Physics 223B: Homework 1

due January 24, 10am in Prof. Balents' mailbox at the KITP

1. **Layers:** Suppose n two-dimensional topological insulators are stacked atop one another. Assuming electrons can tunnel between adjacent layers, and disorder is present, for which n can you *guarantee* the existence of a conducting channel at the boundary/edge? Why?

2. **3d Dirac Symmetries:** In class, we used time-reversal and inversion symmetry to derive the general form for the effective Hamiltonian near a point in which odd and even parity bands exchange. We argued that it took the form of the Dirac Hamiltonian:

$$H = \sum_{\mu=1}^{3} v_\mu k_\mu \Gamma_\mu + m\Gamma_4. \tag{1}$$

The matrices Γ_a $(a = 1 \cdots 5)$, $\Gamma_{a,b} = -\frac{i}{2}[\Gamma_a, \Gamma_b]$ (the brackets indicate commutator, and $a < b = 1 \cdots 5$), and the identity form a set of 16 linearly independent matrices that span the full space of all possible 4×4 matrices. Please determine the transformation properties of all these matrices under time-reversal and inversion symmetries.

3. **Weyl semimetal:** Consider the 3d Dirac Hamiltonian with the coordinates rescaled so that the velocity is uniform:

$$H = v \sum_{\mu=1}^{3} k_\mu \Gamma_\mu + m\Gamma_4. \tag{2}$$

Imagine a perturbation is applied of the form

$$H' = b\Gamma_{1,2}. \tag{3}$$

This corresponds to the formation of a certain type of magnetic order with strength proportional to b.

   (a) Find the energies of the combined Hamiltonian $H + H'$.

   (b) What is the condition that there are states at zero energy? Where do those states occur in momentum space (it should correspond to some discrete points $\vec{k} = \vec{K}_i$).

   (c) In the case in which there are zero energy states, find the dispersion of the excitations around the "nodal" points, i.e. find the energy spectrum for the low energy states to linear order in $\vec{q} = \vec{k} - \vec{K}_i$.