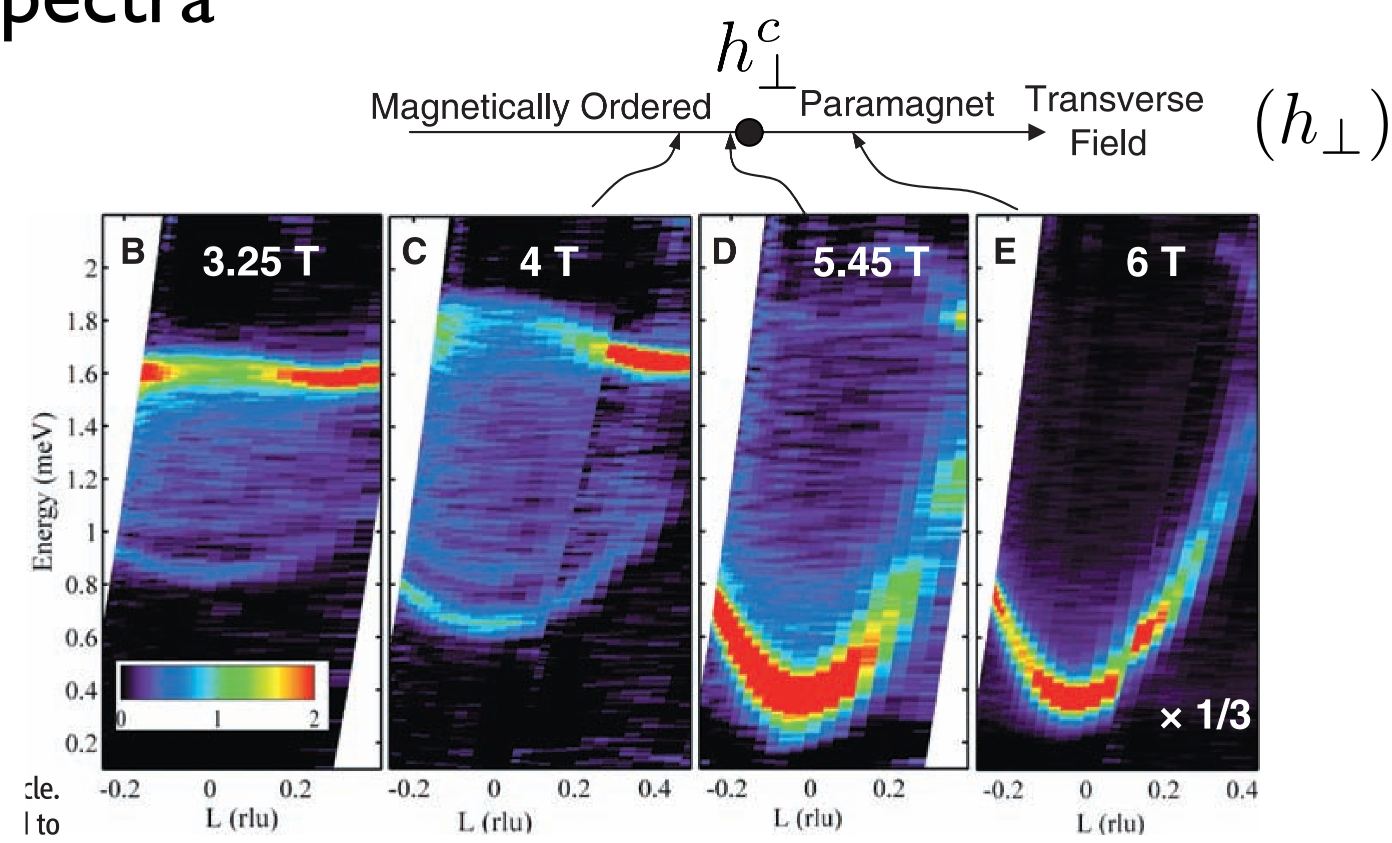
Coldea

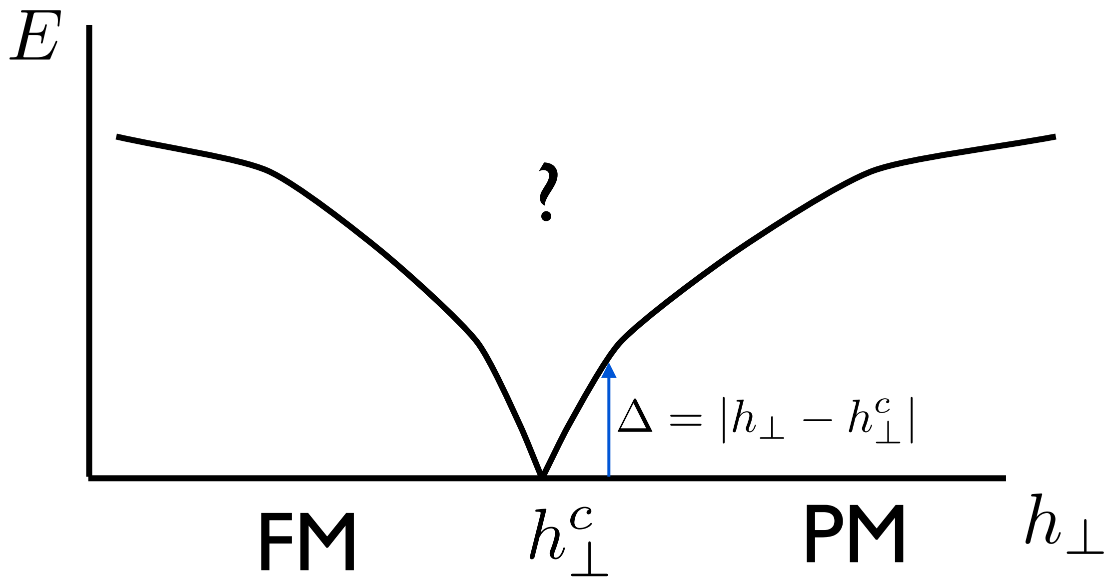
- Spectra



continuum broken into many small dispersion curves

sharply peaked dispersion

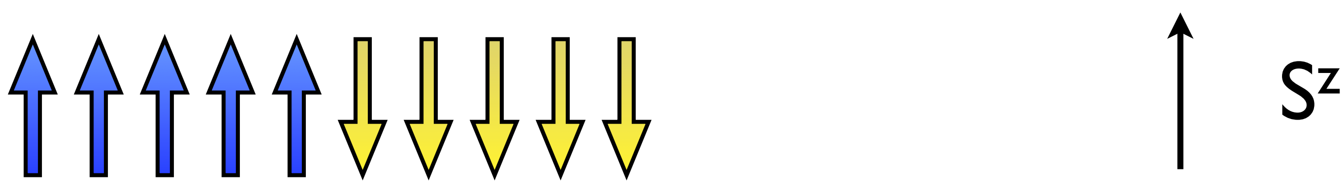
Excitations



- From scaling: expected excitation gap except at QCP
  - what is the nature of the excitations?

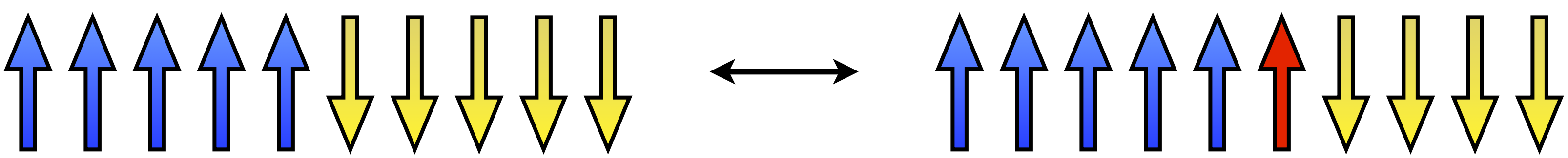
FM phase

- Domain walls



ΔE=J/2 $(h_\perp = 0)$

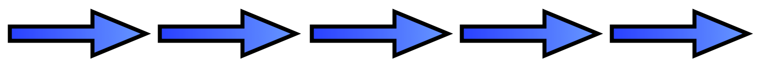
- Hopping



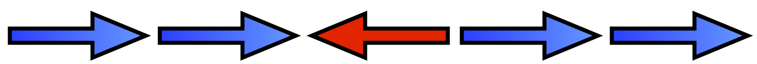
$$\epsilon_{dw}(k) \sim J/2 - h_\perp \cos k$$

PM phase

- J=0: ground state is spins polarized along x

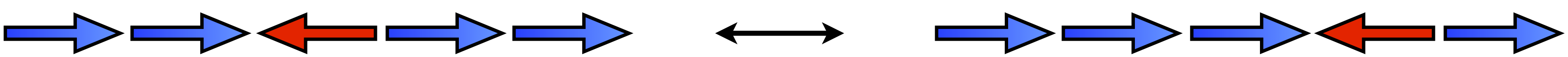


- Excitations are single spin flips



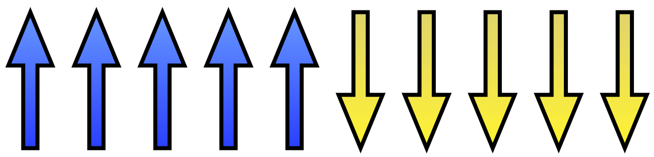
$$\epsilon = h_\perp$$

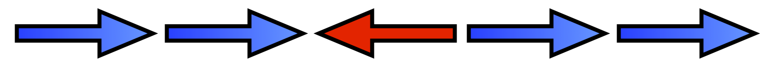
- Hopping



$$\epsilon_{sf}(k) \sim h_\perp - \frac{J}{2}\cos k$$

Local vs Non-local





- Domain wall is *non-local*: a semi-infinite number of spins must be flipped to generate it from the ground state
- The misaligned spin in the x-polarized state is *local*: only one spin needs to be flipped to generate it
  - A neutron can excite a single spin flip, but *not* a single domain wall

Scattering Intensity

- Recall

$E = E_{in} - E_{out}$

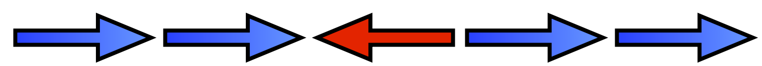
$k = k_{in} - k_{out}$

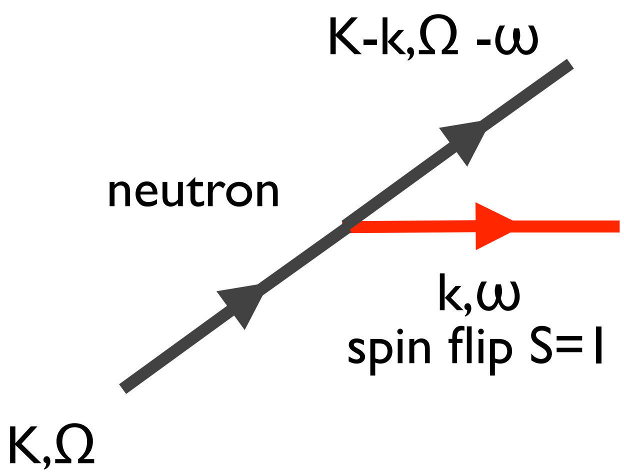
$\Delta S = 1$

measure

$$A(k,E) \sim \sum_n |\psi_n|^2 \delta(E - \epsilon_n(k))$$

- In the paramagnet: neutron creates one spin flip:





$\omega = \varepsilon(k)$

Scattering Intensity

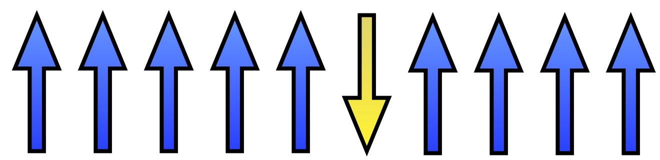
- Recall

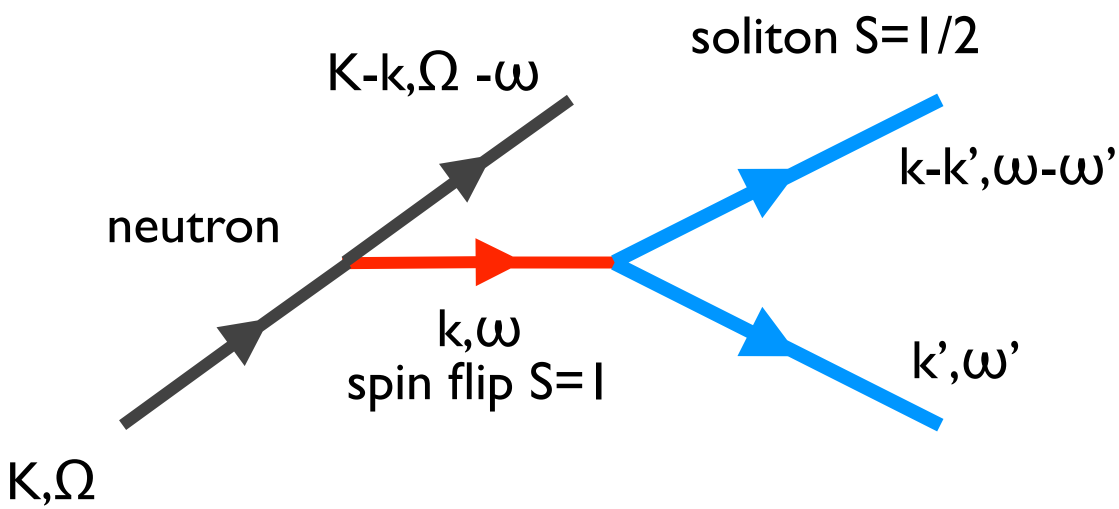
$$E = E_{in} - E_{out}$$
$$k = k_{in} - k_{out}$$
$$\Delta S = 1$$

measure

$$A(k,E) \sim \sum_n |\psi_n|^2 \delta(E - \epsilon_n(k))$$

- In the ferromagnet: neutron creates two domain walls:





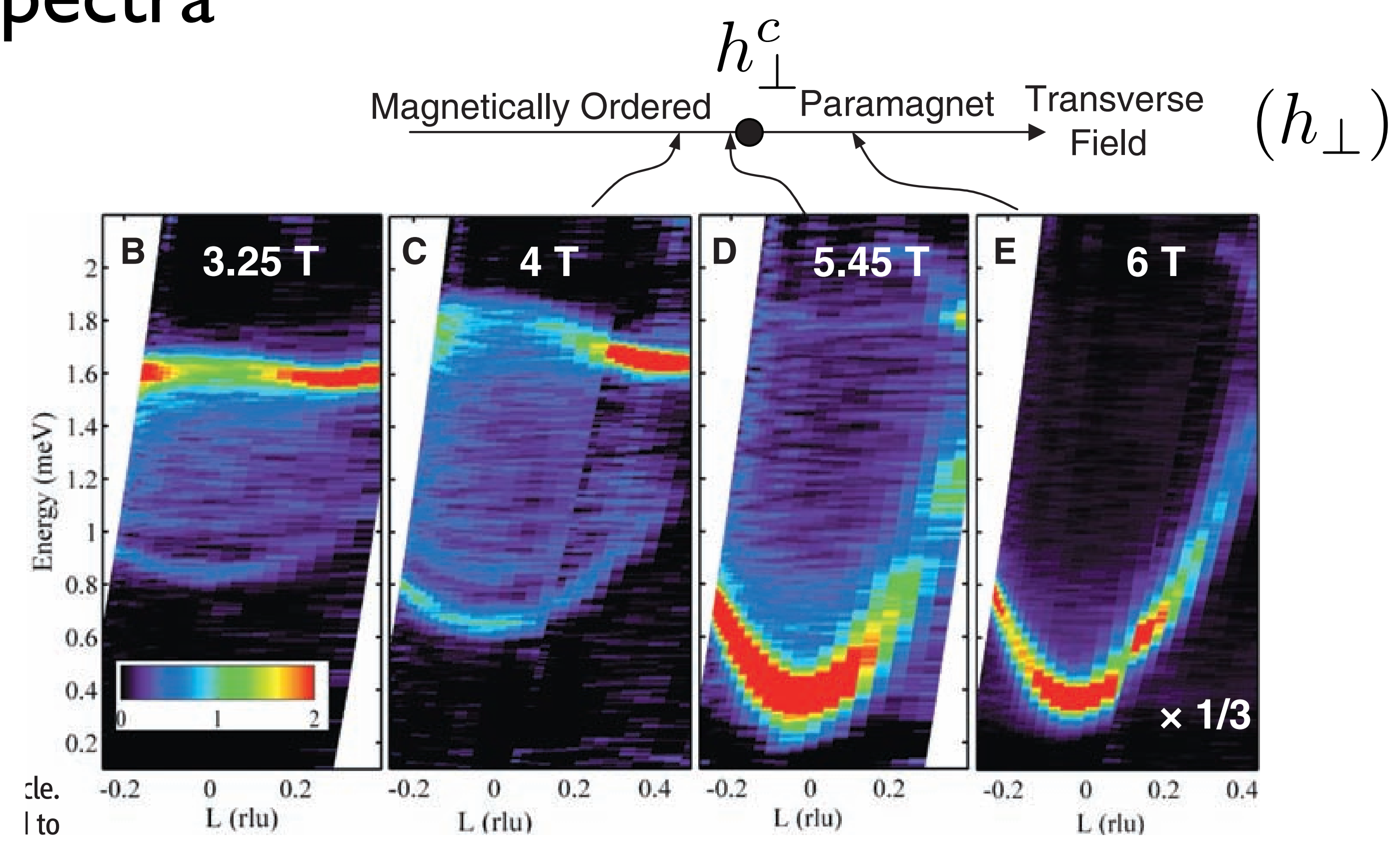
$$\omega = \varepsilon(k') + \varepsilon(k-k')$$

$$A(k,\omega) \sim \int dk' \; f(k') \delta(\omega - \epsilon(k') - \epsilon(k-k'))$$

2-particle continuum

Coldea

- Spectra



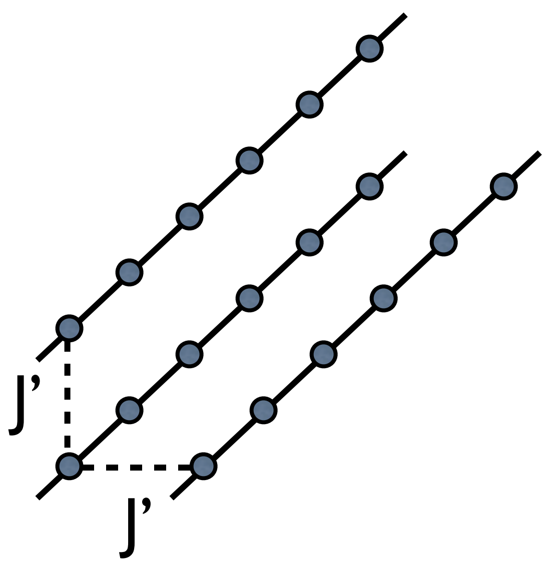
2 soliton continuum
?? why the fine structure ??

single spin flip

Fine structure

- This is due to *three dimensional coupling* between the Ising chains

$$H' = -J' \sum_{n} \sum_{\langle ij \rangle} S^z_{i,n} S^z_{j,n}$$

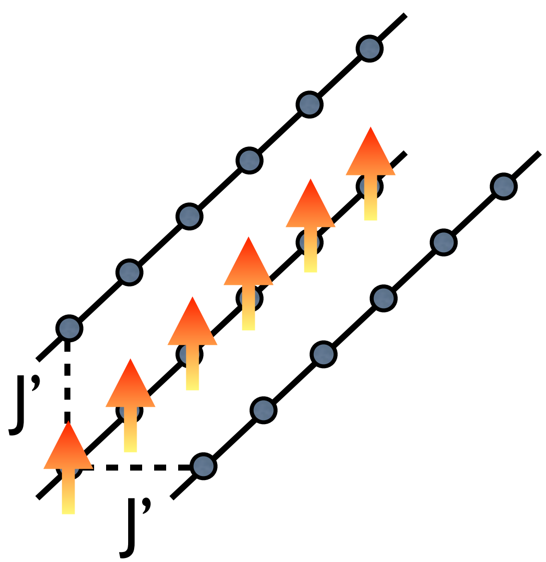


Does very small J' have an effect?

Fine structure

- This is due to *three dimensional coupling* between the Ising chains

$$H' = -J' \sum_{n} \sum_{\langle ij \rangle} S^z_{i,n} S^z_{j,n}$$

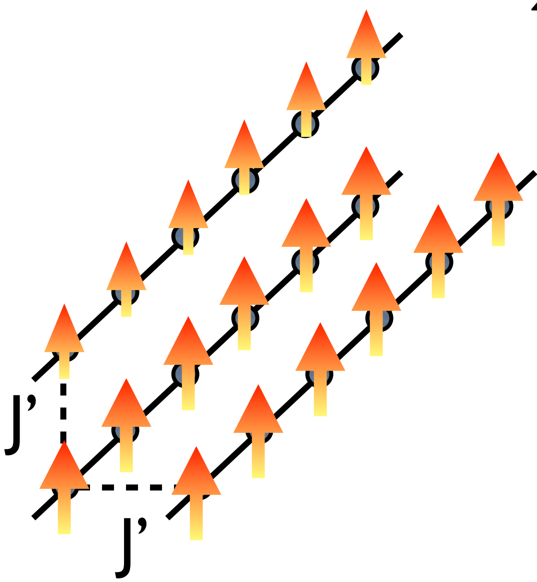


Suppose chains are ferromagnetic

Fine structure

- This is due to *three dimensional coupling* between the Ising chains

$$H' = -J' \sum_n \sum_{\langle ij \rangle} S^z_{i,n} S^z_{j,n}$$

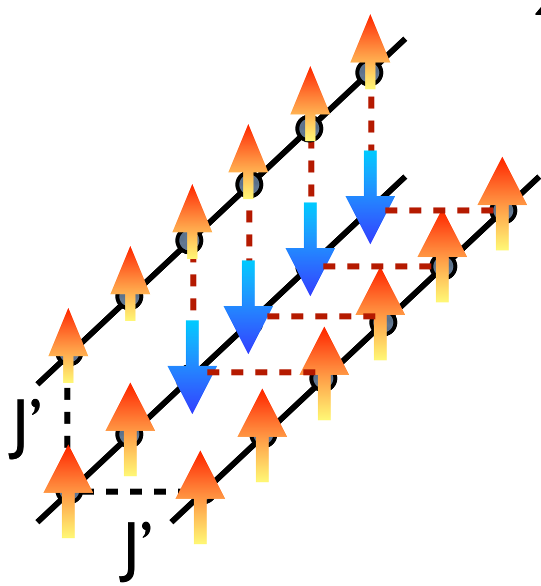


J' prefers they align

Fine structure

- This is due to *three dimensional coupling* between the Ising chains

$$H' = -J' \sum_{n} \sum_{\langle ij \rangle} S^z_{i,n} S^z_{j,n}$$

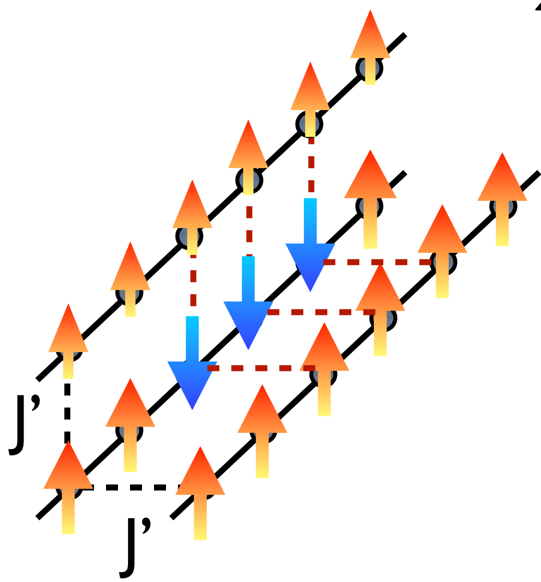


O(J') energy cost *per misaligned bond*: infinite in thermodynamic limit!

Fine structure

- This is due to *three dimensional coupling* between the Ising chains

$$H' = -J' \sum_n \sum_{\langle ij \rangle} S^z_{i,n} S^z_{j,n}$$



pair of domain walls separated by x on the same chain costs an energy ∝ J' |x|: *linear confinement*

Confinement

- Mean field

$$H' \to -h_{\parallel} \sum_{i,n} S^z_{i,n} \qquad h_{\parallel} \propto J' \langle S^z_{i,n} \rangle = J' m$$

- Confining potential

$$V(x) = \lambda |x| \qquad \lambda = h_{\parallel} m$$

- Two particle quantum mechanics

$$H_{\text{eff}} = 2\epsilon_{\text{dw}} - \frac{1}{2\mu}\frac{\partial^2}{\partial x_1^2} - \frac{1}{2\mu}\frac{\partial^2}{\partial x_1^2} + \lambda |x_1 - x_2|$$

Confinement

- Relative coordinate

$$H_{\text{eff}} = 2\epsilon_{\text{dw}} - \frac{1}{\mu}\frac{\partial^2}{\partial x^2} + \lambda|x|$$

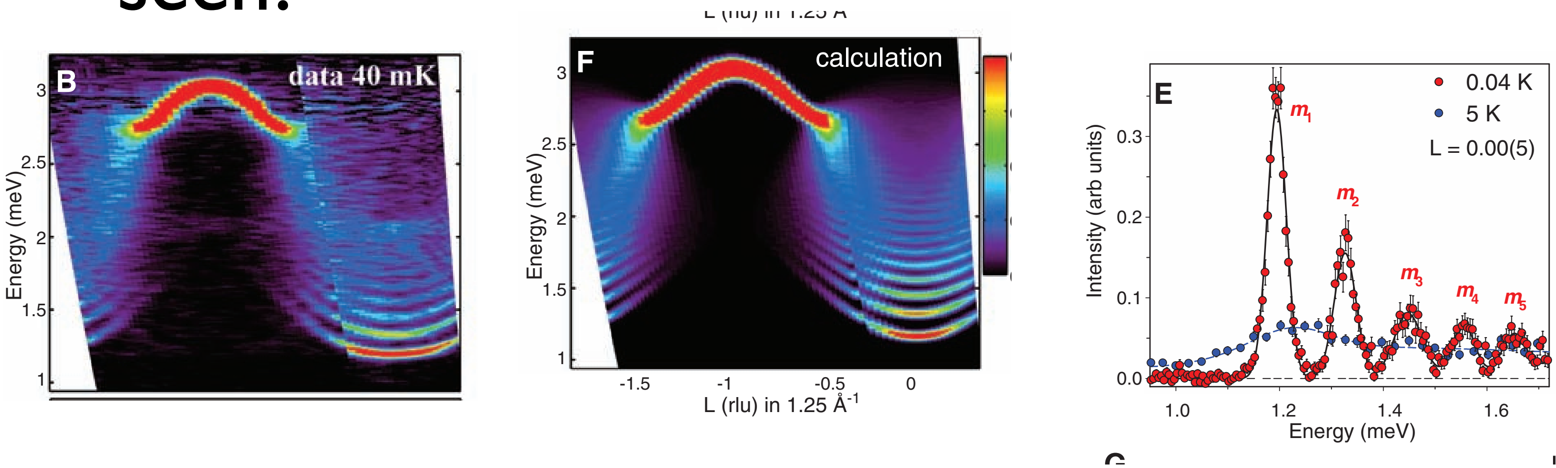
- Standard problem in WKB theory: Airy functions

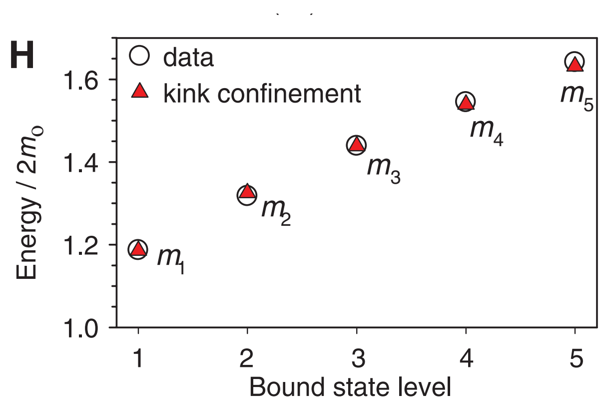
$$E_n = 2\epsilon_{\text{dw}} + z_j(\lambda^2/\mu)^{2/3}$$

- z_j = 2.33, 4.08, 6.78.. zeros of Airy function
- apart from z_j, get this from scaling...

Experiment

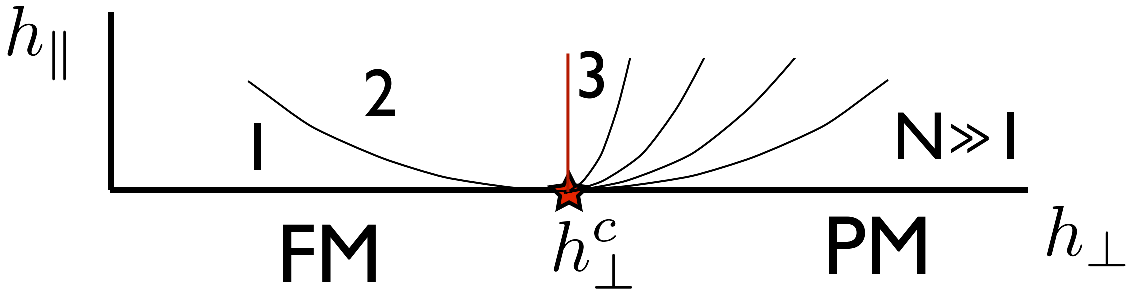
- Airy function levels are very beautifully seen!

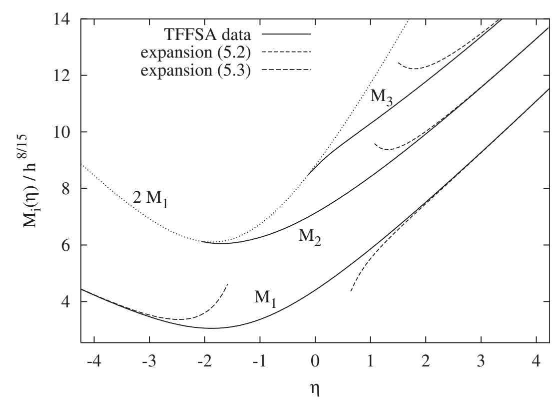




Field evolution?

- Number of bound states evolves with $h_\perp$





Fonseca + Zamolodchikov, 2002

- Precisely at $h_\perp = h_\perp^c$, there is an exact solution
  - Scaling $\epsilon_n \sim c_n (h_{\|}/v)^{8/15}$

$$\epsilon_2/\epsilon_1 = (1+\sqrt{5})/2\,!!$$