Power counting

- Rescaling: $k \to k/b \qquad \omega_n \to \omega_n/b^z$

$$\Phi_{k,\omega_n} \to b^{d+z-d_\Phi}\,\Phi_{bk,b^z\omega_n}$$

$$S = \int \frac{d^d\mathbf{k}\,d\omega_n}{(2\pi)^{d+1}}\left\{\left(k^2 + \frac{|\omega_n|}{k^a} + r\right)|\Phi_{\mathbf{k},\omega_n}|^2\right\} + u\int \frac{d^{3d}\mathbf{k}_i d^3\omega_{n,i}}{(2\pi)^{3d+3}}\,\Phi_{k_1,\omega_{n1}}\Phi_{k_2,\omega_{n2}}\Phi_{k_3,\omega_{n3}}\Phi_{-k_1-k_2-k_3,-\omega_{n1}-\omega_{n2}-\omega_{n3}}$$

$z = 2 + a$

$$-d - z - 2 + 2(d + z - d_\Phi) = 0$$

$$d + a - 2d_\Phi = 0$$

$$d_\Phi = \frac{d+a}{2}$$

k versus x scaling

- Note: Fourier transform

$$\Phi_{\mathbf{k},\omega_n} = \int d^d\mathbf{x}\, d\tau\, e^{-i\mathbf{k}\cdot\mathbf{x} - i\omega_n \tau} \Phi_{\mathbf{x},\tau}$$

- Space-time scaling

$$\Phi_{\mathbf{x},\tau} = \boxed{b^{-d_\Phi}} \Phi'_{\mathbf{x}/b,\tau/b^z}$$

- Hence

$$\Phi_{\mathbf{k},\omega_n} = b^{-d_\Phi} \int d^d\mathbf{x}\, d\tau\, e^{-i\mathbf{k}\cdot\mathbf{x} - i\omega_n \tau} \Phi'_{\mathbf{x}/b,\tau/b^z}$$

$$= b^{-d_\Phi + d + z} \int d^d\mathbf{x}\, d\tau\, e^{-ib\mathbf{k}\cdot\mathbf{x} - ib^z \omega_n \tau} \Phi'_{\mathbf{x},\tau}$$

$$= \boxed{b^{-d_\Phi + d + z}} \Phi'_{b\mathbf{k}, b^z \omega_n}$$

note difference!

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$$z = 2 + a$$

$$d_\Phi = \frac{d+a}{2}$$

$$r \to b^2 r$$

$$u \to b^{2-d-a}u = b^{4-d-z}u$$

u is “irrelevant” when d+a>2

Aside: Classical Case

- Power counting $x \to bx$ $\Phi_x \to b^{-d_\Phi} \Phi_{x/b}$

$$F = \int d^d x \Big\{ (\nabla \Phi)^2 + r\Phi^2 + u\Phi^4 \Big\}$$

- Gradient term $d_\Phi = \dfrac{d-2}{2}$
- RG:

$$r \to b^2 r$$
$$u \to b^{4-d} u$$

u is irrelevant for d>4

Upper critical dimension

- It turns out that for d>4, one gets *mean field behavior*. We call $d_{u.c.}=4$ the *upper critical dimension*
  - This coincides with - and is a consequence of - the fact that *u* is irrelevant, i.e. that the Gaussian fixed point is stable.
  - Below the u.c.d., critical exponents are non-MF like

Classical scaling for d>4

- Correlation length: ν=1/2

$$\xi = b\, g(r\, b^2, u\, b^{4-d}) = |r|^{-1/2} g(\pm 1, u|r|^{(d-4)/2})$$

- Free energy

$$f = b^{-d} \mathcal{F}(r\, b^2, u\, b^{4-d}) = |r|^{d/2} \mathcal{F}(\pm 1, u\, |r|^{(4-d)/2})$$

  - r>0: $f \sim |r|^{d/2}$
  - r<0: u is necessary for stability

$$f \sim |r|^{d/2} [u\, |r|^{(4-d)/2}]^{-1} \sim r^2/u \qquad \alpha=0$$

u is a *dangerously irrelevant* operator