Physics 220: Problem Set 1
due April 14, 2011.

1. **Kardar, Chapter 1, Problem 5.**

2. **Kardar, Chapter 2, Problem 2:** The solution is in the back of the book, so this problem will not be graded.

3. **Coupled orders:** Two different order parameters (which we will denote m and n) may occur in the same system, for instance magnetism and superconductivity, or ferroelectricity and magnetism. If they are completely distinct, i.e. characterize different broken symmetries, then the free energy for the system takes the generic form

$$F = \int d^d\mathbf{x} \left[r_1 m^2 + r_2 n^2 + \frac{u_1}{2} m^4 + \frac{u_2}{2} n^4 + v m^2 n^2 \right]. \tag{1}$$

   Here we neglect gradients and will just perform a saddle point analysis. Assume $u_1, u_2 > 0$ and $v^2 < u_1 u_2$. The parameters r_1 and r_2 may take either sign, and can be tuned independently by varying two external parameters, such as temperature and pressure.

   (a) Draw the phase diagram in the $r_1 - r_2$ plane when the order parameters are uncoupled, $v = 0$.

   (b) Now consider $v \neq 0$. Find the optimal free energy for the four possible states (local minima) of the system: $m = n = 0$, $m \neq 0, n = 0$, $m = 0, n \neq 0$, and $m, n \neq 0$. What are the conditions on r_1, r_2 such that the latter 3 local minima exist?

   (c) By comparing these four free energies, find the phase diagram for $v > 0$.

   (d) Repeat for $v < 0$.

4. **Kardar, Chapter 3, Problem 2:** The solution is in the back of the book, so this problem will not be graded.

5. **Kardar, Chapter 3, Problem 7.**