Physics 223b: Problem Set 3
due 10am, February 11, 2014 in Prof. Balents' mailbox at the KITP

1. **EuO:** Europium oxide is a black, ferromagnetic insulator with the "rock salt" structure, i.e. with Eu and O occupying the two interpenetrating fcc sublattices of a simple cubic lattice. One expects the Eu atoms to be in a Eu^{2+} state.

   (a) Show that Hunds rules predict the total angular momentum $J = S$ for the Europium atom. What is S?

   (b) The Eu spins interact via ferromagnetic exchange interactions $J_1 = 2.4K$ between true nearest neighbors, and $J_2 = 0.48K$ between second neighbors (i.e. between Eu atoms separated by an O atom along one of the principle axes of the conventional simple cubic lattice). Apply MFT to estimate the Curie point T_c of EuO. Compare to the true $T_c = 69.4K$.

2. **Antiferromagnet in a field:** Consider the antiferromagnetic spin Hamiltonian on the square lattice:

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - h \sum_i S_i^z, \tag{1}$$

   where $J > 0$ is the exchange coupling between nearest-neighbor spins, and h is an external magnetic field (measured in energy units) which couples to the spins through the Zeeman interaction.

   (a) In the classical limit, you may treat $\vec{S}_i$ as a vector of length S. Describe (by a few sketches, no need for extensive quantitative calculations) the classical ground state spin configuration as a function of $0 < h/J < \infty$. Show that for $h > h_{sat} = 8JS$, the spins are "saturated", i.e. fully aligned along the z axis.

   (b) Sketch the magnetization (average values of S^z) versus h/J.

   (c) Now let us treat the problem quantum mechanically, with each $\vec{S}_i$ as a spin operator of spin S. For large values of h/J, we may again assume the ground state is the fully aligned one. Calculate the excitation energy of single magnons above this state. From this calculation, predict the saturation field h_{sat} expected in the quantum model, and compare to the classical limit.