Physics 223b: Problem Set 4
due 10am, February 21, 2014 in Prof. Balents' mailbox at the KITP

**Yes, just one problem, since this is somewhat last notice!**

1. **Friedel Oscillations:** Consider a free electron gas (Sommerfeld model) confined in one direction by hard walls (infinite potential barriers) to $0 < x < L$, and infinite in the other two directions, taking $L \to \infty$ eventually. Express your answers below in terms of $x$ and $k_F$.

   (a) Calculate the ground state charge density, $n(\mathbf{r})$, as a function of $x$. Hint: use the formula $n(\mathbf{r}) = \sum_j |\phi_j(\mathbf{r})|^2$, where $j$ sums over occupied states with wavefunction $\phi_j(\mathbf{r})$.

   (b) Show that $n(x)$ vanishes as the wall at $x = 0$ is approached, and has an oscillatory part that decays as a power (what power?) of $x$ into the bulk. These are called Friedel oscillations, and occur any time a metal is locally disturbed, by a surface, an impurity, etc. They can be measured using STM, and have numerous consequences for physical phenomena in metals.

   (c) How is the form of the Friedel oscillations changed for a one-dimensional electron gas ($k_y = k_z = 0$) with an end at $x = 0$?