1 Homework set 3, due March 4

1. Consider a free (left-moving) boson field, which is characterized by the holomorphic operator of weight $(1,0)$, ∂X and by the correlator

$$\langle \partial X(z)\partial X(w)\rangle = \frac{1}{(z-w)^2} \tag{1}$$

Define the mode expansion of ∂X by the following Laurent series

$$i\partial X = \sum_n \alpha_n z^{-n-1} \tag{2}$$

(a) Write α_n as a contour integral around the origin of ∂X.

(b) Calculate the commutator $[\alpha_n, \alpha_m]$ by performing contour integrals in complex variables.

(c) Consider an antiperiodic expansion for ∂X around the origin, characterized by the label n of the oscillators above being a half integer. This is, we have inserted a twist operator $\sigma(0)$ such that

$$\partial X(z)\sigma(0) \sim \frac{1}{z^{1/2}}\mu(0) \tag{3}$$

What is the difference in conformal dimensions $h(\sigma) - h(\mu)$?

(d) Calculate the Greens function

$$\langle \partial X(z)\partial X(w)\rangle_A \tag{4}$$

in the presence of antiperiodic boundary conditions by using the commutation relations of the α_n. If you prefer, you can calculate this correlator using complex analysis techniques as used in class.

(e) Use the definition of the stress tensor

$$T(z) = \lim_{w\to z} \frac{1}{2}\partial X(w)\partial X(z) - \frac{1}{2(z-w)^2} \tag{5}$$

To calculate $\langle T(z)\rangle_A$ from the previous result. Extract the conformal dimension of σ from this result.

2. Consider a free bc-fermion system with propagators given by

$$b(z)c(w) \sim c(z)b(w) \sim \frac{1}{z-w} \tag{6}$$

And with a stress energy tensor given by

$$T = A : b\partial c : +B(: c\partial b :) \tag{7}$$

(a) Calculate the constraints on the values A, B so that c and b are both primary fields, by using the OPE of T with c, b respectively.

(b) If the dimension of c is λ, what is the central charge of the corresponding CFT? (Use the TT OPE to get this result).

(c) Show that if $\lambda = -1$, then $c_{bc} = -26$.

(d) Consider the ghost current $j(z) =: bc : (z)$. For what values of λ is j a primary field?