

Physics 220: Problem Set 1  
due April 12, 2012.

1. Show by direct calculation that in MFT the susceptibility diverges like  $\chi \sim A/(T - T_c)$  and the specific heat has a jump discontinuity, as claimed in class.
2. Use the transfer matrix technique to calculate the spin-spin correlation function for the 1d classical Ising model,  $\langle \sigma_i \sigma_j \rangle$ , as a function of  $|i - j|$  and  $K = \beta J$ . Extract the correlation length as a function of  $K$ .
3. Carry out Curie-Weiss MFT at *zero temperature* for the *quantum* transverse field Ising chain. That is, decouple the exchange term ( $J$  term) to reduce the problem to that of independent spins in both a longitudinal (along  $z$  - this is the effective exchange field) and transverse (along  $x$ ) field. Put each of these spins in its ground state (since it is  $T = 0$ ), and make your solution self-consistent. Find the quantum critical point in mean-field theory and find the longitudinal susceptibility,  $-\frac{1}{L} \frac{\partial^2 E}{\partial h_x^2}$  (here  $E$  is the ground state energy), in the same approximation.