Integrating out c,c†

- Formally

$$Z = \int [d\Phi][dc\, dc^\dagger] e^{-S[\Phi,c,c^\dagger]} = \int [d\Phi] e^{-S_{\text{eff}}[\Phi]}$$

- Fermionic integral may be singular
  - It involves an infinite number of d.o.f.
  - Fermions are gapless: low energy electron/hole excitations mean fermion correlation functions behave like power-laws at large x,τ

J.A. Hertz, PRB **14**, 1165 (1976)

Hertz Theory

- Formally

$$Z = \int [d\Phi] e^{-S_{\text{eff}}[\Phi]}$$

$$e^{-S_{\text{eff}}[\Phi]} = e^{-S_{\text{spin}}[\Phi]} \int [dc\, dc^\dagger] e^{-S_{\text{el}}[c,c^\dagger]} \underbrace{e^{-J_K \int d^d\mathbf{r} d\tau \left(\vec{\Phi}_{\mathbf{r},\tau} e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}\right)\cdot \vec{s}_{\mathbf{r},\tau}}}_{\text{expand this out}}$$

- Result:

$$S_{\text{eff}}[\Phi] = S_{\text{spin}}[\Phi] - \int \frac{d^d\mathbf{k} d\omega_n}{(2\pi)^{d+1}} \frac{\chi_0(\mathbf{Q}+\mathbf{k},\omega_n)}{2} \vec{\Phi}_{\mathbf{k},\omega_n} \cdot \vec{\Phi}_{-\mathbf{k},-\omega_n} + O(\Phi^4)$$

Hertz Theory

- The free electron susceptibility behaves like

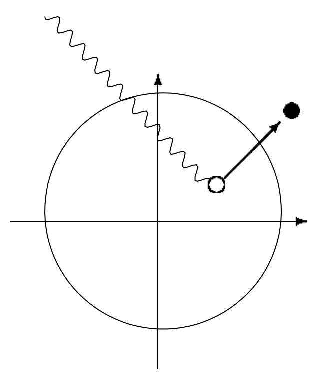
$$\chi_0(\mathbf{Q}+\mathbf{k},\omega_n) \approx c_0 + c_1 k^2 + c_2|\omega_n| \qquad Q \neq 0$$

$$\approx c_0 + c_1 k^2 + c_2 \frac{|\omega_n|}{v_F k} \qquad Q = 0$$

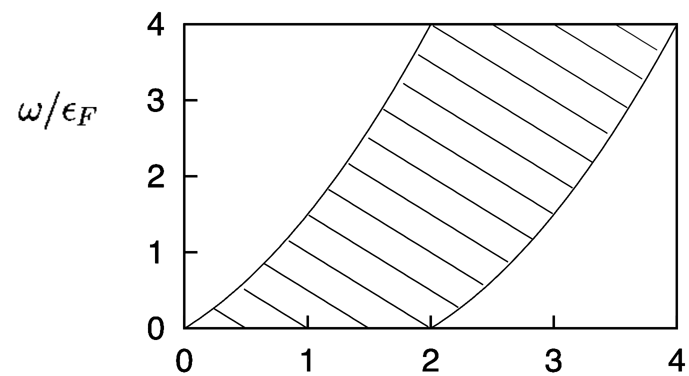
- Importantly, note the *non-analytic* $|\omega_n|$ dependence - this reflects *spin damping*. The spins can exchange energy (and spin) with the electron gas
  - Unfortunately deriving this is a bit complicated, but you would learn it, e.g., in Physics 217b.

Electron-hole pairs

- The non-analytic $|\omega_n|$ term arises because the spin fluctuation can decay into or mix with an electron hole pair at low energy



$Q=k_p-k_h$



Q/k_F

Landau expansion

- Add the fermion term to the Landau theory

$$S = \int \frac{d^d\mathbf{k}d\omega_n}{(2\pi)^{d+1}}\Big\{(k^2 + \frac{|\omega_n|}{k^a} + r)|\Phi_{\mathbf{k},\omega_n}|^2\Big\} + u\int d^d\mathbf{x}d\tau\,|\Phi_{\mathbf{r},\tau}|^4$$

$$= \int \frac{d^d\mathbf{k}d\omega_n}{(2\pi)^{d+1}}\Big\{(k^2 + \frac{|\omega_n|}{k^a} + r)|\Phi_{\mathbf{k},\omega_n}|^2\Big\}$$

$$+u\int \frac{d^{3d}\mathbf{k}_i d^3\omega_{n,i}}{(2\pi)^{3d+3}}\,\Phi_{k_1,\omega_{n1}}\Phi_{k_2,\omega_{n2}}\Phi_{k_3,\omega_{n3}}\Phi_{-k_1-k_2-k_3,-\omega_{n1}-\omega_{n2}-\omega_{n3}}$$

a=0,1 (Q≠0, Q=0)