquigarrow 2$ 

$$W_{12} = -nRT_1 \ln\left(\frac{3V_1}{V_1}\right) = -nRT_1 \ln(3)$$

The - sign indicates the gas did work on the piston.

Since 
$$\Delta E_{int}^{(12)} = W_{12} + Q_{12}$$
 (where  $\Delta E_{int}^{(12)} = 0$  since  $\Delta T = 0$ )  
 $\Rightarrow Q_{12} = nRT_1 \ln (3)$ 

which is heat that flowed *into the gas*. Also

$$\Delta S_{12} = \int_{1}^{2} \frac{dQ}{T}$$

$$\Delta S_{12} = \frac{1}{T_{1}} \int_{1}^{2} dQ$$

$$\Delta S_{12} = \frac{Q_{12}}{T_{1}}$$

$$\Delta S_{12} = \frac{nRT_{1}\ln(3)}{T_{1}}$$

$$\Delta S_{12} = nR\ln(3)$$

$$\Delta S_{12} = 1.1R$$

Const. Vol  $2 \rightsquigarrow 3$ 

$$W_{23} = 0 \quad (\text{constant volume process})$$
  

$$\Rightarrow \Delta E_{int}^{(23)} = Q_{23}$$
  

$$\Delta E_{int}^{(23)} = \frac{5}{2}nR(T_3 - T_1) \qquad C_v = \frac{5}{2}R \text{ for an ideal diatomic gas}$$
  

$$\Delta E_{int}^{(23)} = \frac{5}{2}nR(-0.356)T_1$$
  

$$\Delta E_{int}^{(23)} = -0.89RT_1$$

 $\operatorname{Also}$ 

$$\Delta S_{23} = \int_{2}^{3} \frac{dQ}{T}$$

$$\Delta S_{23} = \int_{2}^{3} \frac{nC_{v}dT}{T}$$

$$\Delta S_{23} = nC_{v} \int_{2}^{3} \frac{dT}{T}$$

$$\Delta S_{23} = \frac{5}{2}nR \ln\left(\frac{T_{3}}{T_{2}}\right)$$

$$\Delta S_{23} = \frac{5}{2}nR \ln\left(\frac{(0.644)T_{1}}{T_{1}}\right)$$

$$\Delta S_{23} = -1.1R$$

Adiabatic  $3 \rightsquigarrow 1$ 

$$\begin{split} W_{31} &= \frac{1}{\gamma - 1} (p_f V_f - p_i V_i) \\ W_{31} &= \frac{5}{2} (p_1 V_1 - p_3 V_3) \\ W_{31} &= \frac{5}{2} (p_1 V_1 - (0.215) p_1 (3V_1)) \\ W_{31} &= 0.89 p_1 V_1 = 0.89 RT_1 \end{split}$$

$$\Delta E_{int}^{(31)} = W_{31} + Q_{31}$$
  
but  $Q_{31} = 0$  adiabatic process  
 $\Delta E_{int}^{(31)} = 0.89 p_1 V_1 = 0.89 R T_1$ 

 $\operatorname{Also}$ 

$$\Delta S_{31} = \int_{3}^{1} \frac{dQ}{T}$$
  
$$\Delta S_{31} = \int_{3}^{1} \frac{0}{T} \quad \text{adiabatic process}$$
  
$$\Delta S_{31} = 0$$

Of course  $\Delta E$  and  $\Delta S$  over a complete cycle should be zero and this checks out.

### HRK 26.41



To minimize confusion we recall that the sign convention is such that heat flowing into the system is positive as is work done on the system.

26.41(a)

Path segment I(i): 
$$\Delta E_{int} = 0$$
  
 $\Rightarrow Q = -W$   
 $Q = p_0 V_0 \ln \left(\frac{2V_0}{V_0}\right)$  isothermal process  
Path segment I(ii):  $\Delta E_{int} = Q$  const. vol. process  
 $Q = \frac{3}{2}nR(4T_0 - T_0) \quad C_v \frac{3}{2}R$   
 $Q = \frac{9}{2}nRT_0 = \frac{9}{2}p_0V_0$ 

Path segment II(i): 
$$\Delta E_{int} = 0$$
  
 $\Rightarrow Q = -W$   
 $Q = -p_0V_0 \ln(2)$  isothermal process  
Path segment II(ii):  $Q = nC_p\Delta T$   
 $Q = \frac{5}{2}R(4T_0 - T_0)$   $C_p = \frac{5}{2}R$  for a monatomic ideal gas  
 $Q = \frac{15}{2}RT_0 = \frac{15}{2}p_0V_0$ 

**26.41(b)** 

Path segment I(i): 
$$W = -p_0 V_0 \ln \left(\frac{2V_0}{V_0}\right) = -p_0 V_0 \ln (2)$$
 isothermal process  
Path segment I(ii):  $W = 0$  const. vol. process

Path segment II(i): 
$$W = -p_0 V_0 \ln \left(\frac{V_0/2}{V_0}\right) = p_0 V_0 \ln(2)$$
 isothermal process  
Path segment II(ii):  $W = p\Delta V$   
 $W = (2p_0)(2V_0 - \frac{1}{2}V_0)$   
 $W = (2p_0)\frac{3}{2}V_0 = 3p_0V_0$ 

26.41(c)

Path segment I(i): 
$$\Delta E_{int} = 0$$
 isothermal process  
Path segment I(ii):  $\Delta E_{int} = nC_v\Delta T$   
 $\Delta E_{int} = \frac{3}{2}R(4T_0 - T_0) = \frac{9}{2}RT_0 = \frac{9}{2}p_0V_0$ 

Path segment II(i): 
$$\Delta E_{int} = 0$$
 isothermal process  
Path segment II(ii):  $\Delta E_{int} = \frac{3}{2}R(4T_0 - T_0) = \frac{9}{2}RT_0 = \frac{9}{2}p_0V_0$ 

26.41(d)

Path segment I(i): 
$$\Delta S = \frac{Q_{I(i)}}{T_0}$$
 isothermal process  
 $\Delta S = \frac{p_0 V_0 \ln(2)}{T_0} = R \ln(2)$   
Path segment I(ii):  $\Delta S = \int \frac{nC_v dT}{T}$   
 $\Delta S = nC_v \ln\left(\frac{4V_0}{V_0}\right)$   
 $\Delta S = nC_v \ln(4) = \frac{3}{2}R(2\ln(2)) = 3R\ln(2)$ 

So total  $\Delta S$  along path I is  $4R \ln(2)$ .

Path segment II(i): 
$$\Delta S = \frac{Q_{II(i)}}{T_0} \text{ isothermal process}$$
$$\Delta S = -\frac{p_0 V_0 \ln(2)}{T_0} = -R \ln(2)$$
Path segment II(ii): 
$$\Delta S = \int \frac{nC_p dT}{T}$$
$$\Delta S = nC_p \ln\left(\frac{4V_0}{V_0}\right)$$
$$\Delta S = nC_p \ln(4) = \frac{5}{2}R(2\ln(2)) = 5R \ln(2)$$

So total  $\Delta S$  along path II is also  $4R \ln(2)$  as it must be since path I and path II have the same start and end points.

#### HRK 26.42

This is an example of irreversible heat transfer. For an object with mass m and specific heat capacity c we have

$$\Delta S = \int \frac{dQ}{T}$$
$$\Delta S = \int \frac{mcdT}{T}$$
$$\Delta S = mc \int \frac{dT}{T}$$
$$\Delta S = mc \ln\left(\frac{T_f}{T_i}\right)$$

#### 26.42(a)

The amount of heat lost by the copper block,  $Q_c$ , must equal the amount gained by the lead block  $Q_l$ 

$$m_c c_c (400 - T_{eqm}) = m_l c_l (T_{eqm} - 200)$$
  
(.05 kg)(387 J/kg)(400 - T\_{eqm}) = (.1 kg)(129 J/kg)(T\_{eqm} - 200)  
$$\Rightarrow T_{eqm} = 320$$

where we have used the heat capacities  $c_c = (387 \ J/kg)$  and  $c_l = (129 \ J/kg)$  for copper and lead respectively.

#### 26.42(b)

We are told that the two block system is insulated so that no heat enters or leaves, therefore Q = 0. We also know that no work is done by (or on) the system therefore W = 0. Using the first Law of thermodynamics  $\Delta E_{int} = Q + W$  we have

$$\Delta E_{int} = 0$$

#### 26.42(c)

For the lead block

$$\Delta S_l = \int_{T_{i,l}}^{T_f} \frac{dQ}{T} = \int_{T_{i,l}}^{T_f} \frac{mc_l dT}{T}$$

and similarly for the copper block

$$\Delta S_l = \int_{T_{i,c}}^{T_f} \frac{dQ}{T} = \int_{T_{i,c}}^{T_f} \frac{mc_c dT}{T}$$

$$\begin{split} \Delta S &= \Delta S_l + \Delta S_c \\ \Delta S &= m_l c_l \ln \left(\frac{320}{200}\right) + m_c c_c \ln \left(\frac{320}{400}\right) \\ \Delta S &= (.1 \ kg)(129 \ J/(kg \ K)) \ln \left(\frac{320}{200}\right) + (.05 \ kg)(387 \ J/(kg \ K)) \ln \left(\frac{320}{400}\right) \\ \Delta S &= 6.06 - 4.32 \\ \Delta S &= 1.745 \ J/K \end{split}$$

#### HRK 26.44

Note that 107°C = 380 K and 18.6°C = 291.6 K, and also  $c_{Al}$  = 900  $J/(kg\ K)$  and  $c_w$  = 4190  $J/(kg\ K)$ 

#### 26.44(a)

The heat lost by the aluminum must equal the heat gained by the water.

26.44(b)

$$\Delta S_{Al} = \int_{T_{i,Al}}^{T_f} \frac{dQ}{T} = \int_{T_{i,Al}}^{T_f} \frac{mc_{Al}dT}{T}$$
$$\Delta S_{Al} = m_{Al}c_{Al}\ln\left(\frac{T_{eqm}}{107}\right)$$
$$\Delta S_{Al} = (0.196)(900)\ln\left(\frac{331}{380}\right)$$
$$\Delta S_{Al} = -24.4 J/K$$

26.44(c)

$$\Delta S_{H20} = m_{H20}c_{H20} \ln\left(\frac{T_{eqm}}{107}\right)$$
$$\Delta S_{H20} = (0.0523)(4190) \ln\left(\frac{331}{291.6}\right)$$
$$\Delta S_{H20} = 27.8 \ J/K$$

26.44(d)

$$\Delta S = \Delta S_{Al} + \Delta S_{H20}$$
$$\Delta S = 3.4 J/K$$

The total change in entropy is positive as expected for any system undergoing an irreversible process.

### HRK 26.45

Since the lake is so much larger than the cube the lake's temperature will not change over the course of this process. The terms in the heat and entropy equations below correspond to (i) warming the ice to ice at  $0^{\circ}C$  (ii) Melting the ice to water at  $0^{\circ}C$  and (iii) heating the water to  $15^{\circ}C$ , respectively. Note that  $c_{ice} = 2220 \ J/(kg \ K)$  and  $L_f = 333,000 \ J/(kg \ K)$ 

$$\begin{aligned} Q_{icc\ cube}^{into} &= Q_{Lake}^{out\ of} \\ m_i c_i (0-T_i) + m_i L + m_i c_w (15-0) &= Q_{Lake}^{out\ of} \\ (0.0126)(2220)(0+10) + (0.0126)(333 \times 10^3) + (0.0126)(4190)(15-0) &= Q_{Lake}^{out\ of} \\ 5267\ J &= Q_{Lake}^{out\ of} \end{aligned}$$

$$\begin{split} \Delta S_{ice} &= m_i c_i \ln \frac{273}{263} + \frac{m_i L_f}{273} + m_i c_w \ln \frac{288}{273} \\ \Delta S_{ice} &= (0.0126)(2220) \ln \frac{273}{263} + \frac{(0.0126)(333 \times 10^3)}{273} + (0.0126)(4190) \ln \frac{288}{273} \\ \Delta S_{ice} &= 1.044 + 15.37 + 2.82 \\ \Delta S_{ice} &= 19.24 \ J/K \end{split}$$

$$\Delta S_{lake} = \int \frac{dQ}{T}$$
  

$$\Delta S_{lake} = \frac{Q_{Lake}^{out \ of}}{T} \text{ isothermal process}$$
  

$$\Delta S_{lake} = \frac{-1076}{288}$$
  

$$\Delta S_{lake} = -18.29 \ J/K$$

Finally