HRK 23.10

Since

$$\frac{pV}{NT} = k,$$

where N is the number of molecules. Since we have exactly one mole that means $N = N_A$. Rearranging,

$$\frac{V}{N_A} = \frac{kT}{p}.$$

This tells us the volume for each molecule.

23.10 (a)

$$\frac{V}{N_A} = \frac{kT}{p}$$

Vol per molecule: $\frac{V}{N_A} = \frac{(1.38 \times 10^{-23})285}{1.01 \times 10^5} = 3.89 \times 10^{-26} m^3$

The edge length of a cube with this volume is

$$L = V^{\frac{1}{3}} = (3.89 \times 10^{-26})^{\frac{1}{3}} = 3.39 \times 10^{-9} m.$$

Now, as suggested in the question, compare this length L with diameter d of a molecule

$$\frac{L}{d} = \frac{3.39 \times 10^{-9}}{3 \times 10^{-10}} = 11.3$$

23.10 (b)

$$\frac{V}{N_A} = \frac{18 \times 10^{-6}}{N_A}$$
$$\frac{V}{N_A} = \frac{18 \times 10^{-6}}{6.02 \times 10^{23}} = 2.99 \times 10^{-29}$$

The edge length of a cube with this volume is

$$L = V^{\frac{1}{3}} = (2.99 \times 10^{-29})^{\frac{1}{3}} = 3.104 \times 10^{-10} m.$$

Now, as suggested in the question, compare this length L with diameter d of a molecule

$$\frac{L}{d} = \frac{3.104 \times 10^{-10}}{3 \times 10^{-10}} = 1.035$$

Interpreting these results, we that in the gas, each molecule has more than 1000 times its own volume to wander around in, while in liquid the molecules are practically side-by-side.

HRK 23.14

Call the smaller vessel A and the larger one B.

Since they are connected by a tube they must have the same pressure (this is Pascal's principle)

$$p_{tot} = p_a = p_b$$

The total volume is

$$V_{tot} = V_a + V_b$$

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The number of moles in each container is n_a and n_b respectively such that

$$n_{tot} = n_a + n_b$$

The number of moles of gas does not change due to heating (this is the no leaks hint). Before heating, the temperature and pressure of both containers are the same so that $T_a = T_b$ as well as $p_a = p_b$.

before heating:
$$n_{tot} = n_a + n_b$$

 $n_{tot} = \frac{p_a V_a}{RT_a} + \frac{p_b V_b}{RT_b}$
 $n_{tot} = \frac{(1.44p_0)(1.22 \times 10^{-3})}{(8.31)(289)} + \frac{(1.44p_0)(3.18 \times 10^{-3})}{(8.31)(289)}$
 $n_{tot} = 0.0738 + 0.1926$
 $n_{tot} = 0.2664 \ mol$

After heating we still have $p_a = p_b$, but it is no longer true that $T_a = T_b$.

after heating:
$$n_{tot} = n_a + n_b$$

 $n_{tot} = \frac{p_a V_a}{RT_a} + \frac{p_b V_b}{RT_b}$
 $0.266 \ mol = \frac{p_{tot}(1.22 \times 10^{-3})}{(8.31)(289)} + \frac{p_{tot}(3.18 \times 10^{-3})}{(8.31)(381)}$
 $0.266 \ mol = p_{tot} \left(\frac{(1.22 \times 10^{-3})}{(8.31)(289)} + \frac{(3.18 \times 10^{-3})}{(8.31)(381)}\right)$
 $0.266 \ mol = p_{tot} (1.51 \times 10^{-6})$
 $\frac{0.266}{1.51 \times 10^{-6}} = p_{tot}$
 $176, 490 = p_{tot}$
 $\Rightarrow p_{tot} = 1.747 \ atm$

Note that after heating we have $n_a = 0.0897$ and $n_b = 0.177$ as compared to $n_a = 0.0738$ and $n_b = 0.1926$ before heating.

HRK 23.17

Method I: Equipartition

We can solve this, like most physicists would, using the equipartition of energy theorem. The molecule has 3 translational degrees of freedom and its translational kinetic energy is

$$K_{trans} = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2).$$

Each term will have an average value of $\frac{1}{2}k_BT$ in thermal equilibrium so

$$\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}k_BT$$

Rearranging we get

 \Rightarrow

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Method II: Manipulating gas laws

Combining

$$\rho = \frac{nM}{V} \quad \text{and} \quad pV = nRT$$

where M is the molar mass, we get

$$\frac{p}{\rho} = \frac{RT}{M}.$$

Since

$$v_{rms} = \sqrt{\frac{3p}{\rho}}$$

then $v_{rms} = \sqrt{\frac{3RT}{M}}$

The punchline is that $v_{rms} \propto \sqrt{T}$. For a gas at different temperatures

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

Table 1 tells us the rms speed of Hydrogen at 300K so

$$v_{1} = \sqrt{\frac{T_{1}}{T_{2}}}v_{2}$$

$$v_{1} = \sqrt{\frac{2.7}{300}}1920$$

$$v_{1} = 182 \ m/s$$

HRK 23.20

We are going to do a similar analysis to the section "Kinetic interpretation of the temperature" in your textbook. The average impulsive force on the wall will be the amount of momentum transferred per unit time. The particles' paths are such that they strike the wall at an angle of 55° from the normal. A number *n* of particles of mass *m* bounce off the wall every second. This means that the momentum transferred to the wall per second (in a direction perpendicular to the wall's surface) is

$$F = n(2mv\cos(55^{\circ}))$$

$$F = n \left[2(3.3 \times 10^{-27})(1 \times 10^{3})(0.5735) \right]$$

$$F = (1.6 \times 10^{23}) \left[3.78 \times 10^{-24} N \right]$$

$$F = 0.6056 N$$

Now divide by the area to find the pressure

$$p = \frac{F}{A} \\ p = \frac{0.6056}{2 \times 10^{-4}} \\ p = 3028 \ Pa$$

HRK 23.33

From Chapter 16 of your textbook, we have

$$\begin{aligned} v_{esc}^{earth} &= \sqrt{\frac{2GM_e}{R_e}} = 1.12 \times 10^4 \ m/s \\ v_{esc}^{moon} &= \sqrt{\frac{2GM_m}{R_m}} = 2.38 \times 10^3 \ m/s \end{aligned}$$

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Method 1: Longer

23.33 (a): Earth

We know that v is a function of T, and we want to find the temperature T^* such that $v(T^*) = v_{esc}$ for the two different molecules i.e.

$$\begin{aligned} v_H(T_H^\star) &= v_{esc}^{earth} \\ \text{and} \ v_O(T_O^\star) &= v_{esc}^{earth} \end{aligned}$$

Use

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

and rearrange to get

$$T = \frac{m\overline{v^2}}{3k}.$$

For hydrogen we have

$$\begin{split} T_{H}^{\star} &= \frac{m_{H_{2}}(v_{esc}^{earth})^{2}}{3k} \\ T_{H}^{\star} &= \frac{m_{H_{2}}(1.12 \times 10^{4})^{2}}{3(1.38 \times 10^{-23})} \quad m_{H_{2}} = (0.002) \frac{kg}{mole} \times \frac{1 \ mole}{6.02 \times 10^{23} \ molecules} \\ T_{H}^{\star} &= \frac{(3.3 \times 10^{-27})(1.12 \times 10^{4})^{2}}{3(1.38 \times 10^{-23})} \\ T_{H}^{\star} &= 10,059 \ K \end{split}$$

For oxygen we have

$$\begin{split} T_O^{\star} &= \frac{m_{O_2}(v_{esc}^{earth})^2}{3k} \\ T_O^{\star} &= \frac{m_{O_2}(1.12 \times 10^4)^2}{3(1.38 \times 10^{-23})} \quad m_{O_2} = (0.032) \frac{kg}{mole} \times \frac{1 \ mole}{6.02 \times 10^{23} \ molecules} \\ T_O^{\star} &= \frac{(5.32 \times 10^{-26})(1.12 \times 10^4)^2}{3(1.38 \times 10^{-23})} \\ T_O^{\star} &= 16(10,059) \ K \\ T_O^{\star} &= 160,951 \ K \end{split}$$

23.33 (b): Moon

A similar calculation, with a different escape velocity,

$$v_H(T_H^{\star}) = v_{esc}^{moon}$$

and $v_O(T_O^{\star}) = v_{esc}^{moon}$

For hydrogen we have

$$T_{H}^{\star} = \frac{m_{H_{2}}(v_{esc}^{moon})^{2}}{3k}$$

$$T_{H}^{\star} = \frac{m_{H_{2}}(2.38 \times 10^{3})^{2}}{3(1.38 \times 10^{-23})}$$

$$T_{H}^{\star} = \frac{(3.3 \times 10^{-27})(2.38 \times 10^{3})^{2}}{3(1.38 \times 10^{-23})}$$

$$T_{H}^{\star} = 454 K$$

For oxygen we have

$$T_{O}^{\star} = \frac{m_{O_2}(v_{esc}^{moon})^2}{3k}$$

$$T_{O}^{\star} = \frac{m_{O_2}(2.38 \times 10^3)^2}{3(1.38 \times 10^{-23})}$$

$$T_{O}^{\star} = \frac{(5.32 \times 10^{-26})(2.38 \times 10^3)^2}{3(1.38 \times 10^{-23})}$$

$$T_{O}^{\star} = 16(454) K$$

$$T_{O}^{\star} = 7,268 K$$

Method 2: Using Ratios

First calculate T_H^{\star} for earth (i.e the temperature at which the average v of hydrogen molecule is equal to v_{esc}). Then, calculate the rest of the quantities using

Ratios:
$$\frac{m_{H_2}}{m_{O_2}} = \frac{1}{16}$$
$$\frac{v_{esc}^{moon}}{v_{esc}^{earth}} = \frac{2.38}{11.2}$$

Method 2:(a) Earth

Use

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

and rearrange to get

$$T = \frac{m\overline{v^2}}{3k}.$$

For hydrogen in Earth's atmosphere we have

Since $T \propto m$ in our expression $T = \frac{m\overline{v^2}}{3k}$, and since $m_{O_2} = 16m_{H_2}$, we immediately have

$$\begin{array}{rcl} T_{O}^{\star} &=& 16(10,059) \ K \\ T_{O}^{\star} &=& 160,951 \ K \end{array}$$

Method 2:(b) Moon

Since $T \propto \overline{v^2}$ in our expression $T = \frac{m\overline{v^2}}{3k}$, and since $v_{esc}^{moon} = \left(\frac{2.38}{11.2}\right) v_{esc}^{earth}$, we immediately have

$$T_H^{\star} = 10,059 \left(\frac{2.38}{11.2}\right)^2 K$$

 $T_H^{\star} = 454 K$

Again, we know that the temperature required for Oxygen must be 16 times as big (the ratio of the masses)

$$T_H^{\star} = (16)454 \ K$$

 $T_H^{\star} = 7268 \ K$

23.33 (c)

This is a more complicated question than it might seem at first. Neither molecule is escaping very quickly, and Hydrogen being lighter has higher speed. Thus the relative abundance of H_2 does not decrease as rapidly with altitude as does that of Oxygen.

HRK 23.36

We will denote subsequent legs of the journey (in the p - V plane) with subscripts *i*, *ii* etc. as we go from the initial configuration to the final configuration.

23.36 (Route 1)

$$W^{(1)} = W_i^{(1)} + W_{ii}^{(1)}$$

$$W^{(1)} = -p(V_f - V_i) + 0$$

$$W^{(1)} = -15(5 - 1)$$

$$W^{(1)} = -60 J$$

Negative 60 J of work done on the gas means the gas did 60 J of work on the piston.

23.36 (Route 2)

Area method: The work done is the area under the curve joining (p_i, V_i) and (p_f, V_f) . In this case the area is comprised of a triangle (with hypotenuse $(p_i, V_i) \rightsquigarrow (p_f, V_f)$) and a rectangle underneath it.

$$|W^{(2)}| = A_{triangle} + A_{rectangle}$$

$$|W^{(2)}| = \frac{1}{2}(10)(4) + (4)(5)$$

$$|W^{(2)}| = 40$$

$$W^{(2)} = -40 J$$

where in the last line, I knew the work done on the gas was negative since the volume increased. Calculus method:

$$W^{(2)} = -\int_{V_i=1}^{V_f=5} p \ dV$$

Looking at the slope of line 2 we see

$$\frac{dp}{dV} = -\frac{10}{4}$$
$$\Rightarrow dV = -\frac{2}{5}dp$$

Use this information to change the equation for work

$$W^{(2)} = -\int_{V_i=1}^{V_f=5} p \, dV$$

$$W^{(2)} = -\int_{p_i=15}^{p_f=5} p\left(-\frac{2}{5}\right) \, dp$$

$$W^{(2)} = \frac{2}{5} \int_{p_i=15}^{p_f=5} p \, dp$$

$$W^{(2)} = \frac{2}{5} \left(\frac{p^2}{2}\right)_{15}^5$$

$$W^{(2)} = \frac{2}{5} \left(-100\right)$$

$$W^{(2)} = -40 \, J$$

23.36 (Route 3)

$$W^{(3)} = W_i^{(3)} + W_{ii}^{(3)}$$

$$W^{(3)} = 0 - p(V_f - V_i)$$

$$W^{(3)} = -5(5 - 1)$$

$$W^{(3)} = -20 J$$

HRK 23.40

The fact that this is an isothermal process means that

$$pV = \text{constant} = nRT$$

 $p_iV_i = p_fV_f$

With this set-up, the work on the gas can be written as

$$W = -\int_{V_i}^{V_f} p dV$$
$$= -nRT \int_{V_i}^{V_f} \frac{dV}{V}$$
$$= -nRT \ln\left(\frac{V_f}{V_i}\right)$$

Thus, we need to find V_f to calculate the work for the isothermal expansion (this is the first leg of the overall process in the pV plane). The second leg (horizontal on the pV diagram) will be easy since p = const in that case $\Rightarrow W = -p(V_f - V_i)$.

Gauge pressure p_g is related to the absolute pressure p and atmospheric pressure p_0 via

$$p_g = p - p_0$$

For the first leg (the isothermal portion)

$$p_{i}V_{i} = p_{f}V_{f}$$

$$\Rightarrow V_{f} = \frac{p_{i}}{p_{f}}V_{i}$$

$$V_{f} = \frac{p_{0} + 103 \ kPa}{p_{0}}(0.142)$$

$$V_{f} = \frac{(101.3 + 103) \ kPa}{(101.3) \ kPa}(0.142)$$

$$V_{f} = 0.2864 \ m^{3}$$

Use information about the initial configuration to find a value for the quantity nRT

$$p_i V_i = nRT$$

$$(204.3 \times 10^3)(0.142) = nRT$$

$$\Rightarrow 29011 = nRT$$

The total work done is the sum over both legs

$$W = W_1 + W_2$$

$$W_{1} = -nRT \ln \left(\frac{V_{f}}{V_{i}}\right)$$

$$W_{1} = -nRT \ln \left(\frac{0.2864}{0.142}\right)$$

$$W_{1} = -(0.70156)nRT$$

$$W_{1} = -(0.70156)(29011)$$

$$W_{1} = -20,353 J$$

$$W_2 = -p(V_f - V_i)$$

$$W_2 = (101.3 \times 10^3)(0.2864 - 0.142)$$

$$W_2 = 14,628 J$$

$$W = W_1 + W_2$$

$$W = -20,353 + 14,628 = -5725 J$$

The negative sign shows that the gas did net work in going around the cycle. i.e. it acted as an engine, taking in heat as it expanded and putting out less heat as it was compressed again.

HRK 23.43

Adiabatic means that no heat flows into or out of the gas. Under these conditions

$$pV^{\gamma} = \text{constant}$$

where γ is a constant (1.4 in this case).

Refer to eqn 30 in your text for extra details on how we arrive at the expression for work done on the gas

$$\begin{split} W &= -\int_{V_i}^{V_f} p \ dV \\ W &= -p_i V_i^{\gamma} \int_{V_i}^{V_f} \frac{dV}{V^{\gamma}} \\ \text{after tidying..} \quad W &= \frac{1}{\gamma - 1} (p_f V_f - p_i V_i) \end{split}$$

23.43 (a)

To find p_f use

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}$$

$$\Rightarrow p_f = \frac{V_i^{\gamma}}{V_f^{\gamma}} p_i$$

$$p_f = \left(\frac{4.33}{1.06}\right)^{\gamma} 1.17$$

$$p_f = \left(\frac{4.33}{1.06}\right)^{1.4} 1.17$$

$$p_f = 8.39 atm$$

Note that I got away with using liters and atmospheres as units, because i was effectively using ratios and the units canceled out.

23.43 (b)

Use

$$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f}$$

$$\Rightarrow T_f = \frac{p_f V_f}{p_i V_i} T_i$$

$$T_f = \frac{(8.39)(1.06)}{(1.17)(4.33)} 310$$

$$T_f = 544 K$$

23.43 (c)

$$W = \frac{1}{\gamma - 1} (p_f V_f - p_i V_i)$$

$$W = \frac{1}{0.4} ((8.39 \ atm)(1.06 \ L) - (1.17 \ atm)(4.33 \ L))$$

$$W = \frac{1}{0.4} ((8.39)(1.01 \times 10^5)(1.06 \times 10^{-3}) - (1.17)(1.01 \times 10^5)(4.33 \times 10^{-3}))$$

$$W = 966 \ J$$

HRK 23.45

Adiabatic implies

$$pV^{\gamma} = \text{constant}$$

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}$$

$$\Rightarrow \frac{nRT_i}{V_i} (V_i)^{\gamma} = \frac{nRT_f}{V_f} (V_f)^{\gamma}$$

$$\Rightarrow T_i (V_i)^{\gamma-1} = T_f (V_f)^{\gamma-1}$$

$$\Rightarrow T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} T_i$$

$$T_f = \left(\frac{1}{1.28}\right)^{0.4} 292 \ K$$

$$T_f = 264.5 \ K$$

bearing in mind, we are told the volume expansion ratio is $\frac{V_f}{V_i} = 1.28$.

HRK 23.48

From the discussion of the kinetic interpretation of temperature, we saw that $E \propto T$, or more specifically

monatomic ideal gas:
$$E_{int} = \frac{3}{2}nRT$$

where n is the number of moles and R is a constant.

monatomic ideal gas:
$$E_{int} = \frac{3}{2}(1.0)(8.314)(273 + 25)$$

 $E_{int} = 3716 J$

since translation (in 3-d) is the only motion possible.

If a diatomic molecule can translate as well as rotate (but not vibrate)

diatomic ideal gas (no vibration):
$$E_{int} = \frac{5}{2}nRT$$

 $E_{int} = \frac{5}{2}(1.0)(8.314)(273 + 25)$
 $E_{int} = 6193 J$

A polyatomic molecule can translate in 3 dimensions but also has 3 rotational degrees of freedom

polyatomic ideal gas (no vibration):
$$E_{int} = \frac{6}{2}nRT$$

 $E_{int} = 3(1.0)(8.314)(273 + 25)$
 $E_{int} = 7433 J$

Note: It turns out that it is reasonable to ignore vibrational degrees of freedom for room temperature.

HRK 23.49

When the gas molecules are not monatomic (like an ideal gas), the energy depends on the degrees of freedom (d.o.f.) available to the molecule

$$E_{int} = \frac{\# \text{d.o.f}}{2} nRT$$

Air is 99% nitrogen and oxygen, both of which are diatomic $(N_2 \text{ and } O_2)$.

The degrees of freedom for a diatomic molecule are 3 translational ones, and 2 rotational ones. If we just concentrate on the portion associated with rotational motion we have

$$E_{int}^{rot} = \frac{2}{2}nRT$$

$$E_{int}^{rot} = (1.0)(8.314)(273 + 25)$$

$$E_{int}^{rot} = 2478 J$$

HRK 23.52

Van der Waals b is related to the diameter d of a molecule (when imagined as a hard sphere) via

$$b = \frac{1}{2}N_A\left(\frac{4}{3}\pi d^3\right)$$

$$\Rightarrow d^3 = \frac{3b}{2\pi N_A}$$

$$\Rightarrow d = \left(\frac{3b}{2\pi N_A}\right)^{\frac{1}{3}}$$

$$\Rightarrow d = \left(\frac{3(32 \times 10^{-6})}{2\pi (6.02 \times 10^{23})}\right)^{\frac{1}{3}}$$

$$\Rightarrow d = 2.94 \times 10^{-10} m$$

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