Physics 220: Problem Set 2 due April 28, 2011.

1. Kardar, Chapter 4, Problem 3.

2. General recursion relations for an n-component model to one loop: Consider the very general model Hamiltonian

$$\beta \mathcal{H} = \int d^d \mathbf{x} \left\{ \frac{K}{2} |\nabla \vec{m}|^2 + \frac{t}{2} |\vec{m}|^2 + \frac{1}{4!} \sum_{abcd} u_{abcd} \, m_a m_b m_c m_d \right\},\tag{1}$$

where $a, b, c, d = 1 \cdots n$, and u_{abcd} is a general rank 4 tensor which can be taken to be symmetric under any permutation of *abcd*.

- (a) Derive the differential recursion relation for u_{abcd} to second order in u, and leading order in $\epsilon = 4 d$.
- (b) Show that the form of the RG equation for u given in class and in Kardar is recovered if you take $u_{abcd} = 8u(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})$.
- (c) Derive the RG equations given in Kardar, chapter 5, problem 4 for the Hamiltonian given in Kardar, chapter 5, problem 3, by using the general equations you derived in part (a) above.
- 3. Kardar, Chapter 5, Problem 6.