HRK 19.2

The frequency, ν, is the number of full oscillations per second. Since the boat completes 12 oscillations in 30 seconds we have

$$\nu = \frac{12}{30} = 0.4\ Hz$$

If it takes $5s$ for a point (in this case, the crest) of the wave to travel $15m$ then

$$v = \frac{15}{5} = 3\ m/s$$

Since $v = \nu\lambda$ we can rearrange and insert the above values to get

$$\lambda = \frac{v}{\nu} = \frac{3}{0.4} = 7.5\ m$$

HRK 19.3

If you sketch a sinusoidal wave you can see that it takes $\frac{1}{4}$ of one full period for the displacement to go from a maximum (in magnitude) to zero (or vice-versa). Hence, the piece of information given in the problem tells us that

$$\text{Period: } T = 4(1.78 \times 10^{-1}) = 7.12 \times 10^{-1}\ s.$$

Once we have the period in its correct units (seconds) we can invert to find frequency

$$\nu = \frac{1}{T} = \frac{1}{7.12 \times 10^{-1}} = 1.4\ Hz.$$

Use the ν we just found, along with $\lambda = 1.38\ m$ to get

$$v = \nu\lambda = (1.4)(1.38) = 1.93 m/s$$

HRK 19.8

Compare the general form for a sinusoidal wave with the description given

$$y = y_{max}\sin(kx - \omega t) \qquad \text{vs.} \qquad y = (2.3 \times 10^{-3})\sin(18.2x - 588t)$$

$$\begin{aligned} \rightarrow y_{max} &= 2.3 \times 10^{-3}\ m \\ \rightarrow k &= 18.2\ \text{rad}/m \\ \rightarrow \omega &= 588\ \text{rad}/s \end{aligned}$$

You will need to remember how the factor of 2π appears in the relative definitions of ω and ν

$$\omega = 2\pi\nu \Rightarrow \nu = \frac{588}{2\pi} = 93.6\ Hz$$

The period is given by

$$T = \frac{1}{\nu} = 193.6 = 1.07 \times 10^{-2}\ s$$

The wavenumber k is given by

$$k = \frac{2\pi}{\lambda} = 18.2 \quad \Rightarrow \quad \lambda = \frac{2\pi}{18.2} = 0.345\ m$$

The velocity is $v = \nu\lambda$ and hence

$$v = (93.6)(0.345) = 32.31\ m/s$$

or, equivalently, use $v = \frac{\omega}{k}$ to get the same result

$$v = \frac{588}{18.2} = 32.31 \ m/s.$$

The transverse speed of a particle in the string (which is completely different from the speed, v, of the traveling wave) is given by the change in vertical displacement (at a particular point) per unit time, or more precisely

$$\begin{aligned} v_{transverse} &= \frac{\partial y}{\partial t} \\ \frac{\partial}{\partial t} y_{max} \sin(kx - \omega t) &= -y_{max}\omega \cos(kx - \omega t) \end{aligned}$$

Since this quantity $v_{transverse} = -y_{max}\omega \cos(kx - \omega t)$ varies with time, we should find the time at which $|v_{transverse}|$ is maximized. That would be when $kx - \omega t = n\pi$ with n being some integer.

$$\max(|v_{transverse}|) = |-y_{max}\omega \cos(n\pi)| = y_{max}\omega = (2.3 \times 10^{-3})(588) = 1.352 \ m/s$$

HRK 19.15

Although this has no relevance until part (f) of the question we should note that the waveform pictured has $y \neq 0$ when $(x = 0, t = 0)$. This means that the waveform is described by an equation of the form

$$y(x,t) = y_{max} \sin(kx + \omega t + \delta) \quad \text{where } \delta = \sin^{-1}\left(\frac{y(x=0,t=0)}{y_{max}}\right) \rightarrow \delta = \sin^{-1}\left(\frac{.04}{.05}\right)$$

By inspecting the picture we see

19.15(a)

The amplitude is $y_m = .05 \ m$ and

19.15(b)

One full cycle of the wave takes $\lambda = 0.4 \ m$ to complete. (Note that this means $k = \frac{2\pi}{\lambda} = 15.7 \ rad/m$)

19.15(c)

The speed of the traveling wave is given by

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{3.6}{2.5 \times 10^{-2}}} = 12 \ m/s$$

19.15(d)

The period T is obtained from $v = \lambda\nu = \frac{\lambda}{T}$. Rearranging slightly,

$$T = \frac{\lambda}{v} = \frac{0.4}{12} = 3.333 \times 10^{-2} \qquad (\Rightarrow \nu = \frac{1}{T} = 30 \ Hz)$$

19.15(e)

Similar to the analysis done in the last question we can show

$$v_{transverse} = y_{max}\omega \cos(kx + \omega t + \delta),$$

and the maximum speed, $\max(|v_{transverse}|)$, occurs when $kx + \omega t + \delta = n\pi$. Therefore

$$\max(|v_{transverse}|) = |(y_{max})(\omega)| = |(y_{max})(2\pi\nu)| = (0.05)\,(2\pi(30)) = 9.43 \ m/s$$

19.15(f)

Using answers (a),(d), $\omega = 2\pi\nu = 188.5$ and $\delta = \sin^{-1}\left(\frac{.04}{.05}\right) = 0.93$ we get

$$y(x,t) = (.05)\sin(15.7x + 188.5t + 0.93)$$

HRK 19.23

The amount of mass contained in a small element of string at point x with linear density $\mu(x)$ is given by

$$\delta m = \mu(x)\delta x$$

19.23(a)

$$\begin{aligned}
dm = \mu(x)\ dx &= kx\ dx \\
\int_0^M dm &= \int_0^L kx\ dx \\
M &= \frac{1}{2}kx^2\Big]_0^L \\
M &= \frac{1}{2}kL^2
\end{aligned}$$

19.23(b)

We know the speed of a traveling wave is given by

$$v = \sqrt{\frac{F_T}{\mu}}$$

but here we must be careful not to forget that that μ varies with x, so consequently the velocity is a function of x:

$$v(x) = \sqrt{\frac{F_T}{kx}}.$$

Rewrite this as a differential equation

$$\frac{dx}{dt} = \sqrt{\frac{F_T}{kx}}.$$

and rearrange so that dx and $\sqrt{x}$ are on one side, and dt is on the other (it does not matter which side of the equation the constant terms go).

Now we can integrate to find the total time, T, for a pulse to traverse the length L.

$$\begin{aligned}
\int_0^T dt &= \int_0^L \sqrt{\frac{kx}{F_T}}\ dx \\
T &= \sqrt{\frac{k}{F_T}}\left(\frac{2}{3}x^{\frac{3}{2}}\right)\Big]_0^L \\
T &= \sqrt{\frac{k}{F_T}}\left(\frac{2}{3}L^{\frac{3}{2}}\right) \\
T &= \sqrt{\frac{4kL^3}{9F_T}} \qquad \text{and, since } M = \frac{1}{2}kL^2, \\
T &= \sqrt{\frac{8L}{9F_T}\left(\frac{kL^2}{2}\right)} \\
T &= \sqrt{\frac{8LM}{9F_T}}
\end{aligned}$$

HRK 19.24

Because of the rotational motion, there will be just enough tension in the string to keep all the little pieces of the string moving in a circle. This is the picture to keep in mind.

Let us concentrate on a small segment, of length δl, of the spinning hoop of string. Similar to the discussion in deriving the wave equation we can rationalize why the force on this segment, directed toward the center of the hoop is given by (see Chapter 19 Figure 9)

$$F_\perp = \frac{F_T \delta l}{R}$$

where F_T is the tension in the string and R is the radius of the hoop.

Since this hoop is assumed to be spinning in the absence of gravity, this is the only force acting on the string. This force $F_\perp$ must therefore be providing the centripetal acceleration that is always present in circular motion.

Let us equate $F_\perp$ with the centripetal force, F_c, on a small segment of string:

$$\begin{aligned} F_\perp = \frac{F_T \delta l}{R} &= \frac{(\delta m) v_0^2}{R} = F_c \\ \Rightarrow \left(\frac{\delta m}{\delta l}\right) v_0^2 &= F_T \\ \Rightarrow \mu v_0^2 &= F_T \end{aligned}$$

We know that the speed of waves in a string, v, is governed by the linear density, μ, of the string, and the tension, F_T in the string

$$v = \sqrt{\frac{F_T}{\mu}}$$

But using $\mu v_0^2 = F_T$ we see that the speed of waves is the same as the tangential speed

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{\mu v_0^2}{\mu}} = v_0.$$

This is rather surprising – what does this mean? Is a wave, traveling in the same direction as the string is rotating, just a permanent deformation of the string? – No! – The wave speed v is *relative* to the string, not absolute. One consequence of this is that a wave, propagating in the *opposite* direction from the direction of rotation, would not be moving in a fixed reference frame!

HRK 19.28

Call the initial distance (at which the observer measures the intensity to be 1.13 W/m^2) r_1, and the second distance from the source r_2.

We know that the intensity measured at a distance r from a source of spherical waves with power P is given by

$$I = \frac{P}{4\pi r^2}$$

$$\begin{aligned} \text{We have } I_1 &= \frac{P}{4\pi r_1^2} \\ \text{and } I_2 &= \frac{P}{4\pi r_2^2} \\ \Rightarrow \frac{I_1}{I_2} &= \left(\frac{P}{4\pi r_1^2}\right)\left(\frac{4\pi r_2^2}{P}\right) \\ \frac{I_1}{I_2} &= \frac{4\pi r_2^2}{4\pi r_1^2} \\ \text{Useful result:} \quad & \boxed{r_2^2 = \left(\frac{I_1}{I_2}\right) r_1^2} \end{aligned}$$

Use this in conjunction with $r_2 = r_1 - 5.3$ to find the radii

$$\begin{aligned} r_2 = \sqrt{\frac{I_1}{I_2}} r_1 &= r_1 - 5.3 \\ r_1\left(1 - \sqrt{\frac{I_1}{I_2}}\right) &= 5.3 \\ r_1 &= \frac{5.3}{0.31525} = 16.82\ m \\ \Rightarrow r_2 &= 16.82 - 5.3 = 11.52\ m \end{aligned}$$

To get the power from this piece of information:
Short way:

$$\begin{aligned} I_1 &= \frac{P}{4\pi r_1^2} \\ 4\pi r_1^2 I_1 &= P \\ 4\pi(16.82)^2(1.13) &= 4017\ W \end{aligned}$$

Unnecessarily Long way:

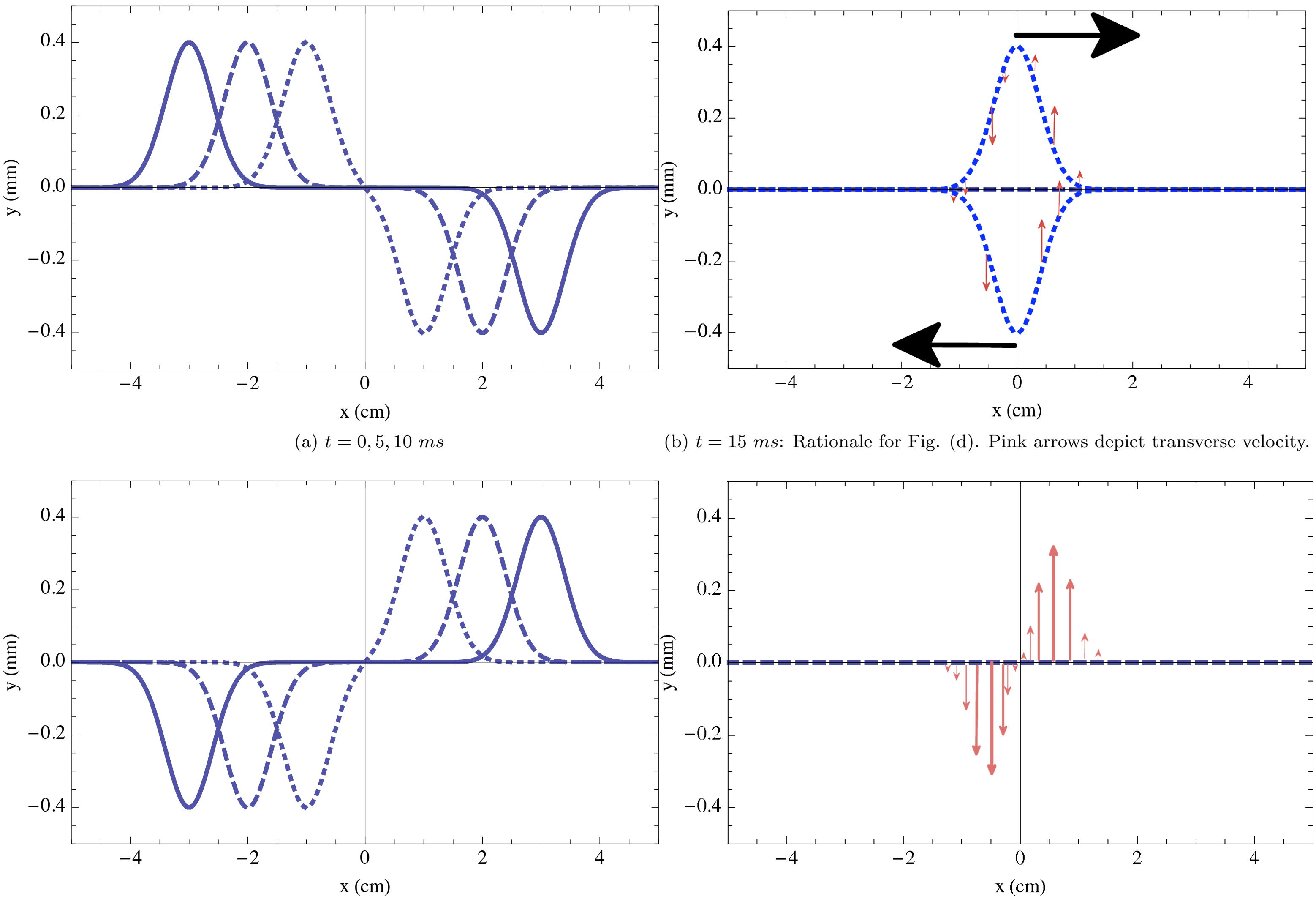
$$\begin{aligned} I_2 - I_1 &= \frac{P}{4\pi r_2^2} - \frac{P}{4\pi r_1^2} \\ I_2 - I_1 &= \frac{P(4\pi r_1^2 - 4\pi r_2^2)}{(4\pi r_1^2)(4\pi r_2^2)} \\ P &= \frac{(I_2 - I_1)(4\pi r_1^2)(4\pi r_2^2)}{(4\pi r_1^2 - 4\pi r_2^2)} \\ P &= \frac{(I_2 - I_1)(4\pi r_1^2)(r_2^2)}{(r_1^2 - r_2^2)} \\ P &= \frac{(2.41 - 1.13)(4\pi(16.82)^2)(11.52)^2}{((16.82)^2 - (11.52)^2)} \\ P &= 4021\ W \end{aligned}$$

HRK 19.33

The waves are moving towards each other, from an initial ($t = 0$) separation of $6cm = .06\ m$. The right-moving wave will move 1 cm to the right every 5 ms, and similarly, the left-moving wave will move 1 cm to the right every 5 ms so they will be exactly superimposed at $t = 15\ ms$ and for e.g. $t = 20\ ms$ or $t = 25\ ms$ they will have passed through each other. We can see this easily from

$$\begin{aligned} x &= vt \qquad \text{where } v = \pm 2m/s \\ x(t = 0ms) &= (\pm 2)(0) = 0\ m \\ x(t = 5ms) &= (\pm 2)(5 \times 10^{-3}) = \pm .01\ m \\ x(t = 10ms) &= (\pm 2)(10 \times 10^{-3}) = \pm .02\ m \\ \mathbf{x(t = 15ms)} &= \mathbf{(\pm 2)(15 \times 10^{-3}) = \pm .03\ m} \\ x(t = 20ms) &= (\pm 2)(20 \times 10^{-3}) = \pm .04\ m \\ x(t = 25ms) &= (\pm 2)(25 \times 10^{-3}) = \pm .05\ m \\ x(t = 30ms) &= (\pm 2)(30 \times 10^{-3}) = \pm .06\ m \end{aligned}$$

and we depict these results below.



(a) $t = 0, 5, 10\ ms$

(b) $t = 15\ ms$: Rationale for Fig. (d). Pink arrows depict transverse velocity.

(c) $t = 20, 25, 30\ ms$

(d) $t = 15\ ms$: Instantaneous transverse velocity, denoted by pink arrows.

Figure 1

The point of the question is to consider what happens at $t = 15ms$ when the principle of superposition tells us that the right-moving and left-moving waveforms completely cancel out (destructively interfere) at that instant, in the center of the piece of string.

The wave equation we are using was derived assuming conservation of energy. Since the displacement from the equilibrium $y = 0$ position is zero at this particular instant then the potential energy contribution is zero and all the energy is kinetic. In Figure 1 (b) and (d) we depict the string at the instant $t = 15\ ms$ – we can see there is no displacement from $y = 0$. The transverse velocity, though, compensates for this and we depict the direction and strength of this velocity at various points using vertical pink arrows.

HRK 19.36

If you glance at problem 27 you will see that the displacement $y(r, t)$ of a medium at a distance r from a point source of spherical waves is given by

$$y = \frac{Y}{r}\sin(kr - \omega t)$$

Here we have two point sources so write out each of their wave equations

$$y_1 = \frac{Y_1}{r_1}\sin(k_1 r_1 - \omega_1 t + \phi_1)$$
$$y_2 = \frac{Y_2}{r_2}\sin(k_2 r_2 - \omega_2 t + \phi_2)$$

The fact that the waves y_1 and y_2 have the same frequency and phase relation at all times means that $\omega_1 = \omega_2 = \omega$, $k_1 = k_2 = k$ and also $\phi_1 = \phi_2 = \phi$. The amplitude of each wave is given by $y_{m,1} = \frac{Y_1}{r_1}$ and $y_{m,2} = \frac{Y_2}{r_2}$ respectively (recall the discussion about why the amplitude has a $\frac{1}{r}$ dependence - this arises from energy conservation considerations) and we are told that the amplitudes of each wave are the same therefore $Y_1 = Y_2 = Y$.

We will add these two waves in a region where $r_1 \approx r_2$:

$$\begin{aligned} y_{total} &= y_1 + y_2 \\ y_{total} &= \frac{Y}{r_1}\sin(kr_1 - \omega t + \phi) + \frac{Y}{r_2}\sin(kr_2 - \omega t + \phi) \end{aligned}$$

We know that both r_1 and r_2 are very close to $r = \frac{r_1+r_2}{2}$. You might be inclined to replace every occurrence of r_1 or r_2 with r in seeking to simplify this problem – however you must be careful.

The insight required to deal with this situation is that we can set $\frac{1}{r_1} \approx \frac{1}{r_2} \approx \frac{1}{r}$ in the denominator, if $r_1 \approx r_2$, but **we must not do so in treating the phases of the two waves**. In the first case (the denominators) we only need $|r_1 - r_2| \ll r$ while in the second case (the phases) we would need $|r_1 - r_2| \ll \lambda$, where λ may be quite small.

Armed with this approximation we can write

$$\begin{aligned} y_{total} &= \frac{Y}{r_1}\sin(kr_1 - \omega t - \phi) + \frac{Y}{r_2}\sin(kr_2 - \omega t - \phi) \\ &\approx \frac{Y}{r}\left(\sin(kr_1 - \omega t - \phi) + \sin(kr_2 - \omega t - \phi)\right) \end{aligned}$$

$$\text{Use: } \sin(A) + \sin(B) = 2\cos\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)$$

by identifying $A = kr_1 - \omega t - \phi$ and $B = kr_2 - \omega t - \phi$.

$$\begin{aligned} \Rightarrow y_{total} &= \frac{2Y}{r}\cos\left(\frac{k(r_1-r_2)}{2}\right)\sin\left(\frac{kr_1 + kr_2 - 2\omega t - 2\phi}{2}\right) \\ \Rightarrow y_{total} &= \frac{2Y}{r}\cos\left(\frac{k(r_1-r_2)}{2}\right)\sin\left(kr - \omega t - \phi\right) \\ y_{total} &= (\text{Amplitude})\sin(\ldots) \end{aligned}$$

which is the typical result for an interference pattern. The $\sin(\ldots)$ term jsut means there is a wave present, having the same wavelength and frequency as the sources. The $\cos(\ldots)$ term shows how the amplitude, and thus the intensity, varies from point to point (which is called an interference effect). This result predicts perfect cancellation whenever $\cos\left(\frac{k(r_1-r_2)}{2}\right) = 0$. While we should expect very small amplitude in such regions, we should not expect to see zero– why not? For perfect cancellation the two source amplitudes must be *exactly equal* at the observation point. Our assumption that $r_1 \approx r_2$ is not exact, and so we should not expect perfect cancelation.

19.36(b)

Obviously destructive interference (imperfect cancellation so amplitude is minimal but not zero) occurs for the regions where $\cos\left(\frac{k(r_1-r_2)}{2}\right) = 0$ and so the argument of the cosine must be some odd multiple of $\frac{\pi}{2}$

$$\begin{aligned}
\text{Cancelation: } \frac{k(r_1 - r_2)}{2} &= \frac{n\pi}{2} \qquad n = 1, 3, 5 \ldots \\
\Rightarrow (r_1 - r_2) &= \frac{n\pi}{k} \qquad n = 1, 3, 5 \ldots \\
\Rightarrow (r_1 - r_2) &= \frac{n\lambda}{2} \qquad n = 1, 3, 5 \ldots \qquad (\text{using } k = \frac{2\pi}{\lambda}) \\
\text{or equivalently } (r_1 - r_2) &= \left(n + \frac{1}{2}\right)\lambda \qquad n = 0, 1, 2, 3 \ldots
\end{aligned}$$

Similarly, constructive interference (amplitude is max) occurs for the regions where $\cos\left(\frac{k(r_1-r_2)}{2}\right) = 1$ and so the argument of the cosine must be some multiple of π

$$\begin{aligned}
\text{Constructive: } \frac{k(r_1 - r_2)}{2} &= n\pi \qquad n = 1, 2, 3 \ldots \\
\Rightarrow (r_1 - r_2) &= \frac{2n\pi}{k} \qquad n = 1, 2, 3 \ldots \\
\Rightarrow (r_1 - r_2) &= n\lambda \qquad n = 1, 2, 3 \ldots \qquad (\text{using } k = \frac{2\pi}{\lambda})
\end{aligned}$$

HRK 19.39

19.39(a)

It is reasonable to assume that the mass per unit length of this string is constant and therefore

$$\mu = \frac{M}{L} = \frac{0.122}{8.36} = 1.46 \times 10^{-2}$$

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{96.7}{1.46 \times 10^{-2}}} = 81.4\ m/s$$

19.39(b)

The longest possible standing wave is the fundamental mode ($n = 1$), where half a wavelength fits between the ends.

$$\begin{aligned}
\lambda_n &= \frac{2L}{n} \\
\lambda_1 &= 2L = 16.72\ m
\end{aligned}$$

19.39(c)

Quick Way: Use the relationship $v = \nu\lambda$ and plug in v from part (a) and λ from part (b)

$$\nu = \frac{v}{\lambda} = \frac{81.4}{16.72} = 4.87\ Hz$$

Unnecessarily Long Way: Recall that for standing waves, the natural frequencies are given by

$$\nu_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$$

and we can rearrange to give the speed of a traveling wave when the n^{th} mode is present

$$v = \frac{2L\nu_n}{n}$$

Equating this with the expression for the speed of a wave in a string under tension gives us a useful relationship

$$\begin{aligned} v = \frac{2L\nu_n}{n} &= \sqrt{\frac{F_T}{\mu}} \\ \Rightarrow \nu_n^2 &= \frac{n^2 F_T}{4L^2\mu} \end{aligned}$$

Now, since we are considering the fundamental mode we insert $n = 1$ and the rest of the values given in the question to solve for the frequency, ν

$$\nu = \sqrt{\frac{F_T}{4L^2\mu}} = \sqrt{\frac{96.7}{4(8.36)^2(1.46\times 10^{-2})}} = 4.87\ Hz$$

HRK 19.47

This problem illustrates what happens when *any* wave goes from one medium to another in which the wave speed is different. Part of the wave is transmitted and part is reflected (perhaps with inversion). That is why it is such an important problem.

19.47(a)

On the left hand side of the knot (where $x < 0$ and $\mu = \mu_1$), we have an incident wave approaching the knot $y_{inc} = A\sin(k_1(x - v_1 t))$ and we also have the reflected wave $y_{ref} = C\sin(k_1(x + v_1 t))$ moving away from the knot. On the right hand side of the knot (where $x > 0$ and $\mu = \mu_2$) we have the transmitted wave $y_{trans} = B\sin(k_2(x - v_2 t))$, moving away from the knot.
Note: We will repeatedly use the facts that $\cos(-\theta) = \cos(\theta)$ and $\sin(-\theta) = -\sin(\theta)$.

Apply the principle of superposition to the waves in the string on the left hand side of the knot to find the total disturbance

$$\begin{aligned} y_{left}(x,t) &= y_{inc} + y_{ref} \\ y_{left}(x,t) &= A\sin(k_1(x - v_1 t)) + C\sin(k_1(x + v_1 t)) \end{aligned}$$

In particular, notice the form this takes *at the knot*

$$\begin{aligned} y_{left}(0,t) &= A\sin(-k_1 v_1 t) + C\sin(k_1 v_1 t) \\ &= \sin(k_1 v_1 t)(-A + C) \\ &= \sin(\omega_1 t)(-A + C) \end{aligned}$$

where in the last line we used $k_1 v_1 = \frac{2\pi}{\lambda_1} v_1 = 2\pi\nu_1 = \omega_1$.

Now let us examine the string on the right hand side of the knot, and in particular what its displacement is at $x = 0$.

$$\begin{aligned} y_{right} &= y_{trans} \\ y_{right}(x,t) &= B\sin(k_2(x - v_2 t)) \\ y_{right}(0,t) &= -B\sin(k_2 v_2 t) \\ y_{right}(0,t) &= -B\sin(\omega_2 t) \end{aligned}$$

With some thought you can convince yourself that $y_{left}(0,t) = y_{right}(0,t)$ must be true for all times t – otherwise segments of the string arbitrarily close to each other (but on either side of the knot which we imagine to be point-like) would have different vertical displacements – i.e. the knot would break. Furthermore, the only possible way to have the string remain intact for all values of t, is to have the two angular frequencies $\omega_1 = k_1 v_1$ and $\omega_2 = k_2 v_2$ be identical. Using these physically motivated conditions we get

$$\begin{aligned}
y_{left}(0,t) &= y_{right}(0,t) \\
\sin(k_1 v_1 t)(-A + C) &= -B \sin(k_2 v_2 t) \\
\sin(\omega_1 t)(-A + C) &= -B \sin(\omega_1 t) \\
\Rightarrow A &= B + C
\end{aligned}$$

19.47(b)

Physical reasoning told us that the displacement y should be the same immediately either side of the knot. Similarly the slope of the string should not change abruptly at the point $x = 0$. The reason is a bit more subtle and holds only if the knot has negligible mass. In that case the net force acting on the knot must vanish. The tension must be the same on both sides, or else the knot would accelerate sideways and so a net $F_y = 0$ implies $T\sin(\theta_1) = T\sin(\theta_2)$ i.e. $\theta_1 = \theta_2$. This gives us the following condition, true for all times t,

$$\left.\frac{\partial y_{left}}{\partial x}\right]_{x=0} = \left.\frac{\partial y_{right}}{\partial x}\right]_{x=0}.$$

Let us use this result to gain some more information about the relationship between A, B and C.

$$\begin{aligned}
\left.\frac{\partial y_{left}}{\partial x}\right]_{x=0} &= \left.\frac{\partial y_{right}}{\partial x}\right]_{x=0} \\
\left.k_1 A \cos(k_1(x - v_1 t)) + k_1 C \cos(k_1(x + v_1 t))\right]_{x=0} &= \left.k_2 B \cos(k_2(x + v_2 t))\right]_{x=0} \\
k_1 A \cos(-k_1 v_1 t) + k_1 C \cos(k_1 v_1 t) &= k_2 B \cos(k_2 v_2 t) \\
k_1 A + k_1 C &= k_2 B \\
k_1(A + C) &= k_2 B
\end{aligned}$$

Now let us use this result in conjunction with $A = B + C$ to get an expression for C which doesn't involve B

$$\begin{aligned}
k_1(A + C) &= k_2 B \\
k_1(A + C) &= k_2(A - C) \\
k_1 C + k_2 C &= -k_1 A + k_2 A \\
C &= \frac{A(k_2 - k_1)}{k_2 + k_1}
\end{aligned}$$

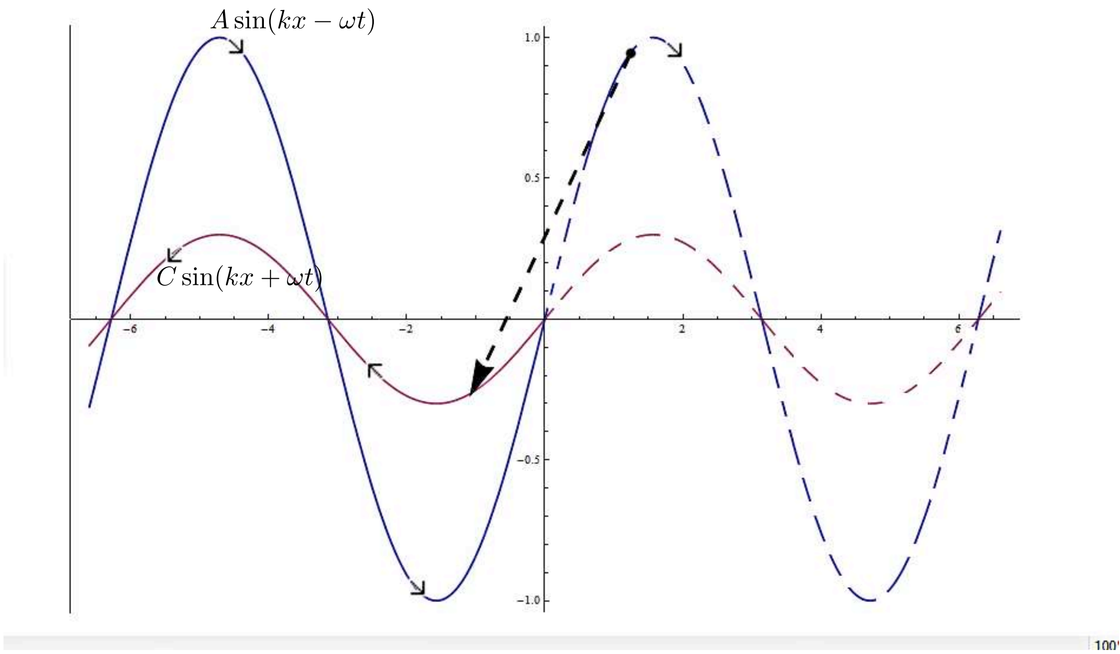
To get the other form use $k_1 v_1 = k_2 v_2 = \omega$ to eliminate the k's

$$\begin{aligned}
C &= A\frac{(k_2 - k_1)}{k_2 + k_1} \\
C &= A\frac{\frac{\omega}{v_2} - \frac{\omega}{v_1}}{\frac{\omega}{v_2} + \frac{\omega}{v_1}} \\
C &= A\frac{\frac{(\omega v_1 - \omega v_2)}{v_1 v_2}}{\frac{\omega v_1 + \omega v_2}{v_1 v_2}} \\
C &= A\frac{(v_1 - v_2)}{v_1 + v_2}
\end{aligned}$$

C is negative when $v_1 < v_2$.

C is positive when $v_2 < v_1$ and this corresponds to the reflected wave being inverted with respect to the incident wave. When a wave goes into a new medium where it slows down (e.g light going from air into glass so that $v_2 < v_1$) the reflected wave is inverted.

As you continue your physics studies, it is more common to see things defined so that $C < 0$ would correspond to an inverted wave (in contrast to this particular problem).



Satisfy yourself that the left-moving wave $C\sin(kx + \omega t)$ is inverted with respect to the right moving wave $A\sin(kx - \omega t)$ by looking at the picture below (Figure 2). Remember that $|C| < |A|$ so the amplitude of the reflected wave is less than that of the incoming wave. The black dashed arrow shows how one particular point from the wave $A\sin(kx - \omega t)$ would get inverted.