Physics 23

Assignment 5 Solutions

(46 Points)

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E27-6

The net electric field flux is related to the total charge inside the die by

$$\Phi_E = \frac{Q_{encl}}{\epsilon_0}. \tag{1}$$

The sides with odd number of spots contribute minus signs since the flux goes into the die through them, while the sides with even number of spots give positive signs. The result is

$$\Phi_E = (-1+2-3+4-5+6)10^3 Mm^2/C = 3 \times 10^3 Nm^2/C = \frac{Q_{encl}}{\epsilon_0}. \tag{2}$$

The charge inside is therefore $2.66 \times 10^{-8} C$.

E27-18

a) (2 Points) At p_1, we can draw a spherical Gaussian surface with radius r=R/2, where R is the radius of the cavity. We immediately note that the only charge enclosed in the surface is q, so that the electric field at p_1 is simply that of a point charge. Therefore

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(R/2)^2} \approx 3.38 \times 10^6 N/c \tag{3}$$

pointing radially outwards.

b) (2 Points) p_2 is inside a conductor, so $E_2 = 0$

E27-26

a) (2 Points) The total charge on the surface is

$$Q = 4\pi R^2 \sigma \approx 1.52 \times 10^{-4} C, \tag{4}$$

where R is the radius of the sphere and σ is the volume charge density.

b) (2 Points) The total electric flux leaving the sphere is due to the the total charge on it. We have

$$\Phi_E = \frac{q_{encl}}{\epsilon_0} \approx 1.72 \times 10^7 Nm/C. \tag{5}$$

c) (2 Points) We can draw a Gaussian sphere right outside of the surface, the electric field is then

$$E = \frac{\Phi_E}{4\pi R^2} = \frac{\sigma}{\epsilon_0} \approx 9.19 \times 10^5 N/C \tag{6}$$

and points radially outward.

P27-2

a) (6 Points) Since the electric field is in the y-direction, we only have flux through the two sides facing the x-z plane. Notating the side closer to the origin by Φ_L and further away by Φ_R, we have

$$\Phi = \Phi_L + \Phi_R = -E_y(y = a)A + E_y(y = 2a)A = (-b(a)^{1/2} + b(2a)^{1/2})a^2 \approx 22.3 Nm/C. \tag{7}$$

b) (2 Points) The total charge inside is

$$q_{encl} = \Phi\epsilon_0 \approx 1.97 \times 10^{-10} C. \tag{8}$$

P27-10 (6 Points)

We have the electric field at the wall and need use it to find the charge density on the source, namely the wire. To do so, we must first establish an expression relating the electric field at the surface a distance r from the center of the wire to the uniform linear charge density λ on the wire.

Draw a cylindrical Gaussian surface of radius r about the wire. The wire is effectively infinitely long, so the electric field points only radially outward in cylindrical coordinates. Using Gauss's Law, we find (the calculation was done in lecture and is also in the textbook, so I will not repeat it a third time here)

$$E = \frac{\lambda}{2\pi\epsilon_0 r}. \tag{9}$$

Using the numbers given, we find $Q = L\lambda \approx 3.6 \times 10^{-9} C$.

P27-12 (6 Points)

The repulsive electro-static force between each pair of particles must be balanced by the attractive gravitational force between them. We therefore have

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = \frac{Gm^2}{r^2}, \tag{10}$$

where r is typical inter-particle distance and m is the mass of each particle. Cancelling r from both sides and putting in numbers given, we find $m \approx 1.86 \times 10^{-9} kg$

P27-14

a) (6 Points) Draw a Gaussian sphere of radius r centered at the center of the spherical charge distribution but with r¡R, where R is the radius of the charged sphere. Since the charge distribution enclosed inside the Gaussian surface is spherically symmetric, we expect the electric field to point purely in the radial direction. Therefore

$$\oint \mathbf{E} \cdot \mathbf{dA} = \frac{q_{encl}}{\epsilon_0} = \frac{4}{3\epsilon_0}\pi r^3 \rho. \tag{11}$$

Our Gaussian surface has its area vector pointing radial outward in all directions (the vector area is perpendicular to the surface itself at every point on the surface). We also expect the electric field to be only in the radial direction as well. Eqn.(11) then becomes

$$\oint E\hat{\mathbf{r}} \cdot dA\hat{\mathbf{r}} = \oint EdA = \frac{4}{3\epsilon_0}\pi r^3 \rho. \tag{12}$$

We integrate the electric field over the spherical Gaussian surface of radius r. We expect the field to have the same magnitude everywhere on the surface because both the surface and the charge distribution inside are spherically symmetric about the same origin. Therefore E is constant as we vary the position on the surface and so the area integral simply becomes the area of the Gaussian sphere:

$$\oint EdA = E\oint dA = E(4\pi r^2) = \frac{4}{3\epsilon_0}\pi r^3 \rho. \tag{13}$$

Solving for E and rewriting in the vector form, we have the desired result:

$$\mathbf{E} = \frac{\rho \mathbf{r}}{3\epsilon_0}. \tag{14}$$

b) (4 Points) At any point inside the cavity $\mathbf{E_c}$, the actual electric field is the field at the that point **without** the cavity $\mathbf{E}$ subtract the field due to the charge that would have been there if there was no missing mass $\mathbf{E_m}$. Let $\mathbf{r}$ be a vector from the center of the big sphere to an arbitrary point inside the cavity, we can write down all the electric fields involved in terms of $\mathbf{r}$ and $\mathbf{a}$ as

$$\mathbf{E_c} = \mathbf{E} - \mathbf{E_m} = \frac{\rho \mathbf{r}}{3\epsilon_0} - \frac{\rho(\mathbf{r} - \mathbf{a})}{3\epsilon_0} = \frac{\rho \mathbf{a}}{3\epsilon_0}. \tag{15}$$

P27-18

a) (2 Points) Since E=0 inside the conductor, the charge on the cavity wall must cancel the point charge inside exactly. Therefore the charge on the wall is $-3\mu C$.

b) (4 Points) From outside of the conductor, we must see the correct TOTAL charge enclosed, namely $+13\mu C$, which then must be the charge on the outer surface.

The phrasing of this problem is somewhat misleading. I gave full points to those who took the net charge of the whole system to be $10\mu C$ but still did the problem correctly.