These are AF $\underline{\text{magnons}}$ $\qquad \varepsilon_k \simeq v|k|$, $\quad v = 2\sqrt{d}\,JS$ $\qquad$ different from k^2 FM

$\underline{\text{QNs?}}$ Neél State $\uparrow \downarrow \uparrow \downarrow \uparrow$

$$\langle S_i^z \rangle = N\varepsilon_i \simeq S\varepsilon_i$$

It is still invariant under rotations around $\hat{z}$ axis
$\rightarrow$ Can label excitations ~~of~~ by their S^z QN

$$a^+ : S^z = +1 \qquad \longrightarrow \text{ same for } \tilde{a}^+, \tilde{b}^+$$
$$b^+ : S^z = -1$$

? What is the $\underline{\text{correction}}$ to the staggered magnetization?

$$\langle S_i^z \rangle_B = S - \langle n_i \rangle = S - \langle b_i^+ b_i \rangle = S - \left(\frac{2}{N}\right)\sum_k \langle b_k^+ b_k \rangle$$

$$\langle S_i^z \rangle - S = -\frac{2}{N}\sum_k \left[\cosh^2\theta_k \langle \tilde{b}_k^+ \tilde{b}_k \rangle + \sinh^2\theta_k \langle \tilde{a}_{-k} \tilde{a}_{-k}^+ \rangle\right]$$

$$= -\frac{2}{N}\sum_k \cosh 2\theta_k \langle \tilde{a}_{-k}^+ \tilde{a}_{-k} + \tilde{b}_k^+ \tilde{b}_k \rangle + \underbrace{\sinh^2\theta_k}_{\frac{1}{2}(\cosh 2\theta_k - 1)}$$

$$\cancel{\text{e.g. } T \to 0} \simeq -2\int_{MBZ} \frac{d^dk}{(2\pi)^d} \left[\underbrace{\frac{d}{\sqrt{d^2 - \gamma_k^2}} \frac{2}{e^{\beta\varepsilon_k} - 1}}_{\text{Thermal excited magnons}} + \underbrace{\frac{1}{2}\left(\frac{d}{\sqrt{d^2-\gamma_k^2}} - 1\right)}_{\text{0-pt fluct.}}\right]$$

e.g. $T \to 0$ $\quad \langle S_i^z \rangle - S = N - S \simeq -\int_{MBZ} \frac{d^dk}{(2\pi)^d} \left(\frac{d}{\sqrt{d^2-\gamma_k^2}} - 1\right) = \begin{cases} -.197 & d=2 \\ -.078 & d=3 \end{cases}$

$$\cancel{\frac{1}{N}\sum_{\substack{k\\ \in MBZ}}\left(\frac{d}{\sqrt{d^2-d_k^2}}-1\right)}$$

$$N = \begin{cases} S - .197 & d=2 \\ S - .078 & d=3 \end{cases}$$

So corrections are naively (bold extrap) 40% : $d=2$ $S=1/2$
16% $d=3$ $S=1/2$

$\longrightarrow$ Tend to think Néel State is stable G.S. in $d=2,3$ even for $S=1/2$

*Confirmed by "QMC" calculations,
& Exact Diagonalization, techniques
Series Expansion

N.B. Mermin-Wagner

[Q: Bloch $T^{3/2}$ law for FM. What is predicted for AF? in 3d]

$\underline{T>0}$ Small k-behavior?

$$N-S \simeq -2\int_{MBZ}\frac{d^dk}{(2\pi)^d}\;\frac{2k_BT}{\varepsilon_k}\;\frac{2JSd}{\varepsilon_k}$$

$\varepsilon_k^2 \sim v^2k^2$

divergent as $k\to 0$ for $d\le 2$.

No Magnetic ^{LR} order in $d=1,2$ at $T>0$
(Real $S=1/2$ AF — La_2CuO_4 do order although "Quasi"-2d
* Coupling in 3rd dim. leads to order)

Much interesting studies looking for materials where moments do not order even as $T\to 0$ "Spin Liquids".

Like ~~for~~ FM, claim $\omega \sim v|k|$ behavior is <u>universal</u> for isotropic AF.

? Semiclassical picture? Again, slow rotation of Neél state

$\nwarrow \downarrow \uparrow \swarrow \nearrow \swarrow$

should be nearly a g.s. and cause ~~a~~ very slow motion of spins

However, need to consider not only

$\vec{n}(\vec{r})$ "coarse-grained" staggered magnetization

but also $\vec{m}(\vec{r})$ " " uniform magnetization

→ Although $\vec{m}$ is <u>small</u>, it also evolves slowly since $\int \vec{m}$ is conserved.

- Moreover, effective local field is $\propto \vec{m}$.

$$\left.\begin{aligned} \partial_t \vec{n} &\sim c_1 \vec{m} \times \vec{n} \\ \partial_t \vec{m} &\sim c_2 \vec{n} \times \nabla^2 n + \underbrace{c_3 \vec{m} \times \nabla^2 m}_{\text{small}} \end{aligned}\right)$$

~~Symmetry~~ Symmetry: Translate by one site $\vec{n} \to -\vec{n}$, $\vec{m} \to \vec{m}$

$$\vec{n} \simeq \left(n_x, n_y, \sqrt{1-n_x^2-n_y^2} \simeq 1\right)$$

$$\partial_t n_x \simeq c_1 m_y$$

$$\partial_t m_y = c_2 \nabla^2 n_x$$

$$\Rightarrow \partial_t^2 n_x = c_1 c_2 \nabla^2 n_x \quad \Rightarrow \quad \omega^2 = c_1 c_2 k^2$$

$$\omega = \sqrt{c_1 c_2}\,|k| \quad \checkmark$$

Magnetism in Conducting Systems (semiconductors & metals)

• Can dope semiconductors w/ low concentration of magnetic or non-magnetic atoms

e.g. Phosphorus (P) doped Si   non-magnetic donor

Mn for Ga in GaMnAs   magnetic donor

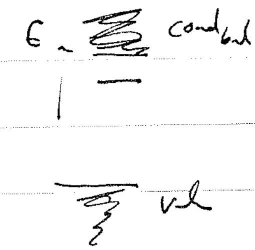
• Many materials have ~~both~~ regular lattice of magnetic atoms **and** conduct e^-s

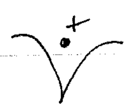
e.g. – CMR manganites from HW2

– most FMs

• Can have AFism w/o any local moments – "SDW"

• Single impurity in semiconductor/~~metal~~



donor  effective Hydrogen atom → localized hydrogenic level

low T : $1e^-$ in ~~the~~ lowest level

has 2 degenerate spin states

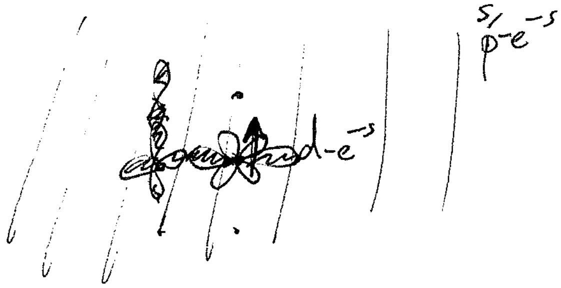
↓ acts like a local moment.

+ Such local moments important whenever impurity density is low

& orbitals only weakly interact → "localized" levels at bottom of "impurity band"

• ~~Single magnetic atom in a metal~~

• Magnetic atom in a metal



<u>Simple model</u>

Free ~~el~~ conduct. e^-s

$$H = \sum_k \varepsilon_k c^\dagger_{k\alpha} c_{k\alpha} + \int_{\vec r} J(\vec r - \vec R)\, \vec S_{\vec R} \cdot \underbrace{\left(c^\dagger \frac{\vec\sigma}{2} c\right)_{\vec r}}_{\vec s(\vec r)} \simeq J a^3 \vec S_R \cdot \vec s(\vec R)$$

very short-range fn

Cond. spin density $\quad \vec s(\vec r) = c^\dagger_\alpha(\vec r) \frac{\vec\sigma_{\alpha\beta}}{2} c_\beta(\vec r)$

$$\cancel{c_\alpha(\vec r) = \frac{1}{\sqrt V}\sum_k e^{-ik\cdot R} c_{k\alpha}}$$

$$\cancel{H = \sum_k \varepsilon_k c^\dagger_{k\alpha} c_{k\alpha}}$$

J could be F or AF & depending upon local physics.

~~If FM → J tries to align $\vec S_R$ + cond. $\vec s$.~~
~~AF → " to form singlet between $\vec S$ + $\vec s$~~

~~However, cond. e⁻ spin is not localized. So to "collect" enough spin to form singlet w/~~

<u>· Effect</u>: If think of $\vec S$ classically, e^-s scatter off it. Forms "Friedel oscillations" (cf. 223a HW) in $\vec s$. Another $\vec S$ can interact w/ this magnetization.

~~[illegible]~~ picture

→ This gives a "mechanism" for interaction of moments in a metal. It is a perturbative ~~perturbative~~ or classical picture of such interactions, since one views spin as "static." Will come back to this.

If scattering is <u>weak</u> ($J \ll \varepsilon_F$) Then can understand this interaction <u>perturbatively</u>.

"R K K Y" Interaction

R: Ruderman
K: Kittel
K: Kasuya
Y: Yosida

<u>Idea</u>: Consider ~~2~~ ~~spins~~ set of local moments with weak coupling J to e-gas.

Then for $J=0$ have degeneracy. Expect this to be split by $J \neq 0$. Can try to use deg. P.T.

at $O(J)$ splitting is zero since $\langle \vec{S}(\vec{r}) \rangle = 0$.

So need $O(J^2)$ → 2nd order deg. P.T.

$$C(r) = \frac{1}{\sqrt{V}} \sum_k c_k e^{ik\cdot r}$$

Spins in metals

d e^-s form moments

p e^-s form a band

$$H \qquad J_{pd} \vec{S}_i \cdot c^+ \frac{\vec{\sigma}}{2} c$$

If J_{pd} weak

$$H_{eff} = - H' \frac{1}{H_0 - E_0} H'$$

evaluated in degenerate GS. manifold of H_0

ie. GS of e gas + free spins.

$$H_0 = \sum_k (\varepsilon_k - \mu) c^+_{k\alpha} c_{k\alpha}$$

$$H' = \frac{J}{V} \sum_{i k k'} \vec{S}_i \cdot c^+_{k\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{k'\beta} \, e^{i(k-k')\cdot R_i}$$

$$= \frac{1}{V} \sum_{kk'} c^+_{k\alpha} c_{k'\beta} \sum_i \vec{S}_i \cdot \frac{\vec{\sigma}_{\alpha\beta}}{2} e^{i(k-k')\cdot R_i}$$

$$H_{eff} = \frac{1}{V^2} \sum_{\substack{k_1 k_1' \\ k_2 k_2'}} - J^2_{pd} c^+_{k_1\alpha} c_{k_1'\beta} \frac{1}{H_0 - E_0} c^+_{k_2\gamma} c_{k_2'\lambda} \sum_{ij} \vec{S}_i \cdot \frac{\vec{\sigma}_{\alpha\beta}}{2} \vec{S}_j \cdot \frac{\vec{\sigma}_{\gamma\lambda}}{2} \, e^{i(k_1 - k_1')\cdot R_i} e^{i(k_2 - k_2')\cdot R_j}$$

$$_e\langle 0 | c^+_{k_1\alpha} c_{k_1'\beta} \frac{1}{H_0 - E_0} c^+_{k_2\gamma} c_{k_2'\lambda} | 0 \rangle_e$$

As usual: 2 choices: $k_1 = k_1'$, $k_2 = k_2'$, $\alpha = \beta$, $\gamma = \lambda$ or $k_2 = k_1'$, $k_1 = k_2'$, $\gamma = \beta$, $\alpha = \lambda$

$\underbrace{k_1, k_2 < k_F}$ $\qquad$ $k_2 > k_F$ $\quad k_1 < k_F$

1st term gives zero:

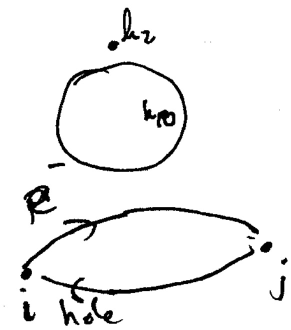
$$\sum_\alpha \vec{\sigma}_{\alpha\alpha} = 0.$$

$$H_{eff} = -\frac{J^2_{pd}}{V^2} \sum_{k_1 < k_F} \sum_{k_2 > k_F} \sum_{\alpha\beta} \frac{1}{\varepsilon_{k_2} - \varepsilon_{k_1}} \sum_{ij} \vec{S}_i \cdot \vec{\sigma}_{\alpha\beta} \vec{S}_j \cdot \vec{\sigma}_{\beta\alpha} \, e^{i(k_1 - k_2)\cdot(R_i - R_j)}$$

$$\sum_{\alpha\beta} \vec{S}_i\cdot\vec{\sigma}_{\alpha\beta}\, \vec{S}_j\cdot\vec{\sigma}_{\beta\alpha} = \sum_{\mu\nu} S_i^{\mu} S_j^{\nu} \left(\sum_{\alpha\beta} \sigma^{\mu}_{\alpha\beta}\sigma^{\nu}_{\beta\alpha} \right) = 2\,\vec{S}_i\cdot\vec{S}_j$$

$$\mathrm{Tr}\,\sigma^{\mu}\sigma^{\nu} = 2\delta^{\mu\nu}$$

$$H_{eff} = -\frac{2 J_{pd}^2}{V^2}\sum_{ij}\vec{S}_i\cdot\vec{S}_j \sum_{k_1<k_F}\ \sum_{k_2>k_F} \frac{e^{i(k_1-k_2)\cdot(R_i-R_j)}}{\varepsilon_{k_2}-\varepsilon_{k_1}}$$

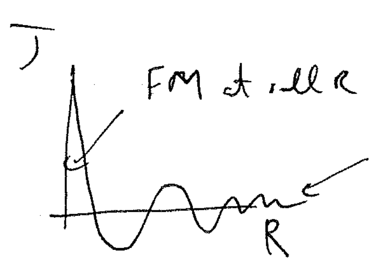


$$= -\sum_{ij} J_{ij}\,\vec{S}_i\cdot\vec{S}_j$$

$$J_{ij} = J(R_{ij}) = \frac{2 J_{pd}^2}{V^2}\sum_{k_1<k_F}\ \sum_{k_2>k_F}\frac{e^{i(k_1-k_2)\cdot\cancel{(R_i-R_j)}\cdot R_{ij}}}{\varepsilon_{k_2}-\varepsilon_{k_1}}$$

For spherical Fermi surface, one finds,

$$J \propto -m k_F^4 J_{pd}^2 \left(\frac{\cos 2k_F R}{(2k_F R)^3} - \frac{\sin 2k_F R}{(2k_F R)^4} \right)$$

- 

- Could mediate FMism if moments are close by compared to λ_F
  (GaMnAs)
- Usually moments dilute: "pseudo-"random" FM or AF interactions depending upon posit of moments
  → "metallic spin glasses", "random-singlet" behavior"

Poll 22.3c

· Another effect: <u>Kondo effect</u>

– What happens for an isolated single spin? RKKY says <u>nothing</u>.

$$H = -\sum_{ij} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + J\, \vec{S}_R \cdot c^{\dagger}_R \frac{\vec{\sigma}}{2} c_R$$

AF

If $J >> \epsilon_F$, clearly can form singlet w/ cond. e^-. "Kondo singlet". Acts like spinless impurity – scatters e^-'s.

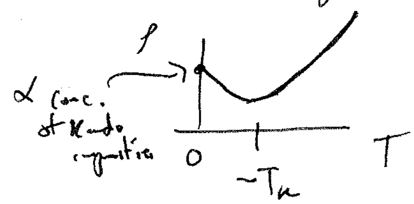
energy required to break singlet $\sim J$

– This is a non-classical effect $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = |s\rangle$

– <u>Remarkably</u>: By going <u>beyond</u> lowest order PT, can show (Kondo, Wilson) that a Kondo singlet forms, even if $J << \epsilon_F$.

However, energy of singlet $k_B T_K \sim \epsilon_F e^{-(\epsilon_F/J)}$
<u>very weak</u>.

"Kondo effect". leads to resistivity minimum $\sim T_K$



– Also occurs in "Quantum Dots" – artificial (magnetic) "atoms" in semiconductors
– Important probably if $k_B T_K \gtrsim J_{RKKY}$