physics 23

Assignment 7 solutions, (58 points)

T. Tao, 12/2/06

E29-4 (4 points)

a) Current = Current density X Cross-sectional area.

$\Rightarrow$ $\boxed{j = \frac{I}{A} \cong 2.6 \times 10^{-5} \ A/m^2}$

b) The current density is basically the total rate of charge flow per unit area,

$\Rightarrow \vec{j} = ne\underbrace{\vec{V}_D}_{\text{Drift}} \Rightarrow V_D = \frac{j}{ne}$ since our

Current flow is 1-D. Putting in $n \cong 8.5 \times 10^{28}/m^3$ for copper,

$\Rightarrow$ $\boxed{V_D \cong 1.91 \times 10^{-15} \ m/s}$

E29-12

(wrong Problem by accident...)

$\Delta V = I R$

R $\rightarrow$ of electrician

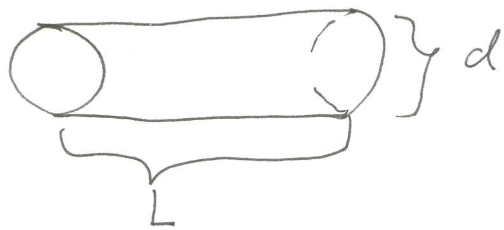
I $\rightarrow$ max. safe current.

$\Rightarrow$ $\boxed{\Delta V \cong 90V}$

$\rightarrow$

E29-20 (4 points)

$$R = \frac{\rho L}{\pi (d/2)^2}$$



$$\Rightarrow R_c = R_i \Rightarrow \frac{\rho_c \cancel{L_c}}{\cancel{\pi}(d_c/2)^2} = \frac{\rho_i \cancel{L_i}}{\cancel{\pi}(d_i/2)^2}$$

$$\Rightarrow \boxed{d_i \cong 2.85 \times 10^{-3}\, m}$$

P29-2 (6 points)

The total current density is the sum of electron and ion (α-particle) contributions

$$\Rightarrow j = j_\alpha + j_e = q_\alpha n_\alpha V_\alpha + q_e n_e V_e$$

$$= -e\, n_e V_e + 2e\, n_\alpha V_\alpha$$

$e \equiv |\text{electron charge}|$

- Now $\vec{V}_e$ and $\vec{V}_\alpha$ are opposite in direction, so:

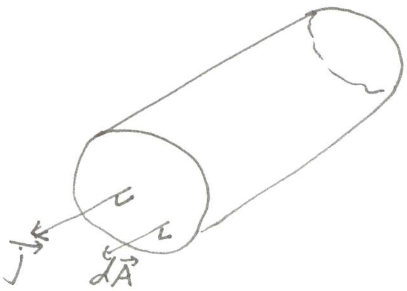
$$j = e\, n_e |V_e| + 2e\, n_\alpha |V_\alpha|$$

$$\Rightarrow \boxed{j \cong 1 \times 10^5\, C/m^2 s}$$

→

P29-6 (6 points)

a) $j = j_0(1 - \frac{r}{R})$, $I = \int \vec{j} \cdot d\vec{A}$



- since $\vec{j} \parallel d\vec{A}$

$$\Rightarrow I = \int j\, dA = \int_0^{2\pi}\int_0^R j_0\left(1 - \frac{r}{R}\right) r\,dr\,d\theta = \frac{2\pi j_0 R^2}{6}$$

- In terms of $A = \pi R^2 \Rightarrow$ $\boxed{I = \frac{j_0 A}{3}}$

b) Now $I = \int j\, dA = \int_0^{2\pi}\int_0^R \frac{j_0 r}{R}\, r\,dr\,d\theta = \frac{2\pi j_0 R^2}{3} \Rightarrow$ $\boxed{I = \frac{2 j_0 A}{3}}$

P29-8 (6 points)

According to HRK Eqn. 29-16, we have:

$$\rho = \rho_0[1 + \alpha_{av}(T - T_0)], \text{ here } T_0 = 20^\circ C$$

- Now ρ, ρ_0 are related to R $\Rightarrow$

$$T = T_0 + \left(\frac{\rho}{\rho_0} - 1\right)\frac{1}{\alpha_{av}}$$

$$= T_0 + \left(\frac{\rho - \rho_0}{\rho_0}\right)\frac{1}{\alpha_{av}}$$

$R \sim \rho$

$$= T_0 + \frac{R - R_0}{R_0}\frac{1}{\alpha_{av}}$$

- Now $R \cong (310\,mA)(2.90\,V)$, $R_0 \cong 1.12\,\Omega$, $\alpha_{av} \cong 4.5 \times 10^{-3}/^\circ C$

$\Rightarrow$ $\boxed{T \cong 1650^\circ C}$

$\rightarrow$

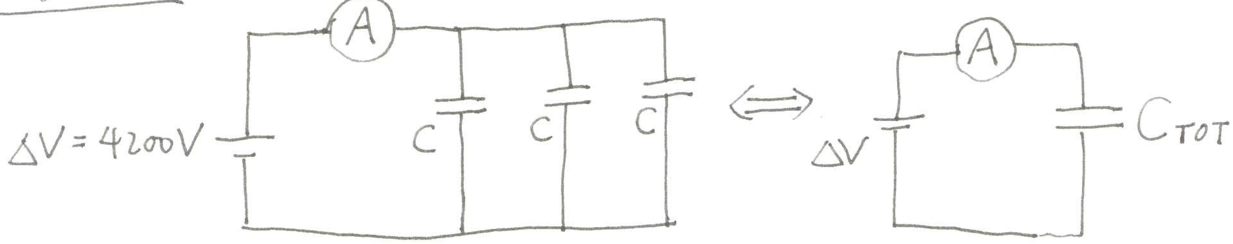
E30-4

a) From Gauss's Law (use $\vec{E}$ to find ΔV in a capacitor),

$$\boxed{C = \frac{\epsilon_0 A}{d} \cong 1.43 \times 10^{-10} F}$$

b) $\boxed{q = C\Delta V \cong 1.66 \times 10^{-8} C}$

E30-12 (4 points)



$\Rightarrow$ $C_{TOT} = 3C$ since they are in parallel $\Rightarrow$ $\boxed{Q = C_{TOT}\Delta V \cong 0.315 C}$

E30-24 (4 points)

- For capacitors in series, $\frac{1}{C_{EFF}} = \frac{1}{C_1} + \frac{1}{C_2}$ *

- The energy stored in the capacitors is (in the $\vec{E}$ fields):

$$U = \frac{1}{2}CV^2 = \frac{1}{2}C_{EFF}V^2 = \frac{1}{2}\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}}V^2$$

$\Rightarrow$ $\boxed{U \cong 7.4 \times 10^{-2} J}$

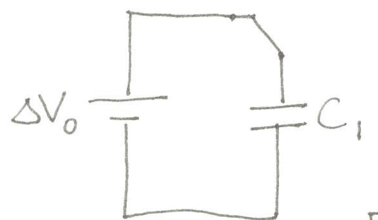
E30-28

a) $C = \frac{\epsilon_0 A}{d} \Rightarrow \boxed{d \cong 6.04 \times 10^{-3}\,m}$

b) $C' = \frac{\kappa_e \epsilon_0 A}{d} = \kappa_e C \Rightarrow \boxed{C' \cong 2.87 \times 10^{-10}\,F}$

P30-6 (10 points)

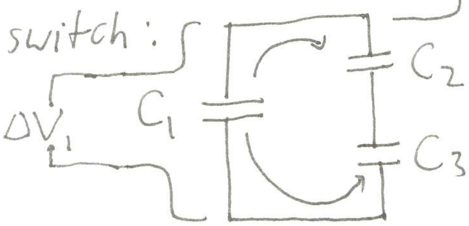
- Initially,



$\Rightarrow C_1 \Delta V_0 = q_0$

→ charges from C_1 flow and accumulate on to C_2, C_3, so that $q_0 = q_1 + q_2 = q_1 + q_3$

- Flip switch:



- The new charges on the capacitors are:

$q_1 = C_1 \Delta V_1$

$\underbrace{q_2 = q_3 = C_{23} \Delta V_1,}_{}$ where

C_1, C_{23} are in <u>parallel</u>

$\Rightarrow$ same voltage drop.

$$C_{23} = \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}}$$

- Now ΔV_1 is also the total voltage drop in the circuit

$$\Rightarrow \underbrace{q_0}_{\text{total charge still}} = C_{TOT} \Delta V_1 = (C_1 + C_{23}) \Delta V_1$$

$$\Rightarrow \Delta V_1 = \frac{q_0}{C_1 + \left[\frac{1}{C_2} + \frac{1}{C_3}\right]^{-1}}$$

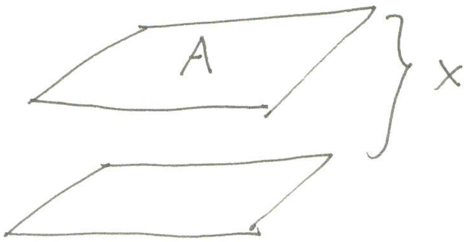
$$\Rightarrow q_2 = q_3 = \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}} \frac{q_0}{C_1 + \left[\frac{1}{C_2} + \frac{1}{C_3}\right]^{-1}} \Rightarrow \boxed{\cancel{q_2 = q_3 \cong q}}$$

$$\Rightarrow \boxed{q_2 = q_3 = \frac{\Delta V_0}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}}$$

using $q_0 = C_1 \Delta V_0$, and $\boxed{q_1 = q_0 - q_2 = q_0 - q_3}$

→

P30-14 (6 points)



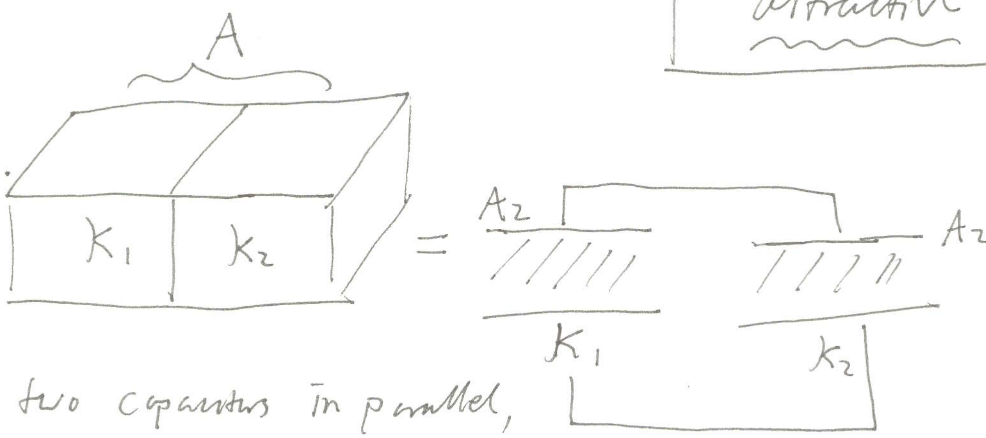
$$U = \frac{1}{2} C V^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} q^2 \frac{A\epsilon_0}{x}$$

$\downarrow$

TOTAL electrostatic energy

$$\Rightarrow \vec{F} = -\vec{\nabla} U \Rightarrow F = -\frac{\partial U}{\partial x} \Rightarrow \boxed{F = -\frac{q^2}{2A\epsilon_0}, \text{ attractive}}$$

P30-18 (6 points)



– Same as two capacitors in parallel, each completely filled with dielectric:

$$\Rightarrow C_T = C_1 + C_2 = \frac{K_1 \epsilon_0}{d}\frac{A}{2} + \frac{K_2 \epsilon_0}{d}\frac{A}{2}$$

$$\Rightarrow \boxed{C_T = \frac{\epsilon_0 A}{2d}(K_1 + K_2)}$$