Superfluidity & Superconductivity

Kamerlingh Onnes - ~~Leiden~~ - Superconductivity in Mercury 1911
Kapitza - Superfluidity in ^{4}He 1938

Related phenomena: "flow w/o resistance/dissipation below "T_c""

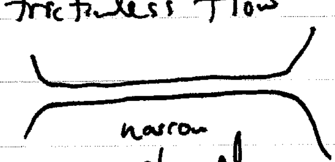
difference ^{4}He is neutral atom
e^-s have charge.

In many ways superfluidity conceptually simpler but $T_c^{He^4} = 2.17\,K$
while $T_c^{Hg} = 4.2\,K$

Hence could discover SCivity with He refridgeration
But SCivity is much more "ubiquitous"
Roughly: most atoms are too heavy. ~~QMs only below few K for He. For larger atoms gets too weak.~~
→ Small KE. For He, QMs only below few K
other atoms usually solidify (KE loses to PE) at $T \approx 0$ low T
~~or stay gaseous~~

Superfluidity

Properties: frictionless flow

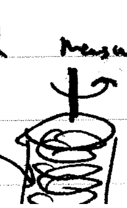


no friction up to v_c (depends on channel width)

Classical measurement

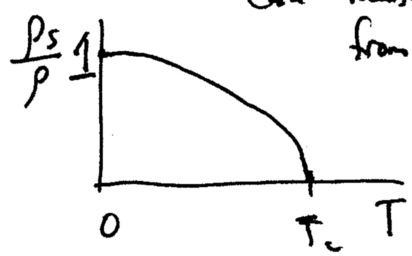
torsional pendulum → Freq. gives mom. of inertia

^{4}He

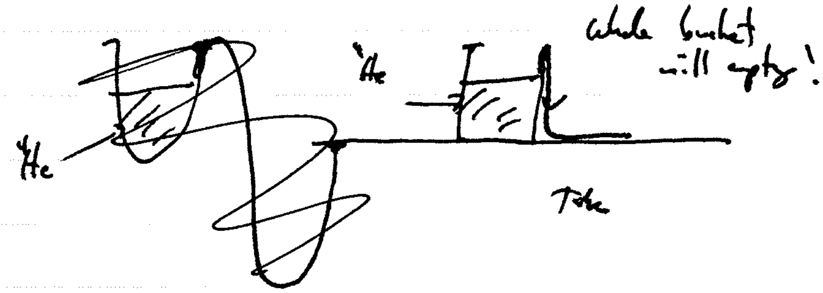


below T_c apparent "mass" of Helium drops.
→ "superfluid component" doesn't oscillate.

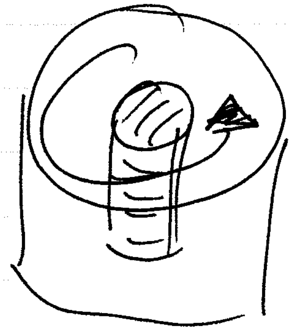
Can measure this
from Torsional Pendulum
"Andronikashvili expt"



another consequence



Persistent Current



will flow for as long as you keep it cold ($T<T_c$).

"2nd Sound" - heat pulse in superfluid propagates ballistically like a wave rather than diffusively

Basic physics → "Bose-Einstein Condensation".
- take w/ a grain of salt: ^4He atoms are bosons but not non-interacting
- nevertheless, weakly interacting or non- gives some intuition
(weakly interacting Bose gas is quantitatively good for BECs in atomic traps)

Probably you have seen this: Bose-Einstein function $n = \frac{1}{e^{\beta(\varepsilon-\mu)}-1}$

$\varepsilon = \frac{k^2}{2M}$

naively

$$\boxed{\rho = \sum_k \frac{1}{e^{\beta(\epsilon_k-\mu)}-1}} \qquad \text{fixes } \mu(T)$$

densely $$\rho = \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\beta\left(\frac{k^2}{2m}-\mu\right)}-1} \quad (*)$$

$\mu \le 0$ clearly. as $T \downarrow$ $\beta \uparrow$

need $|\mu|$ close to zero to keep ρ fixed.

problem: can't make μ any smaller than zero.

Turns out the integral (*) is convergent for $\mu=0$

hence (*) $\Rightarrow$ $$\rho \le \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\beta k^2/2m}-1} \propto T^{3/2}$$

$k \to \sqrt{T}\,k$

which implies there is some T where it is inconsistent.

What happens? well really there is $\vec{k}=0$ state.

at some point $\mu \sim O(1/\mathrm{Vol})$ very small

and $\langle N_{k=0} \rangle = n_B(k=0) \propto \mathrm{Vol}$.

* finite fract of particles in one $k=0$ Q-State.

at $T \to 0$ this is all the particles

$$|GS\rangle = \left(a^\dagger_{k=0}\right)^N |0\rangle$$

$\Rightarrow$ "Off-diagonal LRO"

Boson creation operator in real space

$$\psi^\dagger(\vec{x}) = \frac{1}{\sqrt{V}} \sum_k e^{i\vec{k}\cdot\vec{r}} a^\dagger_k$$

(creates particle in $\hat{x}$ eigenstate $|x\rangle$)

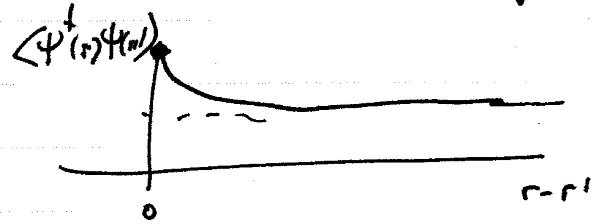
$$\langle G|\psi^\dagger(\vec r)\,\psi(\vec r')|G\rangle = \frac{1}{V}\sum_{kk'} e^{ik\cdot r - ik'\cdot r'}\langle G|a_k^\dagger a_{k'}|G\rangle$$

$$= \frac{N}{V} = \rho$$ <u>independent</u> of $\vec r, \vec r'$ (and in particular $\vec r - \vec r'$)

more generally at $T>0$

$$\langle \psi^\dagger(r)\psi(r')\rangle = \frac{1}{V}\sum_k e^{ik\cdot(r-r')}\, n(\varepsilon_k)$$

$$= \frac{N_{k=0}}{V} + f(r-r')$$



ODLRO

density matrix

<u>Crudely</u> might say $\langle \psi^\dagger(r)\psi(r')\rangle \xrightarrow[|r-r'|\to\infty]{} \langle\psi^\dagger(r)\rangle\langle\psi(r')\rangle$

$= |\langle\psi\rangle|^2$

uncorrelated

~~not quite correct~~

by analogy with $\langle \vec S_r \cdot \vec S_{r'}\rangle \longrightarrow \langle \vec S_r\rangle\cdot\langle \vec S_{r'}\rangle$

in FM $= |M|^2$

Magnetic LRO

Gives rise to notion: $\psi(\vec r)$ is <u>order parameter</u>

<u>Physics</u>: Condensate wavefunction: Macroscopic # of particles occupy this wavefunction. One wf then controls macroscopic behavior.

Landau Analysis

dimensionally. Otherwise not so obvious!

$$F[\psi] = \int d^3r \left[r|\psi|^2 + u|\psi|^4 + \frac{\hbar^2}{2m^*}|\nabla\psi|^2 \cdots \right]$$

$r > 0 \quad T > T_c$

$r < 0 \quad T < T_c$ expect $|\psi| \simeq \sqrt{\frac{-r}{u}} \equiv \psi_0$

But phase is not determined.

Write $\psi \simeq \psi_0 e^{i\theta}$ $\qquad |\psi_0|^2 = n_s$ number density

$$F \simeq \int d^3r \left[\frac{\hbar^2 n_s}{2m^*} (\nabla\theta)^2 \right]$$

Slow variations of order parameter cost little energy. Just like magnons.

Physics? Current. $\partial_t \rho + \nabla \cdot J = 0$ mass current.

$$\vec{J}(\vec{r}) = \sum_i M\vec{v}_i\, \delta(r - r_i) = \frac{\hbar}{2i}\left(\psi^\dagger(\vec{r}) \nabla \psi(r) - \nabla\psi^\dagger(r)\psi(r)\right)$$

$$\simeq \hbar n_s \vec{\nabla}\theta \equiv \rho_s \vec{v}_s$$

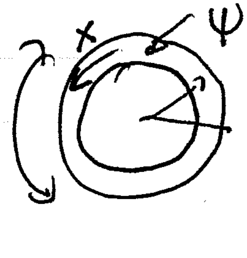
$\rho_s = M n_s$ mass density

$$\boxed{\vec{v}_s = \frac{\hbar \vec{\nabla}\theta}{M}}$$

$\vec{v}_s$ gives superfluid velocity.

Persistent Current $\quad \psi = \psi_0 e^{i\theta(x)}$

circumference L



$$v_\varphi = \frac{\hbar\, \partial_x \theta}{M}$$

average $\frac{1}{L}\int dx\, v = \frac{\hbar}{M}\frac{1}{L}\int_0^L dx\, \partial_x \theta$

$$= \frac{\hbar}{M} \frac{\theta(L) - \theta(0)}{L}$$

But ψ must be single-valued.

$$\Rightarrow \theta(L) = 2\pi \underbrace{N}_{\text{integer}} + \theta(0)$$

$$\Rightarrow \quad \bar{v} = \frac{\hbar}{M} \frac{2\pi N}{L}$$

or

$$\oint \vec{v} \cdot d\vec{r} = 2\pi N \left(\frac{\hbar}{M} \right)$$

"Quantized Circulation"

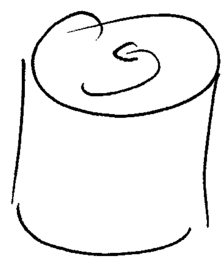
Note: Since N is discrete, circulation <u>cannot</u> decrease continuously!

*Only way for circulation to decay is for $|\psi| \to 0$ somewhere so θ is not well-defined.

This costs a lot of free energy (will return to this) and so is <u>extremely</u> improbable at low T.

Vortex

What happens if you take a bucket, rotate it, cool, then stop?

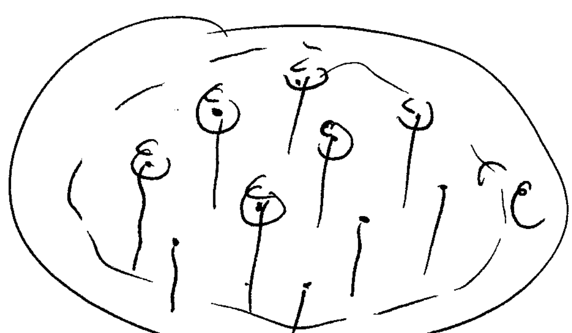


^4He is rotating!

~~Vortex~~ How is it possible? $\int \vec{v} \cdot d\vec{l} = \frac{\hbar}{m} \int \vec{\nabla}\theta \cdot d\vec{l} \neq 0$

Means $|\psi| \to 0$ somewhere inside!

What actually happens? Array of "vortices" $|\psi| \to 0$



$\int \vec{\nabla}\theta \cdot d\vec{l} = 2\pi$ around each vortex ($\nabla\theta \propto 1/r$)

flow adds up to look like rigid rotation!

$$\frac{m}{\hbar} \oint_C \vec{v} \cdot d\vec{l} = \oint_C \vec{\nabla}\theta \cdot d\vec{l} = 2\pi N_{\text{vortices inside}}$$

Clearly, moving vortex through C lowers circulation.