SOLUTIONS

Physics 22 Midterm #2 Spring 2010

Instructions:

First, this is a closed book exam. That means no notes, no books, and no use of a calculator.

Second, do not open your exam until I say it is OK to begin.

Third, while waiting to begin, please PRINT your name CLEARLY on your blue book.

Advice:

The problems are not arranged in increasing order of difficulty, so look for the ones you can do most easily first.

Please start each problem on a new page in your blue book, and please DO NOT do algebra in your head. First, you WILL make a mistake, and second you WILL NOT get any partial credit. Write it down, line-by-line as you do it. Also, please try to be NEAT, REALLY NEAT. This will help your grade, although in a perfect world maybe it shouldn't.

There are 4 problems in all, so try to work intensely for the entire 50 minutes. If you finish early, check your work instead of leaving. You can always find a mistake if you really try!

1. (20 points) A brass gear has an accurate cylindrical hole of diameter 5.000 cm bored along its axis, where the hole diameter is measured at 20° C. You would like to place the gear on a steel shaft whose diameter is 5.010 cm, as also measured at 20° C. Find the temperature at which it is just possible to place the gear on the shaft, assuming both the gear and the shaft are at the same temperature. The thermal expansion coefficient of brass is 19×10^{-6} K^{-1}, and that of steel is 11×10^{-6} K^{-1}. As usual, put the numbers in, but don't waste time doing arithmetic.

For the gear, the diameter of the hole as a fcn of T will be $\phi_G = 5.000\,[1+\alpha_B(T-20)]$, where T is in °C. (OR convert 20 °C to 293 K, and work in Kelvin.)

For the shaft $\phi_S = 5.010\,[1+\alpha_S(T-20)]$.

They have different expansion coefficients, with that of the gear being larger, so we will have to heat them until $\phi_G = \phi_S$ at the proper T.

$$5.000\,[1+\alpha_B(T-20)] = 5.010\,[1+\alpha_S(T-20)]$$

$$5.000 + 5\alpha_B(T-20) = 5.010 + 5.010\,\alpha_S(T-20)]$$

$$\text{so } (5\alpha_B - 5.01\alpha_S)(T-20) = 0.01 \text{ cm}$$

$$(5\times19 - 5.01\times11)(T-20)\times10^{-6} = 0.01 \text{ cm}$$

$$(T-20) = \frac{10^4 \text{ cm}}{39.89\ (\text{cm/K})}$$

This K means the size of the degrees, NOT the scale!

$$T-20 = 0.025\times10^4\ \cancel{K} = 250\ °C$$

$$\boxed{T = 270\,°C}$$

$$\begin{array}{r} 19 \\ \times 5 \\ \hline 95 \end{array} \qquad \begin{array}{r} 11 \\ \times 5.01 \\ \hline 55+0.11 \end{array} \qquad \begin{array}{r} 95 \\ -55.11 \\ \hline 39.89 \end{array}$$

2. Three moles of Helium, which is an ideal gas, are allowed to expand from an initial volume of 10 Liters to a final volume of 40 Liters. During the expansion, the temperature of the gas is held fixed at 27° C.

a. (10 points) Find the number of Joules of work the gas does on the piston during this expansion. In case you forgot, the gas constant R = 8.31 Joules/Mole Kelvin. Don't do the arithmetic; just be sure you put in the numbers correctly.

b. (10 points) How many Joules of heat did the gas absorb during the expansion? State the reasoning for your answer briefly.

Isothermal, $T = 27 + 273 = 300K$ (300.15 to be picky)

$W = \int_{V_i}^{V_f} P\,dV$, $PV = nRT$ so $P = \frac{nRT}{V}$

$$W = nRT \int_{V_i}^{V_f} \frac{dV}{V} \quad (T = \text{CONST})$$

$$= nRT\left[\ln V_f - \ln V_i\right] = nRT \ln V_f/V_i = nRT \ln 4$$

$$W = 3\text{ moles} \times 8.31 \frac{J}{\text{mole }K} \times 300K \ \ln 4$$

$= 10{,}368$ Joules

3. All systems are in thermal equilibrium at temperature T for this problem. Your answers should be based on classical physics, not on quantum mechanics.

a. (10 points) What is the rms speed of a particle of mass m that can only move in the x-y-plane?

b. (10 points) What is the average rotational kinetic energy of a rigid diatomic molecule made of point masses?

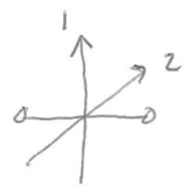
c. (10 points) What is the average vibrational energy of a diatomic gas molecule?

(a.) $E = K + \cancel{U}^{0} = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 \Rightarrow 2$ quadratic terms

$\therefore \langle E \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} k_B T + \frac{1}{2} k_B T = k_B T$

so $\langle v^2 \rangle = \frac{2 k_B T}{m}$ $\quad v_{rms} \equiv \sqrt{\langle v^2 \rangle} = \sqrt{\frac{2 k_B T}{m}}$

(b.) There are 2 possible axes of rotation, 1 and 2



I is the same for both.

$\therefore E = K + \cancel{U}^{0} = \frac{1}{2} I \omega_1^2 + \frac{1}{2} I \omega_2^2$

$\therefore \langle E \rangle = \frac{1}{2} k_B T + \frac{1}{2} k_B T = k_B T$

(c.) $E_{VIB} = K + U = \frac{1}{2} m' \dot{r}^2 + \frac{1}{2} k r^2$ $\quad r =$ displacement from Eq. pos.

$\therefore \langle E_{VIB} \rangle = \frac{1}{2} k_B T + \frac{1}{2} k_B T$

$= k_B T$

4. An object with moment of inertia I can only rotate about a fixed axis, and it is in thermal equilibrium at temperature T. Recall that the kinetic energy of rotation is $K_{ROT} = \frac{1}{2} I\omega^2$.

a. (15 points) Find the properly normalized probability density for the angular velocity ω.

[Hint: $\int_{-\infty}^{\infty} e^{-a^2x^2}\,dx = \frac{\sqrt{\pi}}{a}$]

b. (15 points) Write down the integral that equals the probability the cylinder will have angular speed less than the rms value. Don't even think about trying to do the integral, but do express the rms angular speed in terms of I, k_B, and T.

$$W(\omega) = c\,e^{-E/k_BT} = c\,e^{-I\omega^2/2k_BT}$$

It must have some angular velocity between $-\infty$, and ∞, and the meaning of $W(\omega)$ is that $W(\omega)\,d\omega$ is the probability it has angular velocity in the range ω to $\omega + d\omega$. So we must have

$$c\int_{-\infty}^{\infty} e^{-I\omega^2/2k_BT}\,d\omega = 1 = c\,\frac{\sqrt{\pi}}{\sqrt{I/2k_BT}}$$

$$\therefore 1 = c\sqrt{\frac{2\pi k_BT}{I}} \qquad c = \sqrt{\frac{I}{2\pi k_BT}}$$

$$\boxed{W(\omega) = \sqrt{\frac{I}{2\pi k_BT}}\,e^{-I\omega^2/2k_BT}}$$

b. $\frac{1}{2}I\omega^2 = \bar{E} \Rightarrow \langle\omega^2\rangle = \frac{k_BT}{I}$ $\qquad \omega_{rms} = \sqrt{\frac{k_BT}{I}}$

so $$P(|\omega| < \omega_{rms}) = \sqrt{\frac{I}{2\pi k_BT}}\int_{-\sqrt{k_BT/I}}^{+\sqrt{k_BT/I}} e^{-I\omega^2/2k_BT}\,d\omega$$