Classical scaling for d>4

- Order parameter

$$m \sim b^{-(d-2)/2}\mathcal{M}(\pm 1, u|r|^{(d-4)/2})$$

- m vanishes for r>0 and again is singular for r<0 ($m \sim u^{-1/2}$)

$$m \sim b^{-(d-2)/2}[u|r|^{(d-4)/2}]^{-1/2} \sim |r|^{1/2}$$

β=1/2

Back to Hertz

critical point is “trivial” ?

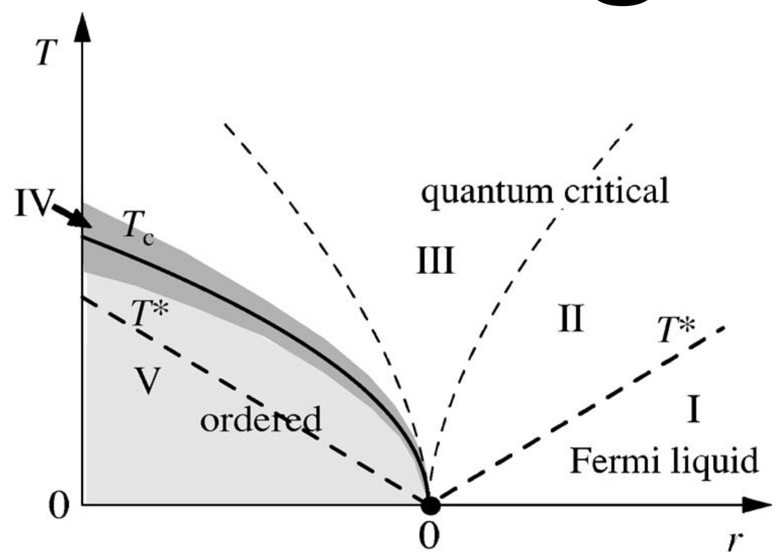
$$S = \int \frac{d^d\mathbf{k} d\omega_n}{(2\pi)^{d+1}} \left\{ (k^2 + \frac{|\omega_n|}{k^a} + r)|\Phi_{\mathbf{k},\omega_n}|^2 \right\} + u \int \frac{d^{3d}\mathbf{k}_i d^3\omega_{n,i}}{(2\pi)^{3d+3}} \Phi_{k_1,\omega_{n1}} \Phi_{k_2,\omega_{n2}} \Phi_{k_3,\omega_{n3}} \Phi_{-k_1-k_2-k_3,-\omega_{n1}-\omega_{n2}-\omega_{n3}}$$

- Additional ingredient for QCP: Temperature scaling:
  - relative to renormalized low energy scale, temperature *increases* under RG

$$k_B T \to b^z \, k_B T$$

  - Also seen from action $S = \int_0^\beta d\tau \cdots$

“Fan” diagram



- Two relevant perturbations of QCP
  - r: deviation from critical point at T=0
  - T: temperature

$$r \to b^2 r \qquad\qquad k_B T \to b^z\, k_B T$$

Quantum critical scaling

- Example: energy density

$$\varepsilon \sim b^{-(d+z)}\mathcal{E}(r\,b^2, k_B T\,b^z, u\,b^{4-d-z})$$

- Let's sit *at* the QCP (r=0) and raise temperature

$$\varepsilon \sim b^{-(d+z)}\mathcal{E}(0, k_B T\,b^z, u\,b^{4-d-z})$$

$$\sim (k_B T)^{\frac{d+z}{z}}\tilde{\mathcal{E}}(u\,(k_B T)^{\frac{d+z-4}{z}}) \;\; \sim (k_B T)^{\frac{d+z}{z}}$$

- Specific heat

$$c_v \sim \partial\varepsilon/\partial T \sim T^{d/z} \qquad \sim T^{3/2} \;\; \text{for 3d AF}$$

Quantum critical scaling

- Thermal expansion coefficient

$$\alpha = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_p = -\frac{1}{V}\left.\frac{\partial S}{\partial p}\right|_T$$

- We can deduce entropy scaling from specific heat

$$S \sim \int_0^T dT' \, \frac{C(T')}{T'} \sim T^{3/2}$$

- Hence

$$S \sim T^{3/2}\mathcal{S}(rT^{-2/z})$$

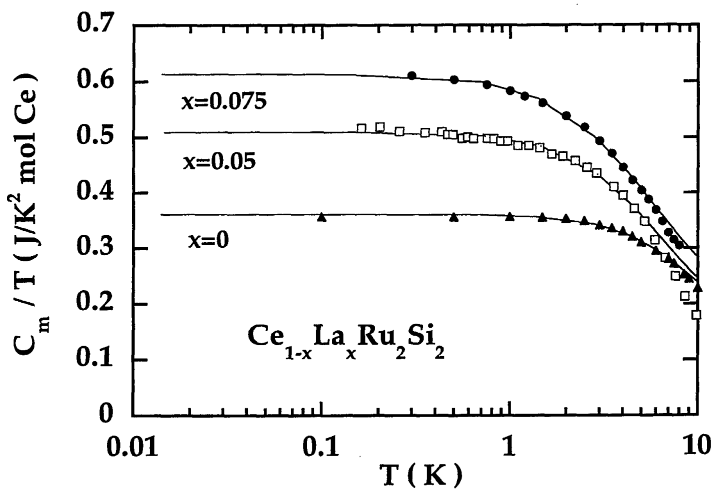
- For a pressure tuned transition then r ~ p

$$\alpha \sim \frac{\partial S}{\partial r} \sim T^{1/2}$$

(it is usually linear in a metal)

$Ce_{1-x}La_xRu_2Si_2$

- This seems to be one of the rare examples where Hertz theory works



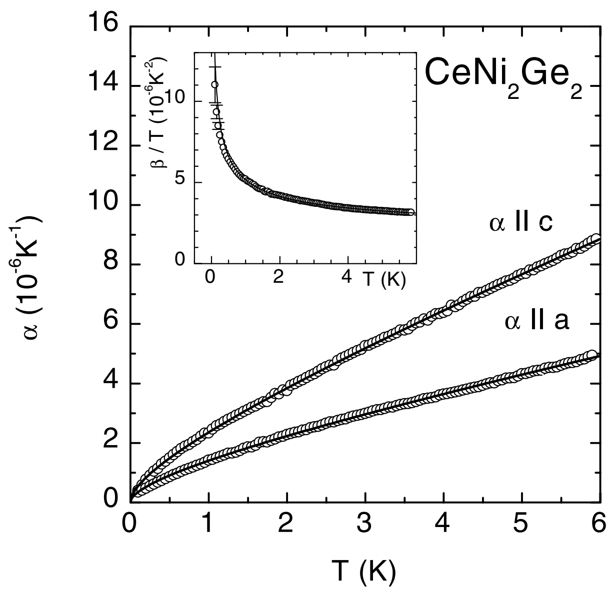
S. Kambe *et al*, JPSJ **65**, 3294 (1996)

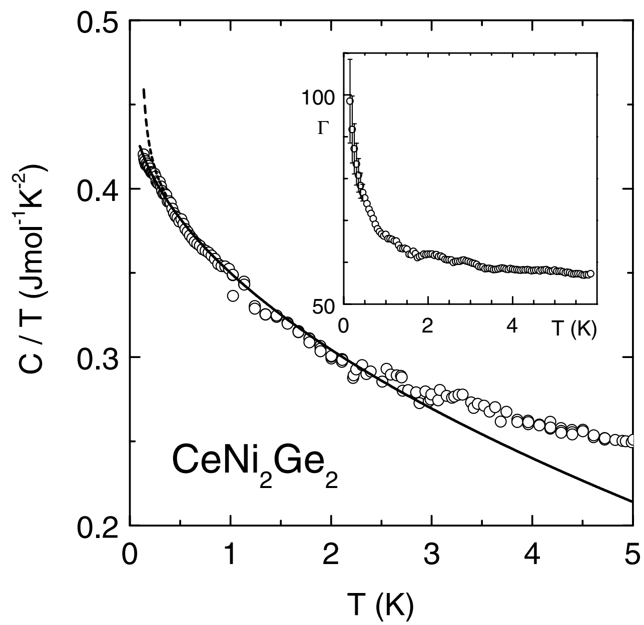
$$c_v \sim \gamma T - cT^{3/2}$$

Fit is to a (slightly) more sophisticated theory which includes r $\neq 0$

$CeNi_2Ge_2$

- Believed to be “close” to an AF QCP at ambient pressure





$$\alpha \sim aT^{1/2} + bT$$

$$c/T \sim \gamma - cT^{1/2}$$