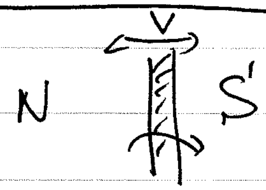
~~Ways to~~

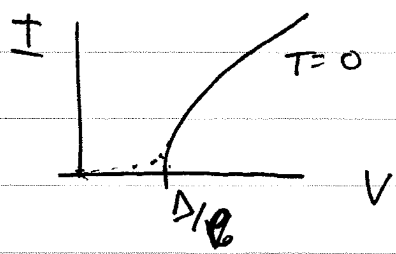
Exptl Consequences of BCS Theory

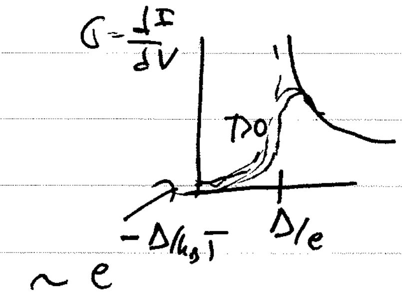
N-S Tunneling:



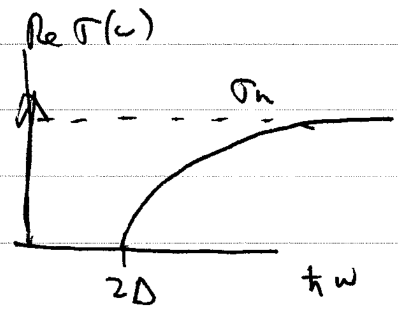
$$H_{tun} = \sum_{kq} t_{kq}\, c^\dagger_{kN} c_{qS} + h.c.$$

$I \propto$ Rate $\propto$ # states DOS of e^{-}s with $\varepsilon < V$ in sc.





Optics



Missing weight?

$$\int_0^\infty \mathrm{Re}\,\sigma\, d\omega = \frac{\pi n e^2}{2m} \text{ conserved}$$

Goes into $\omega = 0$ peak.

$\approx$ like Drude peak

(Recall $\sigma_{Dru}(\omega) = \dfrac{ne^2/m}{1/\tau - i\omega}$

$\mathrm{Re}\,\sigma = \left(\dfrac{\frac{1}{\tau}\, ne^2/m}{\frac{1}{\tau^2} + \omega^2}\right)$)

But w/ width $1/\tau \to 0$

(SC e^{-}s don't scatter)

Thermal Conductivity

* Condensate carries **no entropy** (Just one state).
  Cooper pairs at $p=0$.

* e^- contribution to thermal conductivity (cf. Wiedemann Franz) is due to QPs.

  Hence $\kappa_e \sim e^{-\Delta/k_B T}$

Many other properties similarly exponentially suppressed for $T \ll T_c$
- Spec. heat
- $\lambda(0) - \lambda(T)$ (Related to loss of SF density)
  → ~~[illegible]~~

⋮

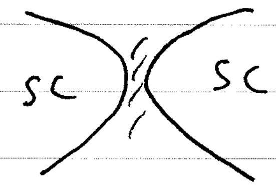
~~Many~~ These can be used to distinguish s-wave from d-wave (ie. detect nodes)

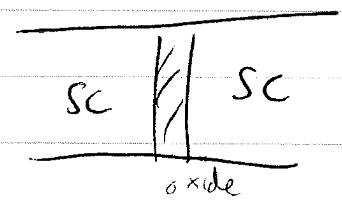
BCS theory also predicts these things for $T \lesssim T_c$ not just $T \ll T_c$.

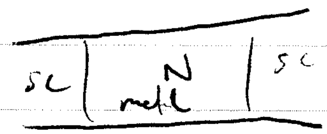
(II)

Josephson Effect

Suppose you bring 2 SCs nearby, separated by a "weak link", i.e. a non-SC region







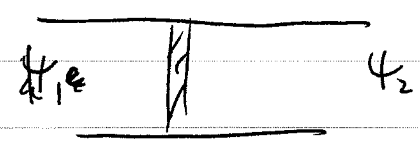
(1962) ← Very late in SC timescale

<u>Josephson</u>: It is possible for a ~~supe~~ supercurrent to flow <u>across</u> the barrier by "tunneling"

He did it by explicitly considering tunneling of e^-s from 1 SC to another.

The effect is very general.

<u>Assume</u>: each SC is ordered, and characterized by an order parameter (pair field) $\psi_1, \psi_2 = |\psi| e^{i\phi_{1,2}}$



$\phi_{1,2} \simeq$ constant in space.
(Can relax this for larger junctions)

In general, motion of e^-s from $1 \Leftrightarrow 2$ is always possible (tunneling or otherwise) & will lead to dependence of the free energy on the relative phase of 1,2 presumably $\phi_1-\phi_2=0$ in min energy state.

So we should expect

$$F = F_{GL}^{(1)} + F_{GL}^{2} \quad "- E_J \cos(\phi_1-\phi_2)"$$

("" ⇒ periodic. 1st term Fourier series. Usually good approx esp. near T_c)

Actually this is not gauge-invariant. Should really be

$$F_J = -E_J \cos\Big(\underbrace{\phi_1-\phi_2 - \frac{2e}{\hbar c}\int_1^2 \vec{A}\cdot d\vec{\ell}}_{\gamma\ =\ \text{gauge-invariant phase difference}}\Big)$$

So the free energy depends on γ.

But there is a relation between γ + the voltage. Schrödinger

~~scalar potential~~

$$\cancel{i\hbar\partial_t\Psi = -eA_0 + H}$$

$$\partial_t\phi_i = -\underset{\text{energy}}{\epsilon_i}/\hbar = +2e\frac{\varphi_i}{\hbar}$$

scalar potential.

$$\partial_t\gamma = +\frac{2e}{\hbar}(\varphi_1-\varphi_2) + \frac{2e}{\hbar c}\int_1^2 \partial_t\vec{A}\cdot d\vec{\ell}$$

$$= +\frac{2e}{\hbar}\int_1^2 d\vec{\ell}\cdot\Big(-\vec{\nabla}\varphi - \frac{1}{c}\partial_t\vec{A}\Big) = +\frac{2e}{\hbar}\int_1^2 \vec{E}\cdot d\vec{\ell} = \frac{2eV_{12}}{\hbar}$$

$$\boxed{\partial_t\gamma = \frac{2eV_{12}}{\hbar}}$$

~~Knowing $\partial_t \gamma = \frac{eV}{\hbar}$, we can get the current.~~

~~Current: General: For small change in vector potential,~~

~~$\Delta F = \int$~~

Work done in changing γ a little bit

$$dF = \oint I V \, dt = I \frac{\hbar}{2e} \partial_t \gamma \, dt$$

$$= I \frac{\hbar}{2e} d\gamma$$

But $\quad dF = d\left(-E_J \cos\gamma\right) = + E_J \sin\gamma \, d\gamma$

$$\Rightarrow \boxed{I = \frac{2e E_J}{\hbar} \sin\gamma}$$

Thus by maintaining a phase difference $\gamma \neq 0$ across JJ, get $I \neq 0$

maximum current "Critical current" $I_c = \frac{2e E_J}{\hbar}$

$$\boxed{I = I_c \sin\gamma}$$

How big is I_c or E_J? Depends on barrier obviously.

You may expect for a <u>tunnel</u> <u>barrier</u> (Not for a Normal metal barrier) need $\underline{\underline{2}}$ e^-s to tunnel from one SC to another (to establish coherence of <u>pair</u>)