Taylor Series Problems

1. Use a Taylor series expansion around $x = 0$, to show that $\frac{1}{1+x} \simeq 1 - x + x^2 - x^3 + x^4 + \ldots$ To do this you need to be organized. Use the following table to achieve this, by working out each part before you try to add them up.

f, f', f'', etc.	Value at x = 0	Resulting term
$(1+x)^{-1}$		

This series will not converge for $|x| \geq 1$. Do you see why not? What happens to our original function for x = -1? It blows up to infinity. This singularity at x = -1, sets the so-called "radius of convergence" at $|x| < 1$.

2. Use a Taylor series expansion to show that $Sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$
Hint: Use the same sort of table to avoid errors! Can you write down the rest of the series by recognizing the pattern? This one converges for any angle because the factorials get big so fast.

3. Use a Taylor series expansion to show that $Cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots$

4. Show that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$. Now let $x = i\theta$ and prove Euler's theorem [$e^{i\theta} = Cos(\theta) + i\,Sin(\theta)$], by showing that the two sides have the same Taylor series expansions.