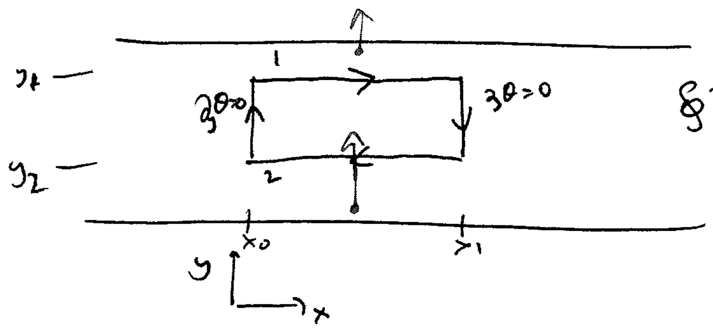
top view

Math



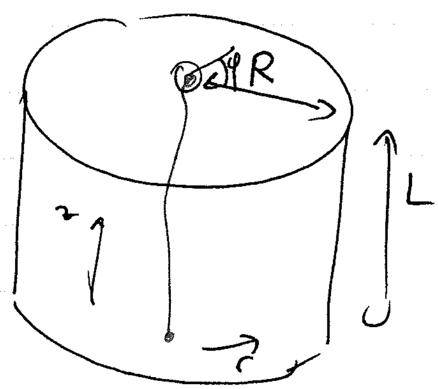
$$\oint \nabla\theta \cdot dl = 2\pi N$$

$$\Rightarrow 2\pi N = \int_{x_0}^{x_1} \partial_x\theta(x,y_1) - \int_{x_0}^{x_1}\partial_x\theta(x,y_2)$$

$= 0$ unless vortex

when vortex fully crosses system, circulation has reduced by one quantum.

Free energy cost?



$$F = \int \frac{\hbar^2 n_s}{2m^*}(\nabla\theta)^2 = \frac{\hbar^2 n_s L}{2m^*}\int_a^R dr\, r\, \frac{2\pi}{r^2}$$

$$\nabla\theta = \frac{\hat{\varphi}}{r}$$

core size

$$F = \underbrace{\frac{\hbar^2 \pi n_s L}{m^*}}_{\text{crude estimate}} \ln\left(R/a\right)$$

proportional to height L of sample.

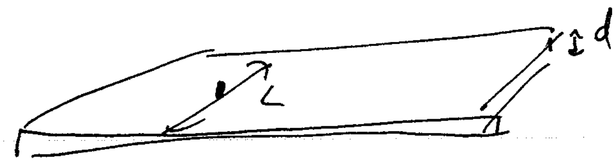
$n_s \sim \frac{1}{a^3}$ spacing between He atoms

$$F \sim \frac{\hbar^2}{m^* a^2}\,\frac{L}{a} \sim \underbrace{\frac{\hbar^2}{m_e a^2}}_{10^4 K}\underbrace{\left(\frac{m_e}{m^*}\right)}_{10^{-5}}\underbrace{\frac{L}{a}}_{\left(\frac{L}{Å}\right)}$$

$\sim .1K/Å$

So energy cost of vortex is prohibitive for $L > 1\mu$ (Rate $\sim e^{-F/k_BT}$ huge.)

Thin SF Film



$$E = \frac{\hbar^2 n_s d}{2m^*} \ln\left(\frac{L}{a}\right)$$

Should add in entropy of vortex $\quad E - TS$

$$S = k_B \ln\left(L/a\right)^2 = 2k_B \ln\left(L/a\right)$$

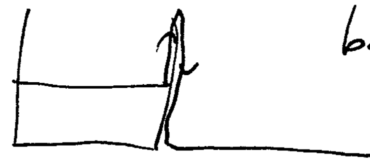
* expect vortices entropically favorable for $\quad 2k_BT > \frac{\hbar^2 n_s d}{m^*}$

Kosterlitz-Thouless SF transition

Kelin Kosik

Some other effects

* Mentioned "film flow"



bucket empties

- This is just the absence of friction
  - Normally, film ~~is~~ stabilized by Van der Waals but due to friction will break up and then collapse by surface tension.
  - In SF, no friction to break it up. Just ~~sip~~ "super-siphon"

* 2nd Sound
  - Ballistic/wavelike heat propagation
  - Physics: Can have heat flow w/o mass flow

$\longrightarrow v_n$

$\longleftarrow v_s$

Counter-propagate normal & superfluid component.

SF component is single quantum state $\rightarrow$ No entropy.

$\rightarrow$ No heat.

Heat is convected along w/ $\vec{v}_n$.

Superconductivity

* Discovered earlier (1911) than SF
* Many similar properties
  - Zero resistance $T < T_c$
  - Persistent Current
  - Perfect diamagnetism – SCs expel field

    others
* Like SF → suggests Bose Condensation of some charged boson.

Text – e^{-}s Fermions → Cannot Macroscopically occupy state

→ Suggests at least 2 fermions form "bound state"

$F + F = B$.

"Cooper Pair" → Cooper showed that e^{-}s w/ attractive int. always form B.S. near FS.

? Where does attraction come from? Coulomb clearly repulsive!
- In conventional SCs, known to come from e^{-}-phonon int
- In other SCs (high-T_c, heavy fermion, ruthenates...) not so clear.

e.g. e^{-}-phonon. e^{-}s near FS see screened int.

→ Screening is accomplished by mobile charges
- Other e^{-}s (Thomas Fermi)
- Ions.

* Since ions are slow, can only screen for freq. $\omega \lesssim \omega_D$.

For these frequencies, should consider both sorts of screening to calculate $V_{sr}(\vec{q})$

$\varepsilon V_{sc}(q) = V_{ext}(q)$ (1) defines total dielec. const. ε.

If consider screening by e^-s

$$\varepsilon_{el} V_{sc}(q) = V_{ext}(q) + V_{ion}(q) \qquad (2)$$

or by ions

$$\varepsilon_{ion} V_{sc}(q) = V_{ext} + V_{el} \qquad (3)$$

and $V_{sc} = V_{ext} + V_{el} + V_{ion}$

(2)+(3)−(1): $(\varepsilon_{ion} + \varepsilon_{el} - \varepsilon) V_{sc} = V_{ext} + V_{el} + V_{ion} = V_{sc}$

$$\Rightarrow \boxed{\varepsilon = \varepsilon_{ion} + \varepsilon_{el} - 1}$$

now we want to understand interaction of 2 low-energy e^-s
$\omega \lesssim \omega_D$ with $k, k' \sim k_F$  $\rightarrow q = k - k' \sim 2k_F$

For e^-s this is low frequency $\omega \lesssim \omega_D \ll v_F q \sim \varepsilon_F$

For phonons $\omega \lesssim \omega_D \approx v_s \cancel{q} k_D$.
but $v_s q \lesssim v_s k_D$ so consider the $q \simeq 0$ for phonons.

both very crude approx.:

$\underline{e^-s}$: low frequency → static screening $\varepsilon_{el} \simeq 1 + \frac{k_{TF}^2}{q^2}$

$\underline{\text{ions}}$ ~~phonons~~: $q \to 0$: Plasma frequency of ionic plasma $\Omega_p^2 = \frac{Zm}{M} \omega_p^2$ $\quad \varepsilon^{ion} = 1 - \Omega_p^2$

Given $\varepsilon = 1 + \frac{k_{TF}^2}{q^2} - \frac{\Omega_{pe}^2}{\omega^2}$

* For $\omega < \Omega_{pe}$, $\underline{\underline{\varepsilon < 0}}$ $\Rightarrow V_{sc} = \frac{4\pi e^2}{\varepsilon q^2} < 0$

This is very crude, but nevertheless, a more sophisticated treatment gives attraction at low ω.

~~Most~~ Much of the physics of SCivity is in the consequences of any attractive interaction.

- Many (but not all) ~~details depend do~~ aspects of SCivity do not depend upon detailed form of the interaction