1 Homework set 2, due Feb 10

1. Consider an open string with Neumann boundary conditions on both ends. In lightcone quantization, at level one, one has a massless vector particle with $D-2$ polarizations (in the end $D = 26$ for the critical string). At level two, one can show that one has a single representation of the Lorentz group, associated to a symmetric traceless tensor of $SO(D-1)$ (the $SO(D-1)$ is called the little group of a massive particle): this splits into a symmetric representation of $SO(24)$ and a vector representation of $SO(24)$, given by

$$a^i_{-1}a_{-1}|0\rangle \tag{1}$$
$$a^j_{-2}|0\rangle \tag{2}$$

   Show that at level 3 and level 4 one recovers complete irreducible representations of the Lorentz group. For this, you have to show that the multiplets form complete representations of the little group $SO(25)$, and check their decompositions under $SO(24)$. (You can find a list of representations in page 114 of Green-Schwartz-Witten. However, you have to justify the answer and explain how these are put together.)

2. Consider an open string in 26 Dimensions with the left endpoint ending on a D-brane at $X^{25} = 0$, and the right end with a free endpoint (Neumann boundary conditions).

   (a) Give the appropriate decomposition of the field into normal modes for X^{25}.

   (b) What are the allowed energies for X_{25} oscillators?

   (c) What is the zero point energy associated to the direction X^{25}?

3. Consider an open string in 26 Dimensions with the left endpoint ending on a D-brane at $X^{25} = 0$, and the right end ending at $X^{25} = R$.

   (a) Show that fixing X^{25} to have those boundary conditions requires a classical gradient in X^{25}.

   (b) Give the appropriate decomposition of the field into normal modes for X^{25}.

(c) What are the allowed energies for X_{25} oscillators?

(d) What is the zero point energy associated to the direction X^{25}? Include both the classical energy stored in the gradient, and the quantum zero point energy.

4. Verify that

$$\partial\bar{\partial}\log|z|^2 = \partial\frac{1}{\bar{z}} = \bar{\partial}\frac{1}{z} = 2\pi\delta^2(z,\bar{z}) \tag{3}$$

5. Consider a theory of a free scalar field, with the standard correlator between the X fields. Define a (modified) stress energy tensor by the formula

$$T(z) = -\frac{1}{2} : \partial X \partial X : (z) + \beta\partial^2 X \tag{4}$$

(a) Show that

$$T(z)T(0) \sim \frac{c}{2z^4} + \frac{2T(z)}{z^2} + \frac{\partial T(z)}{z} \tag{5}$$

for some appropriate value of c.

(b) Calculate the operator product expansion

$$T(z) : \exp(ikX) : (0,\bar{0}) \tag{6}$$

(c) Calculate the operator product expansion

$$T(z)\partial X(0) \tag{7}$$