Ginzburg-Landau Theory

Like SF, Assume "Bosonic ~~state~~ condensate" $\Psi(\vec{r})$
except that this boson is **charged** w/ charge $2e$.

$$F = \int d^3r \left\{ \frac{1}{2m^*} \left| \left(\frac{\hbar}{i}\vec{\nabla} + \frac{e^*}{c}\vec{A} \right)\Psi \right|^2 + \alpha |\Psi|^2 + \frac{\beta}{2} |\Psi|^4 + \frac{B^2}{8\pi} \right\}$$

$$B = \nabla \times A$$

difference from SF : $e^* \neq 0$ $\quad e^* = 2e$

initially not obvious!

(Can take (convention $m^* = 2m$)

* Consequence : $j = \frac{+e^*}{2m^*}\left[\Psi^*\left(\frac{\hbar}{i}\vec{\nabla} + \frac{e^*}{c}\vec{A} \right)\Psi + h.c. \right]$

$$\vec{j} = \frac{\hbar e^*}{2im}\left(\Psi^*\vec{\nabla}\Psi - \vec{\nabla}\Psi^*\Psi \right) + \frac{(e^*)^2}{m^*c}\vec{A}|\Psi|^2$$

Careful: Gauge Symmetry : $\Psi \rightarrow \Psi e^{i\chi}$

$$\vec{A} \rightarrow \vec{A} + \frac{\hbar c}{e^*}\nabla\chi$$

Can use this to "fix a gauge" when convenient.

~~e.g.~~ but phase alone has ambiguous meaning.

In SCing state , $\Psi = \sqrt{n_s^*}\, e^{i\theta}$

$$\vec{j} = \frac{\hbar e^* n_s^*}{m^*}\left(\vec{\nabla}\theta + \frac{e^*\vec{A}}{\hbar c} \right)$$

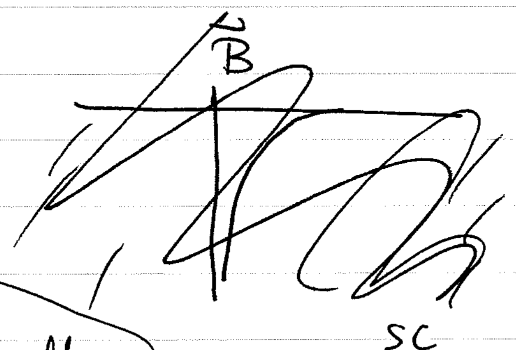
Maxwell

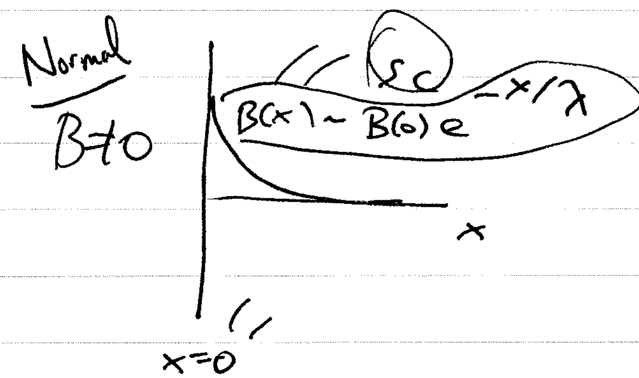
$$\nabla\times B = \frac{4\pi}{c} j$$

$$\nabla\times\nabla\times B = \frac{4\pi}{c}\nabla\times j \qquad e^* = 2e \quad m^* = 2m \quad n_s^* = \frac{n_s}{2}$$

$$\nabla\cdot B = 0 \;\Rightarrow\; \nabla^2 \vec{B} = \frac{4\pi (e^*)^2 n_s^*}{m^* c^2}\vec{B} = \frac{4\pi e^2 n_s}{m c^2}\vec{B} \equiv \frac{1}{\lambda^2}\vec{B}$$

This is the Helmholtz eqn. and implies $\vec{B}$ is screened.





$\lambda \sim 500Å$ typically

* Magnetic field is screened from SC over a length λ. "London penetration depth"

* This is actually thermodynamic

- Might expect if $\sigma = \infty$, applied field could not penetrate
- But actually field is expelled for $T < T_c$ $(B < B_c)$

Reason $\quad \Psi = \sqrt{n_s^*} \quad (\theta = 0 \text{ gauge})$

$$F = \int d\vec{r} \left[\frac{e^{*2} n_s^*}{2m^* c^2} |\vec{A}|^2 + \text{Const} + \frac{B^2}{8\pi} \right]$$

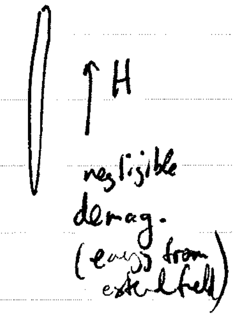
if $B = \nabla\times A \neq 0$, $\vec{A}$ must grow linearly w/ x e.g. $A_x = B_z y$

$\Rightarrow \infty$ energy/unit volume.

Expulsion of field = "Meissner Effect"

(Really applies to ideal cylinder)

Quantitatively consider Gibbs Free Energy



$$G = F - \int \frac{HB}{4\pi}\, d^3r \qquad g = f - \frac{HB}{4\pi}$$

· Meissner state: $B = 0$ $\quad g_s = f_s = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 \equiv g_s$

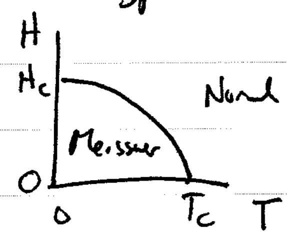
$$\left(\alpha < 0:\ |\psi|^2 = -\frac{\alpha}{\beta}\right) \quad g_s = f = -\frac{\alpha^2}{2\beta} \equiv -\frac{H_c^2}{8\pi}$$

· Normal State $\psi = 0$ $\quad g = \frac{B^2}{8\pi} - \frac{HB}{4\pi} \underset{B=H}{=} -\frac{H^2}{8\pi}$

Clearly $g_N > g_s$ if $H < H_c$ $\quad$ Thermodynamic

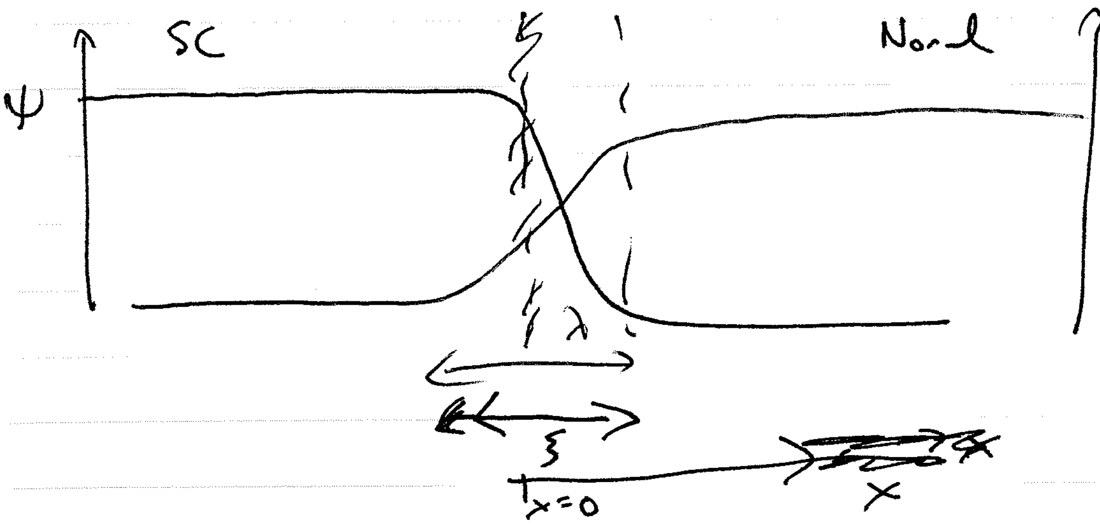
$\qquad g_n < g_s$ $\quad H > H_c$ $\quad$ Critical field H_c.

This truly describes the SCing critical field for so-called "Type I" SCs.



However, it is not always the case.

Consider interface between a SC~~ing~~ + normal region. Imagine we choose
$H = H_c$, so that they have the same energy density.



ξ is the length over which Ψ decays in the ~~norm~~ SC state near an edge
It is different from λ.

To see this, suppose $B = 0$ (simple)

$$\frac{F}{A_{rea}} = \int dx \left[\frac{\hbar^2}{2m^*} \left| \frac{d\Psi}{dx} \right|^2 + \alpha |\Psi|^2 + \frac{\beta}{2} |\Psi|^4 \right]$$

Minimize: Euler-Lagrange equation

$$\Psi \to \Psi + \delta\Psi \qquad \frac{\delta F}{\delta \Psi} = 0 \qquad \alpha = -|\alpha| < 0$$

$$\Rightarrow -\frac{\hbar^2}{2m^*} \frac{d^2\Psi}{dx^2} - |\alpha|\Psi + \beta |\Psi|^2 \Psi = 0$$

In SC $|\Psi|^2 = -\frac{\alpha}{\beta}$ let $\Psi_\infty = \sqrt{-\frac{\alpha}{\beta}} = \sqrt{\frac{|\alpha|}{\beta}}$ define $\Phi = \frac{\Psi}{\Psi_\infty}$

Then

$$-\frac{\hbar^2}{2m^*}\frac{d^2\Psi}{dx^2} - |\alpha|\Psi + \overbrace{\beta\psi_\infty^2}^{|\alpha|}|\Psi|^2\Psi = 0$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m^*|\alpha|}\frac{d^2\Psi}{dx^2} - \Psi + |\Psi|^2\Psi = 0$$

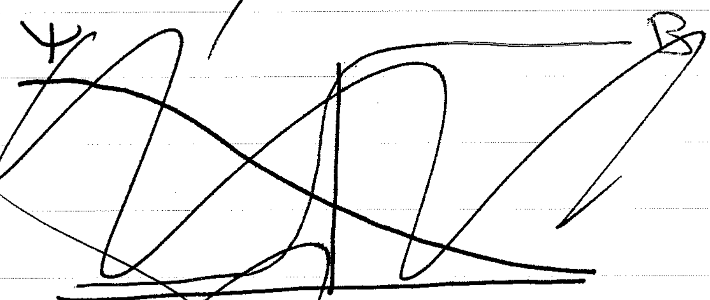
$$\xi^2\frac{d^2\Psi}{dx^2} = -\Psi + |\Psi|^2\Psi \;\cancel{= \frac{\partial}{\partial\Psi}\left(\frac{\Psi^2}{2} + \frac{\Psi^4}{4}\right)}$$

$$\boxed{\xi^2 = \frac{\hbar^2}{2m^*|\alpha|} \qquad \text{"coherence length"}}$$

Clearly width of variation in Ψ is determined by ξ.

Important parameter $\kappa = \frac{\lambda}{\xi}$

$\kappa << 1$
$\xi >> \lambda$



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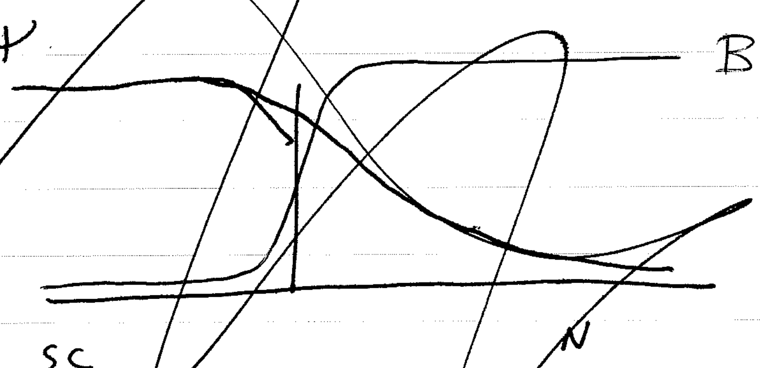


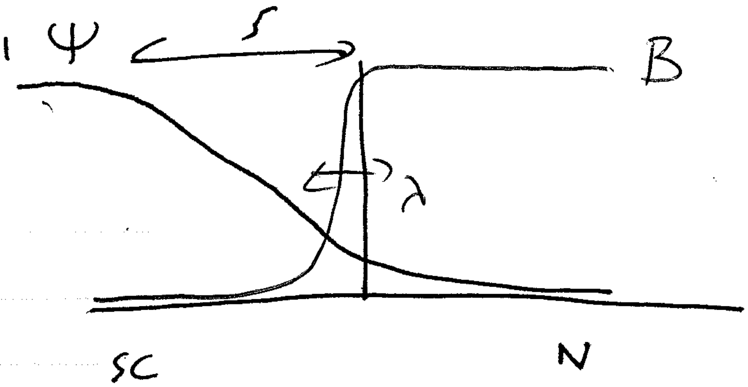
Large region where

Important parameter $\kappa = \lambda/\xi$ Recall $\mathcal{G} \sim -\alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{B^2}{8\pi} - \frac{H\cdot B}{4\pi}$

W

$\kappa \ll 1$

$\xi \gg \lambda$

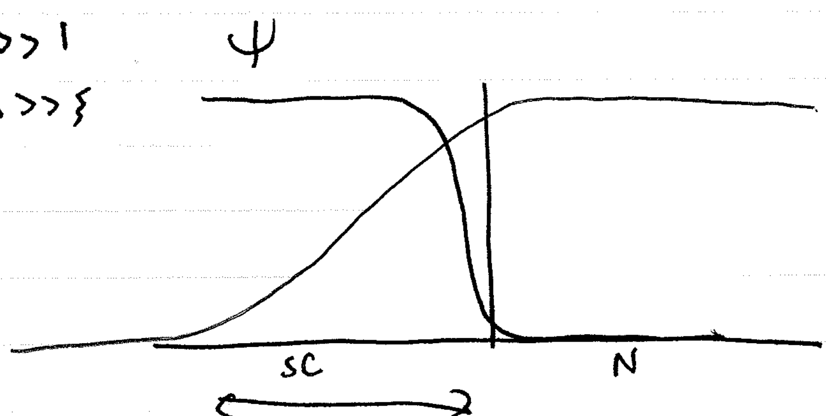


region of width $\sim \xi - \lambda$ where lose fract of condensate energy <u>and</u> magnetic energy.

So surface energy $\sim (\text{Area}) \times (\xi - \lambda) H_c^2$

$\kappa \gg 1$

$\lambda \gg \xi$



region of width $\lambda - \xi$ where gain condensation energy & magnetic energy.

$\Rightarrow$ surface energy is <u>negative</u>.

Can show surface energy is negative for $\kappa > \frac{1}{\sqrt{2}}$

Most low T_c: $\lambda \approx 500$ Å, $\xi \approx 3000$ Å, $\kappa \simeq \frac{1}{6}$ or less, type I

however: Nb, high-T_c $\kappa \gg 1/\sqrt{2}$

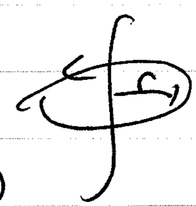
For $\kappa > \frac{1}{\sqrt{2}}$ have "type II" SC.

Then clearly for $H \lesssim H_c$, interfaces will <u>spontaneously</u> form between SC + N regions.

This will happen until the SC is divided into microscopically "small domains".

In fact, instead of domains, Abrikosov showed that what forms is a <u>vortex lattice</u>.

Recall SF vortex



$\oint \vec{v}\cdot d\vec{l}$ is quantized

far from vortex core ($r \gg \xi$)

here $\Psi = \Psi_\infty e^{i\theta(\vec{r})}$ where $\oint \vec{\nabla}\theta \cdot d\vec{l} = 2\pi n$

Free energy $f = \int \frac{\hbar^2 n_s^*}{2m^*}\left(\nabla\theta - \frac{e^* A}{\hbar c}\right)^2 + \frac{B^2}{8\pi}$

* To minimize $\left(\nabla\theta - \frac{e^* A}{\hbar c}\right)^2$, want $\vec{A} = \frac{\hbar c}{e^*}\vec{\nabla}\theta = \frac{\hbar c}{2e}\nabla\theta$ ~~(will be true~~

~~$\Rightarrow \oint A\cdot d\vec{l} =$~~

This is always true far enough away from core ($r \gg \lambda$)

Flux Quantum $\Rightarrow \oint \vec{A}\cdot d\vec{l} = \frac{\hbar c}{2e}\int \nabla\theta\cdot d\vec{l} = \frac{hc}{2e} n \equiv n\varphi_0$

$= \int B\cdot d\hat{n}$

note factor of 2! $\varphi_0 = \frac{hc}{2e} \simeq 2\times 10^{-7}$ G cm² SC Flux Quantum.