Physics of the Diffuse Universe

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Outline for Week 1

- Overview of the Interstellar Medium
- Introduction to Diffuse Gas
- Line emission processes
- Spectroscopic notation

Read Draine chapters 1, 2, 3, 4, and 17.

These notes include Figures from Dopita and Sutherland "Astrophysics of the Diffuse Universe" chapters 1 and 2 as well as Draine's book.

Why Study the Interstellar Medium (ISM)?

- Interstellar gas forms stars.
- Stars are the dominant source of energy in galaxies.
- Hence the physics of the ISM determines the visible appearance of galaxies.

What Is between the Stars?

- Gas phase atoms, ions, and molecules with velocity distributions that are nearly thermal.
- Solid particles less than 1 um in size, i.e., dust.
- Electrons and ions with kinetic energies far greater than thermal, i.e., cosmic rays.
- Photons from CMB, stars, and the above.

Cont'd

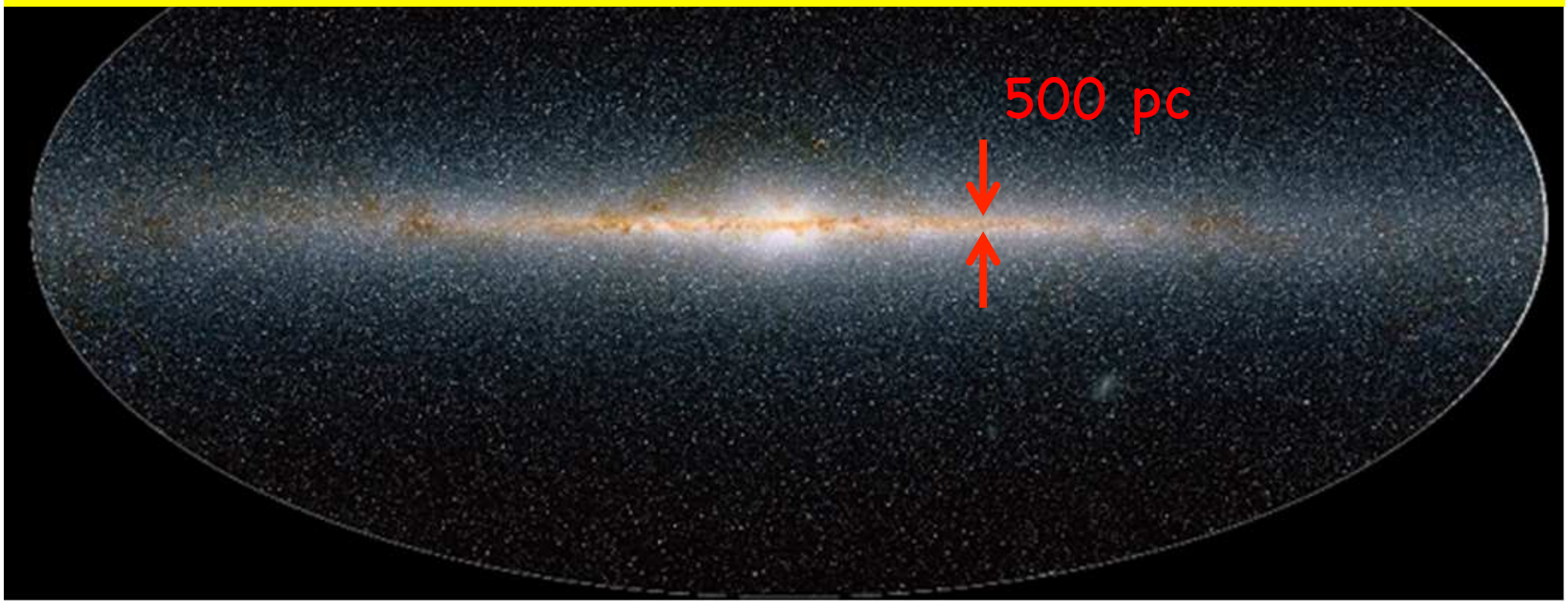
- Interstellar magnetic field resulting from electric currents in the ISM.
- Gravitational field due to all the matter in the galaxy; the contribution of the ISM can lead to regions of self-gravitating clouds.
- Dark matter particles to the extent that they interact non-gravitationally with any of these components.

Milky Way: 2 Micron All Sky Survey

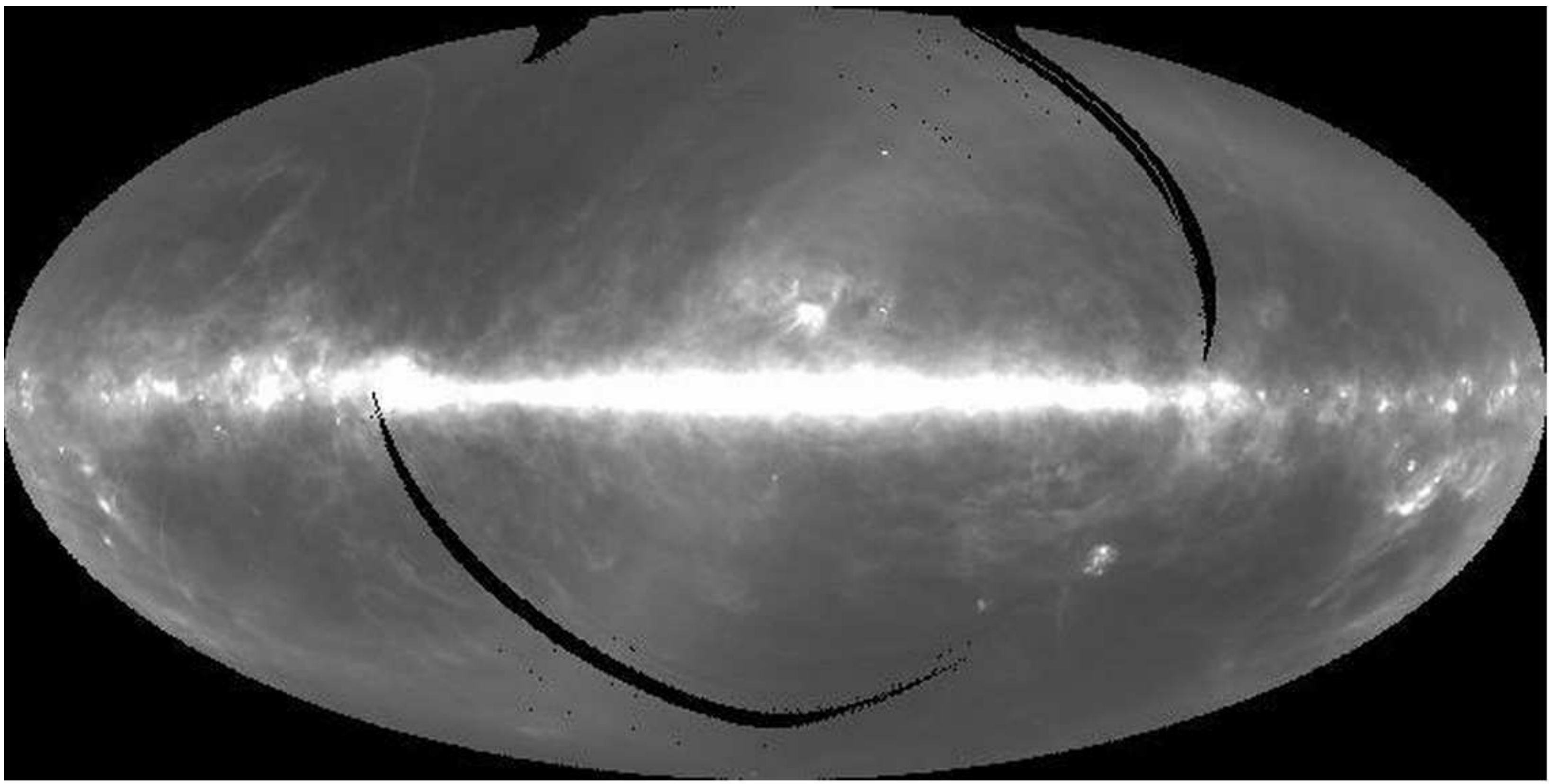
Within 15 kpc of the center:

- $5 \times 10^{10}\ M_0$ of stars
- $5 \times 10^{10}\ M_0$ of dark matter
- $7 \times 10^{9}\ M_0$ of interstellar gas

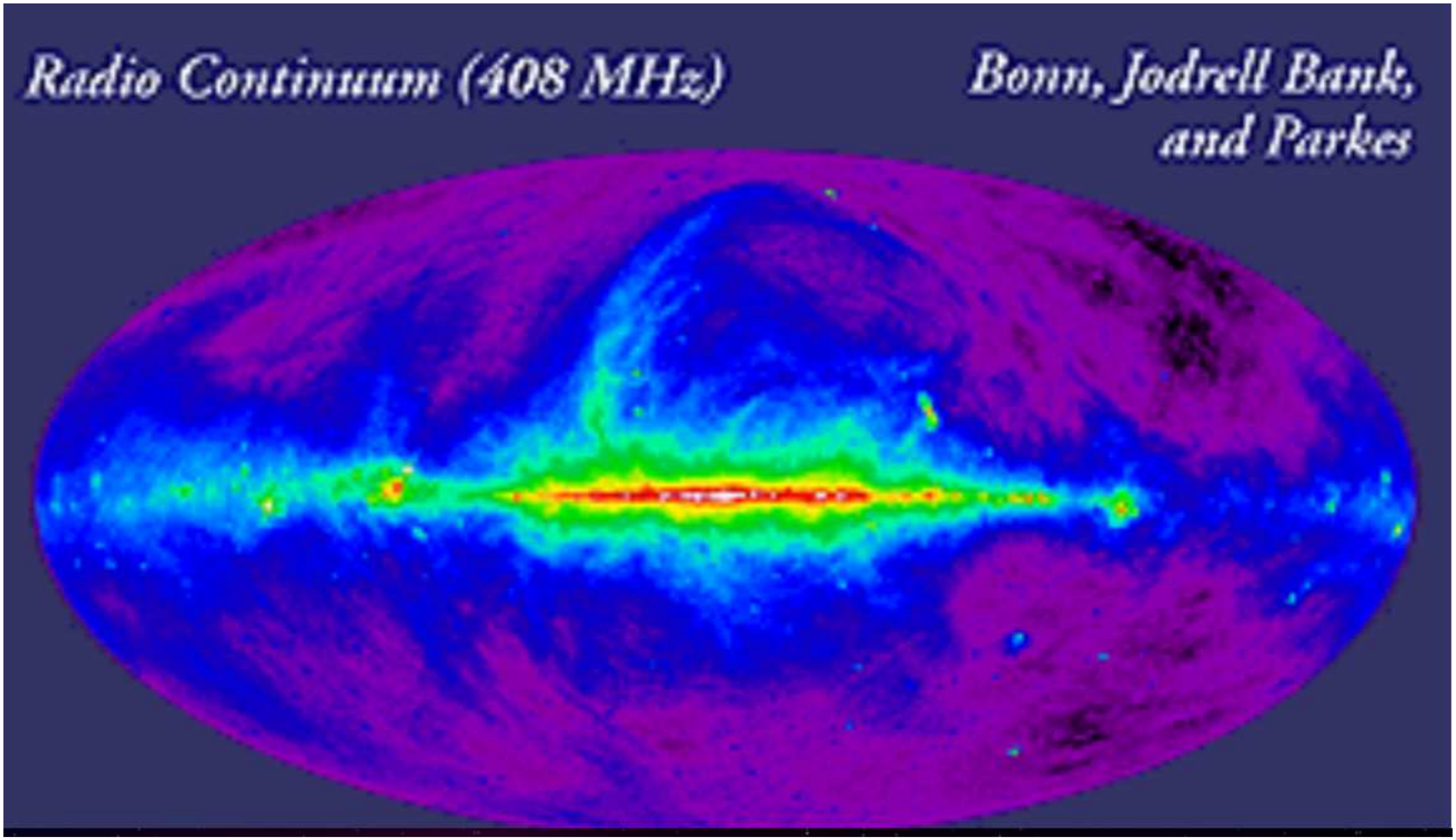
The obscuration produced by the dust is visible in this composite 1.2, 1.65, 2.2 um image.



Milky Way: 100 um

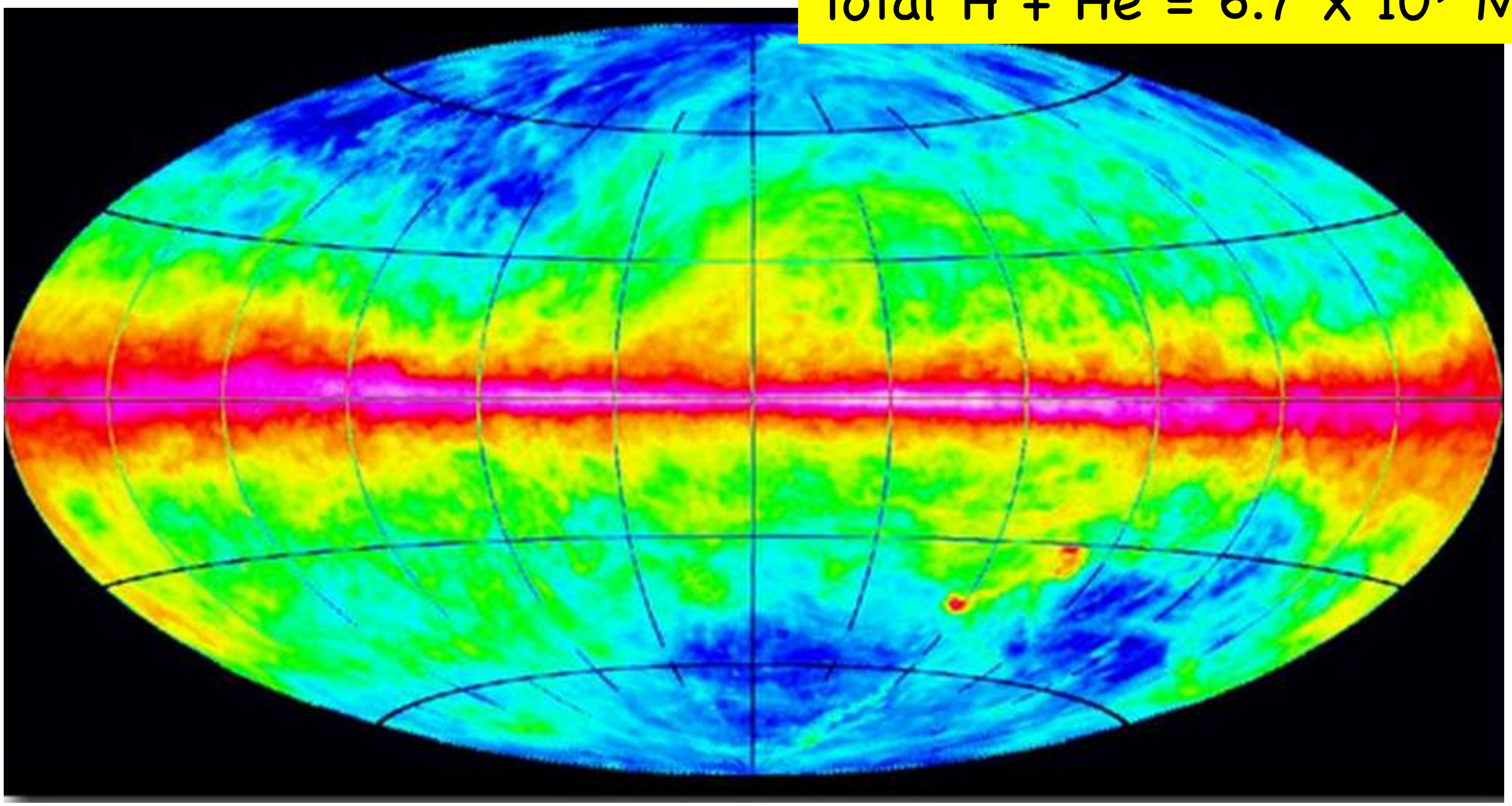


Milky Way: Synchrotron Emission

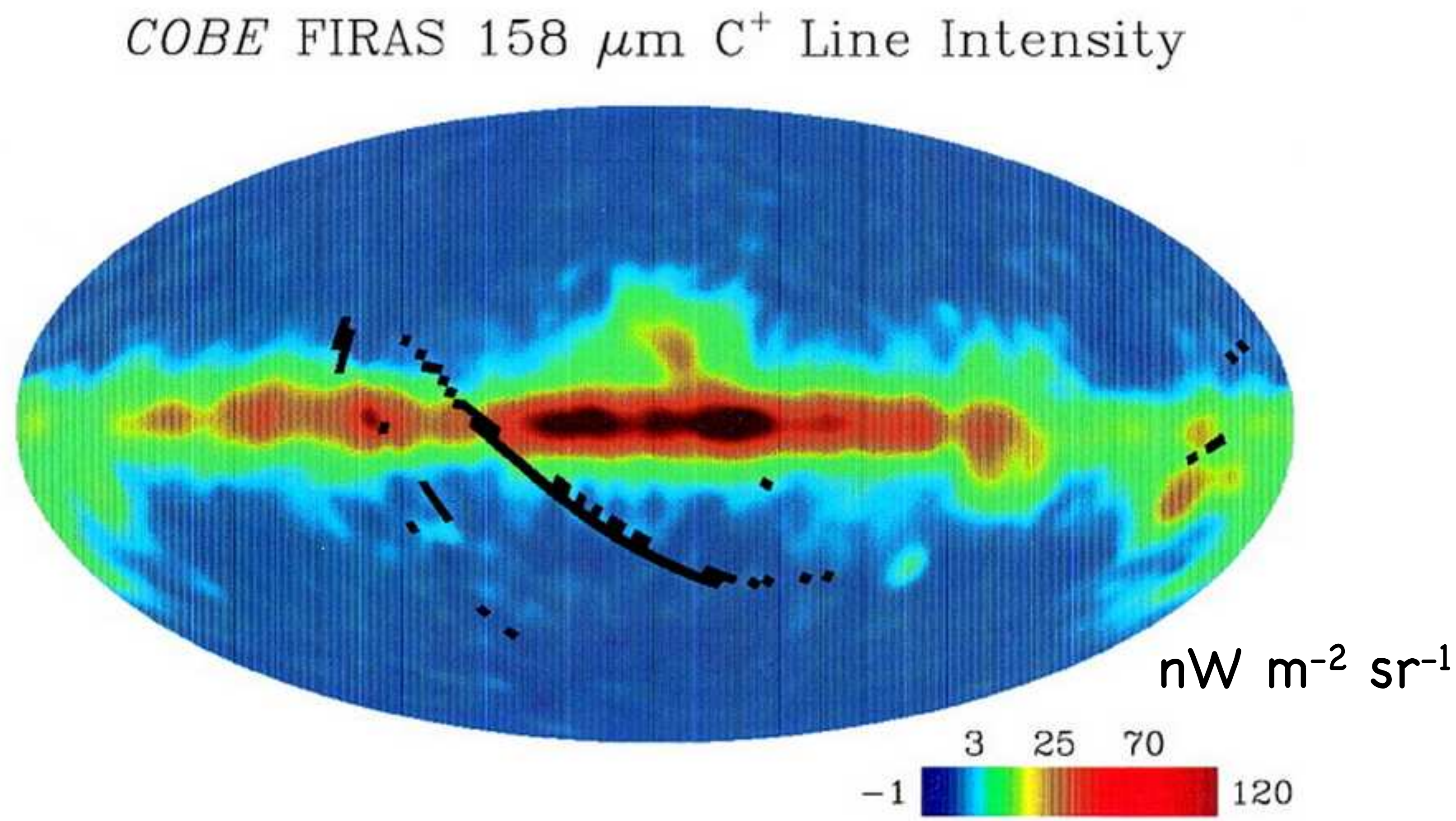


Millky Way:
Atomic Hydrogen
(LAB Survey)

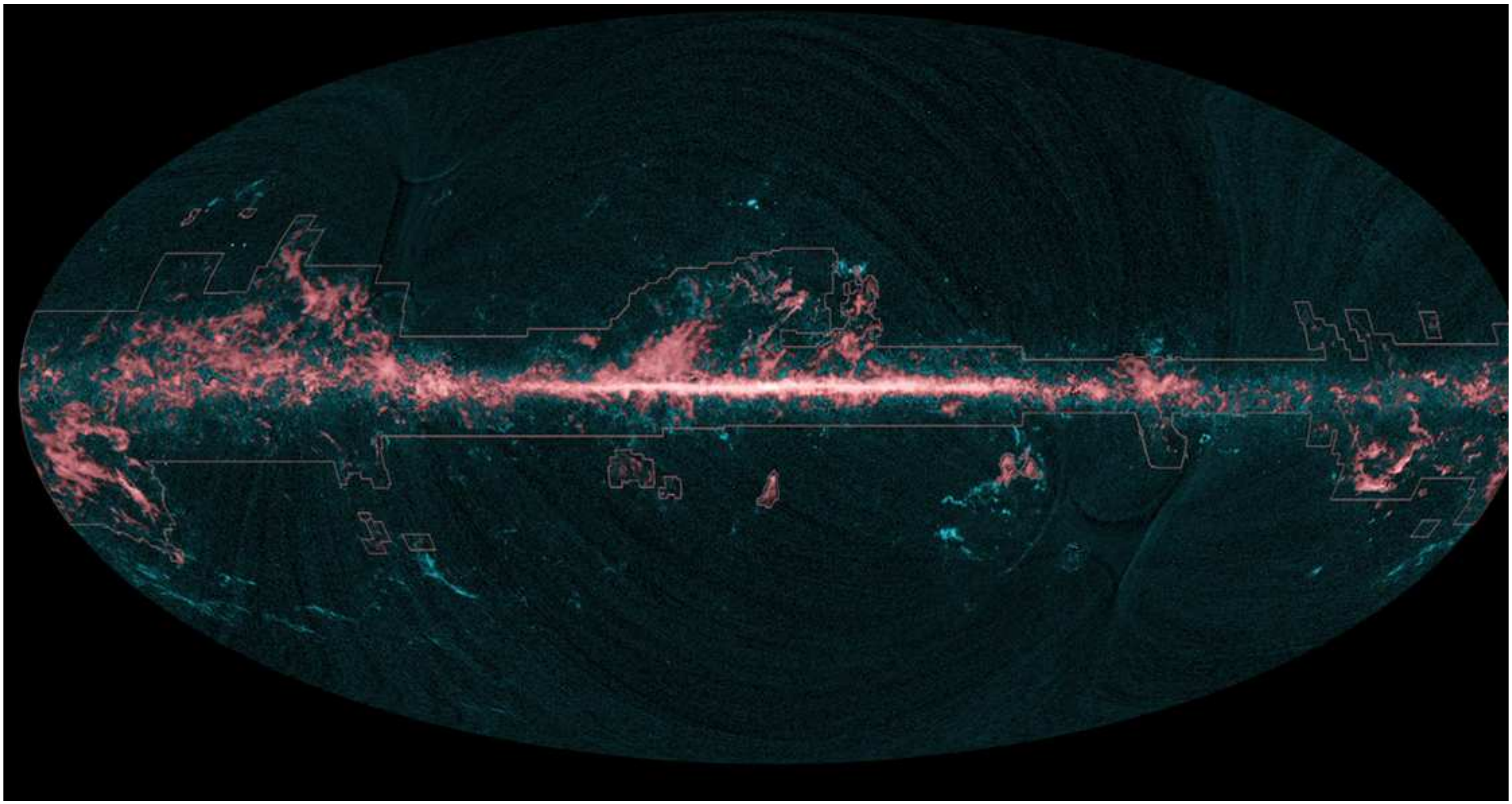
Mass H I = $2.9 \times 10^9\ M_0$
Mass H II = $1.12 \times 10^9\ M_0$
Mass H_2 = $0.84 \times 10^9\ M_0$
Total H = $4.9 \times 10^9\ M_0$
Total H + He = $6.7 \times 10^9\ M_0$



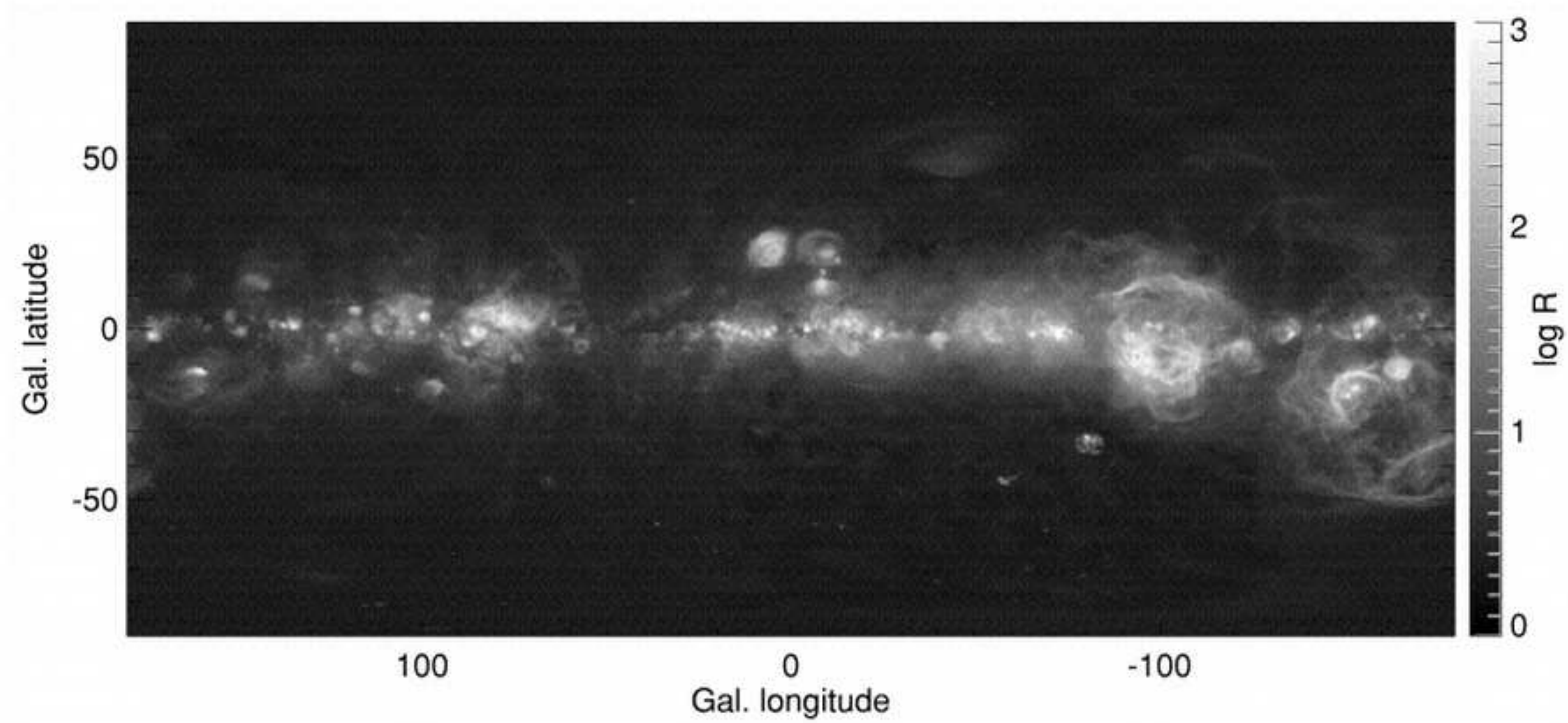
Milky Way: C^+ emission as [CII] 158 um (Fixsen+1999)



Milky Way: Molecular Gas as Traced by CO and Observed by Planck



Milky Way: Ionized H (Finkbeiner 2003)



Phases of Interstellar Gas

- Warm HI
- Cool HI
- Diffuse H_2 (similar to HI)
- Dense H_2 (opaque; often self-gravitating)
- Ionized HII at 10^4 K
- Coronal Gas; Ionized HII at $T > 10^{5.5}$ K

Elemental Composition

- Primarily H and He from the Early Universe
- Additional 1-2% heavy elements (Z > 2), a.k.a, "metals"
  - Based on solar photospheric abundances and meteorites
  - These metals are VERY important for the temperature, ionization state, and chemistry of the ISM.
  - Many diagnostic spectral lines rely on these metals.

Energy Densities (eV/cm^3)

CMB energy density increases as $(1 + z)^4$, so the rough equipartition at z=0 is coincidental.

Thermal energy	0.49
Bulk kinetic energy	0.22
Cosmic ray energy	1.39
Magnetic energy	0.89
CMB	0.265
FIR emission from dust	0.31
Starlight	0.54

Energy Densities (eV/cm^3)

Gas motions build up the magnetic field, so it is not surprising that $B^2/8\pi \sim 0.5\ \rho v^2$.

Thermal energy	0.49
Bulk kinetic energy	0.22
Cosmic ray energy	1.39
Magnetic energy	0.89
CMB	0.265
FIR emission from dust	0.31
Starlight	0.54

Energy Densities (eV/cm^3)

The gas and dust are heated by the stars, and cosmic rays are accelerated in supernova shock fronts, so their energy densities are directly coupled to the starlight.

• Thermal energy	• 0.49
• Bulk kinetic energy	• 0.22
• Cosmic ray energy	• 1.39
• Magnetic energy	• 0.89
• CMB	• 0.265
• FIR emission from dust	• 0.31
• Starlight	• 0.54

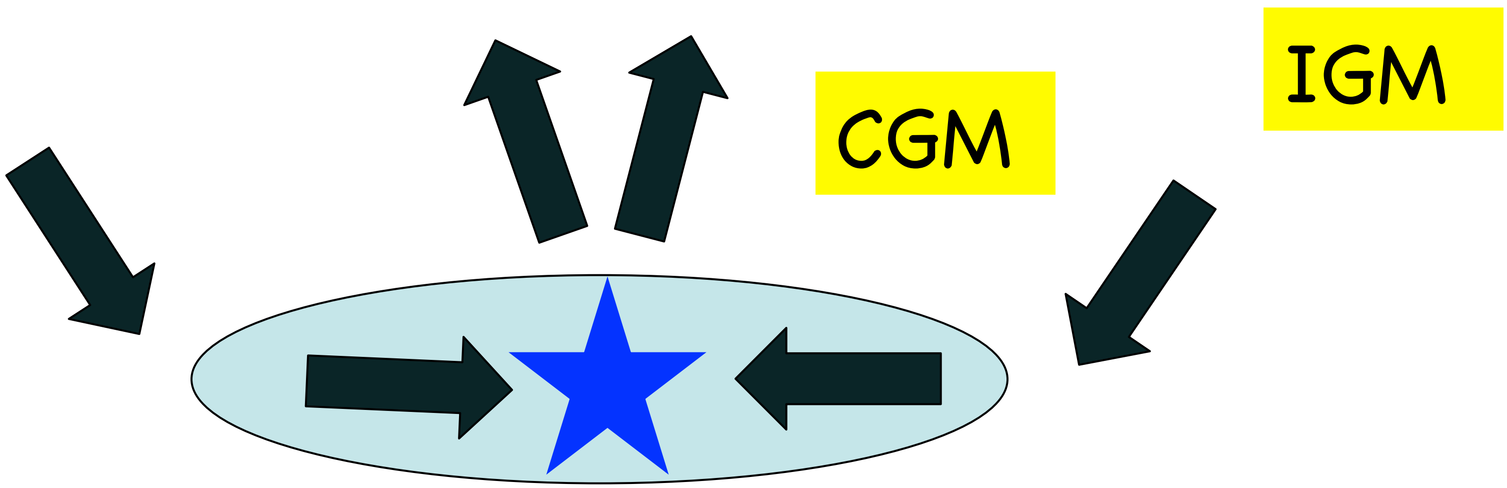
Energy Densities (eV/cm^3)

If any of these components grows much larger than the gravitational binding energy, then hydrostatic equilibrium is disrupted. All galaxies drive winds at some time.

• Thermal energy	• 0.49
• Bulk kinetic energy	• 0.22
• Cosmic ray energy	• 1.39
• Magnetic energy	• 0.89
• CMB	• 0.265
• FIR emission from dust	• 0.31
• Starlight	• 0.54

Where Does the Galaxy End?

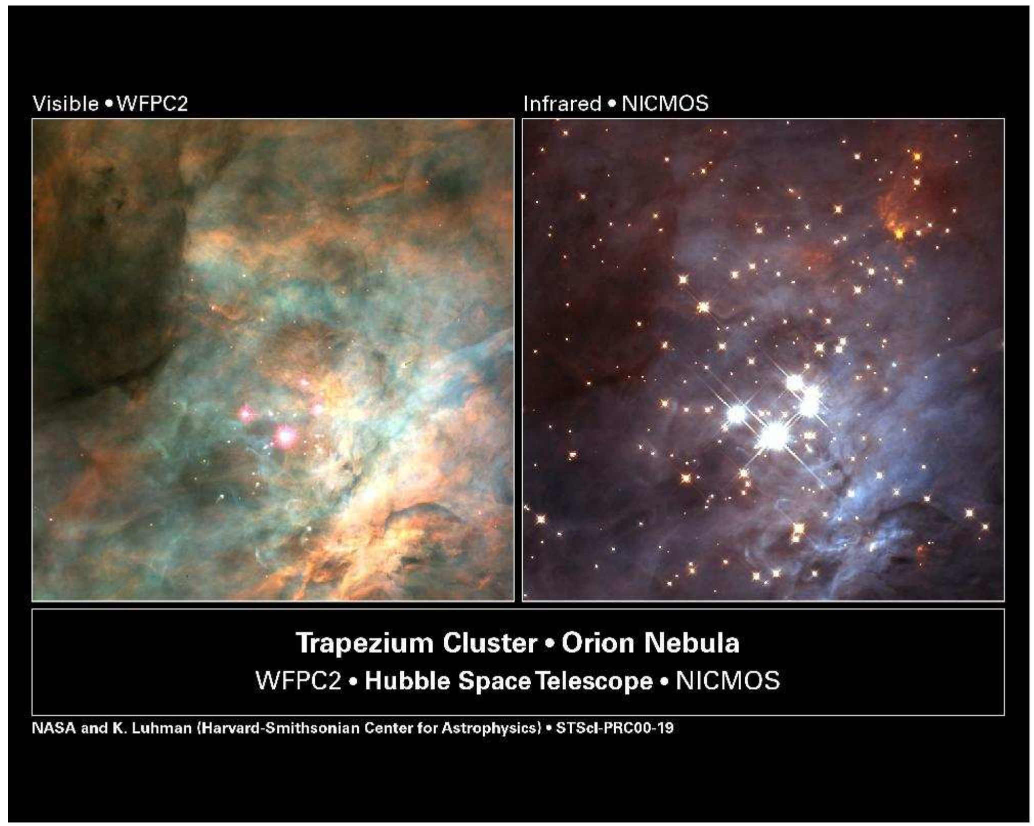
- All the ISM constituents are present between galaxies, and the same physical processes apply to studying the Intergalactic Medium (IGM).



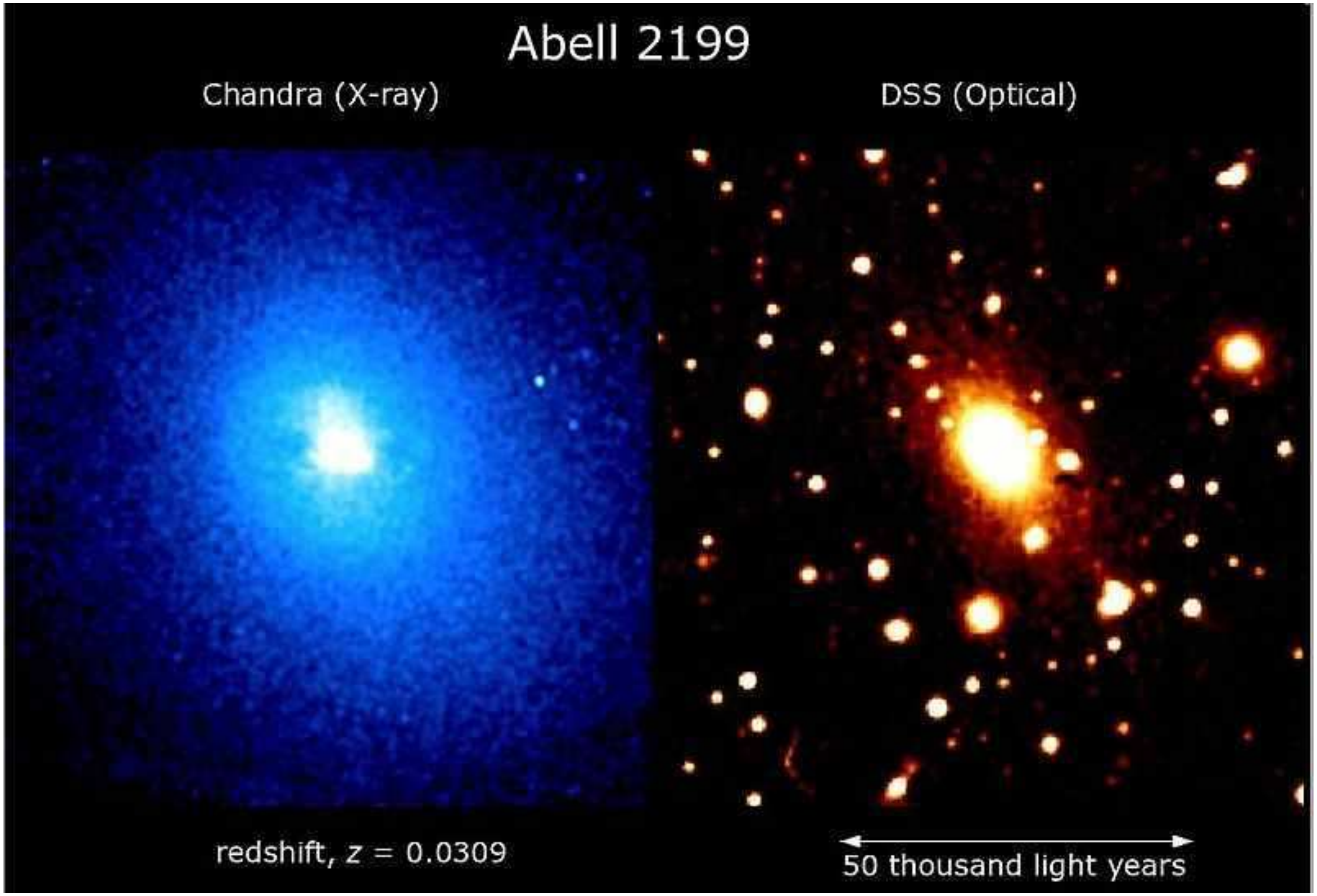
What is the Diffuse,Universe?

- It is the low density diffuse plasma …
  - Around stars
  - Within disks of spiral galaxies
  - Around active galaxies
  - In clusters of galaxies
  - In intergalactic space
- Found over an extraordinary range of scales
- Contains most of the baryons in the universe!

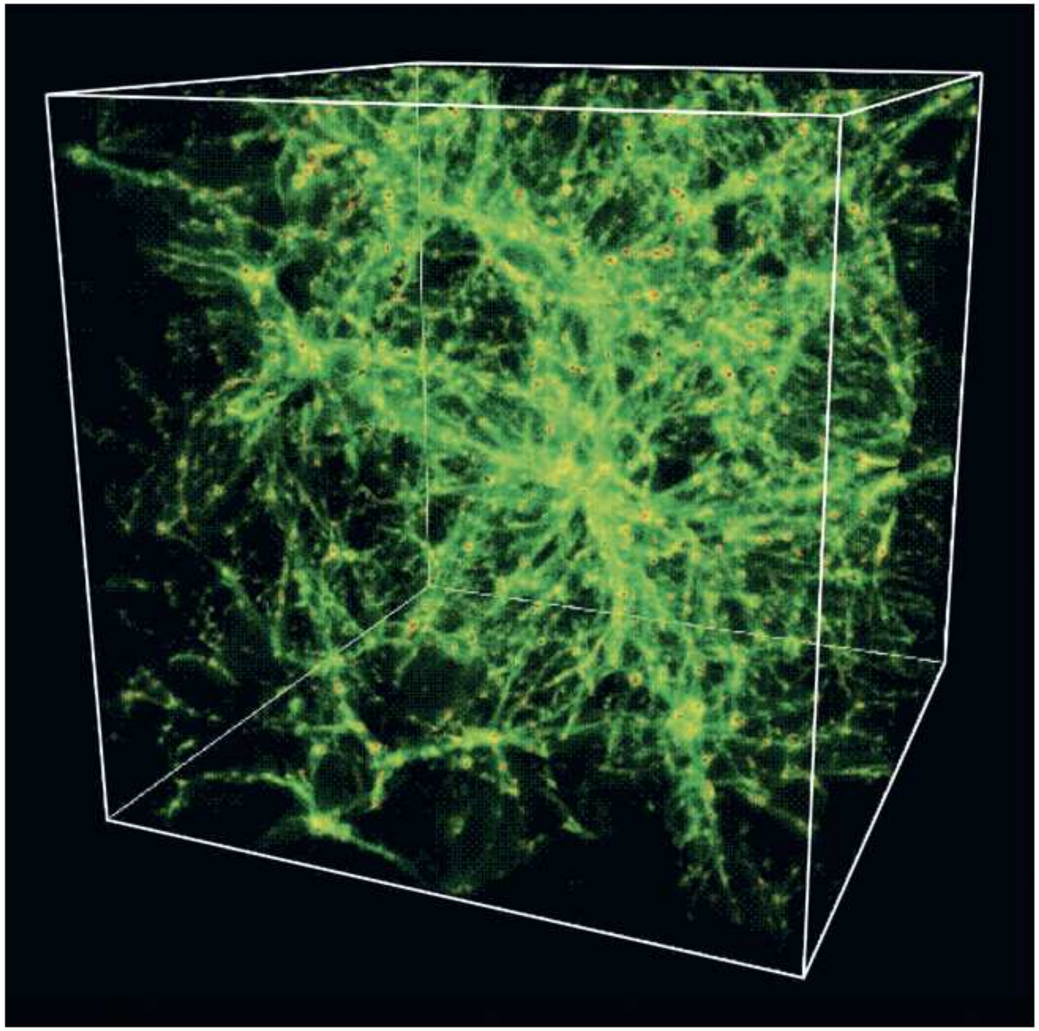
Nestled in the center of M42 is a group of stars, known as the Trapezium, which have formed from the gas in the nebula.The stars of the Trapezium are young blue stars. It is their energy which makes the nebula glow. *Can you point to the Trapezium?*



Intracluster Gas

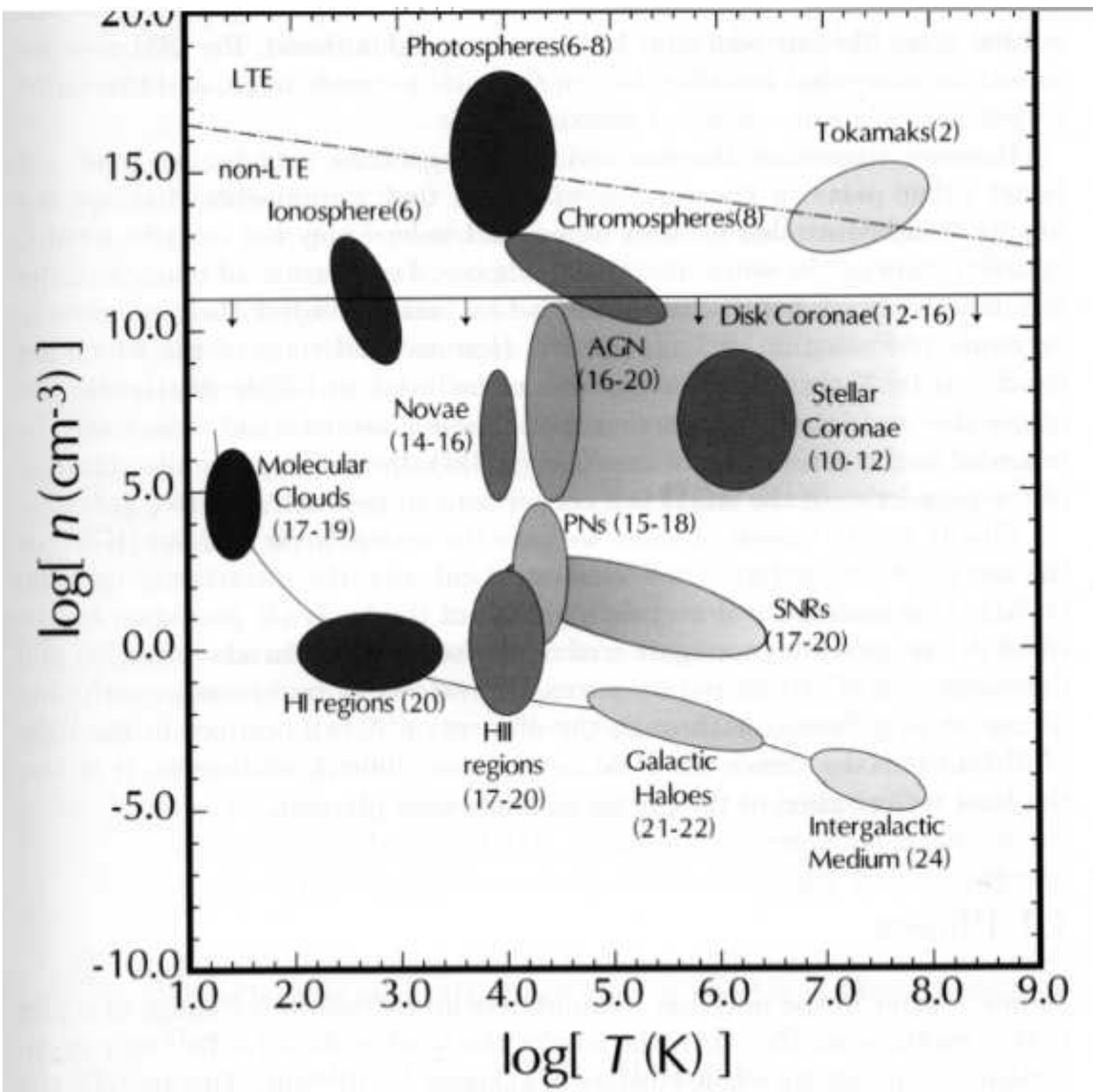


Missing Baryons



The density distribution of baryons at low redshift
from the simulation of Cen & Ostriker (2006)

Physical Parameters



- HII Regions n ~ 1 cm-3 (much more rarified than the best laboratory vacuums)
- HII Region T ~ 1e4 K

What determines the physics of these cosmic gas clouds?

- Does Local Thermodynamic Equilibrium (LTE) apply?
- Much, much lower collision rate than in planetary atmospheres or stellar interiors
- LTE requires
  - Population of excited states given by Boltzmann equilibrium
  - Particle energies distributed according to the Maxwell Distribution
  - Ionization balance given by Saha Equation
  - Photon energies described by Planck Function

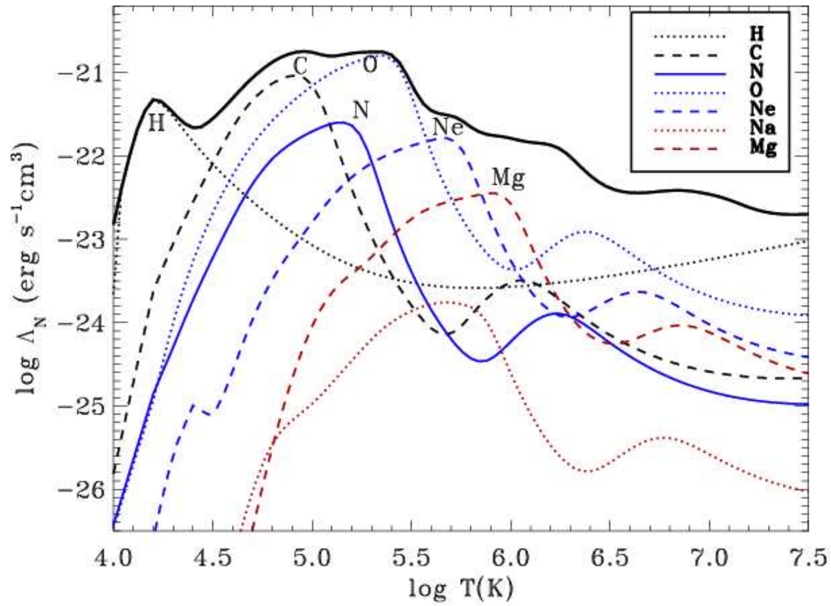
Phases of Diffuse Gas

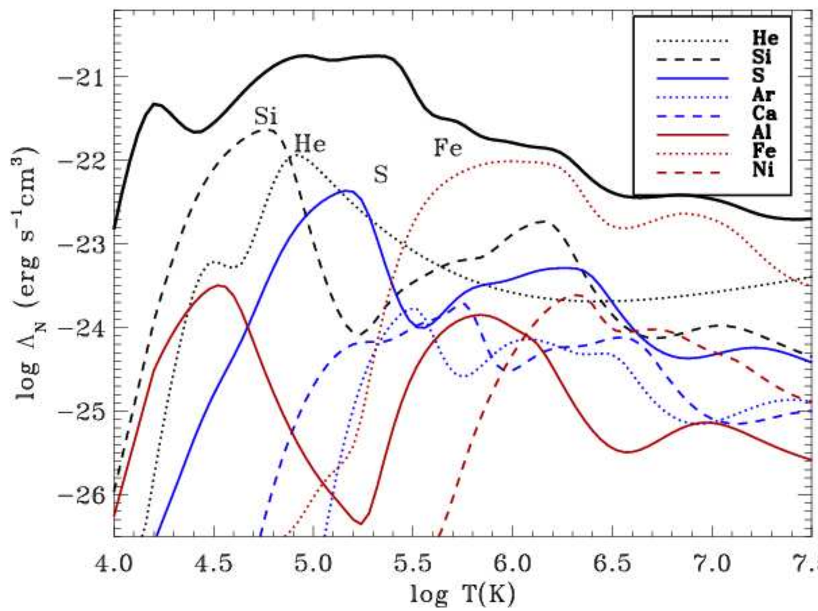
- Interstellar Medium (ISM) in galaxies
  - Dynamic equilibrium between cloud collapse into stars and energy/momentum input from stars; a self-regulating process known as "feedback."
  - At high redshift, this regulation may involve the entire circumgalactic medium; the rapid accretion of cold gas may be balanced by high star formation rates and massive galactic winds
  - Stable balance of heating and cooling at a given pressure can often be reached at more than one temperture giving rise to "multiphase structure."

Phases of Diffuse Gas

- Cooling timescale depends on the internal energy of the gas cloud and how fast it can radiate energy
- Rate of radiation is a well-understood function of density, temperature, and metallicity (I.e., heavy element content)
- These phases co-exist in a dynamic equilibrium in galaxies
  - Molecular medium
  - Cold neutral medium (CNM)
  - Warm neutral medium (WNM)
  - Hot Ionized medium (HIM)

Cooling Curve





Observability of Diffuse Gas

- Emission
  - Governed by binary collisions between atoms, ions, molecules, or electrons
  - Local emissivity (erg/s/cm2/sr) varies as the square of the local density
  - Define the "emission measure"
  - $EM = \int n_e^2 \, dl$
  - Units? [BB]
  - Sky Brightness lowest at high E, so better constrast in ultraviolet.
  - Detection? Detailed analysis?
- How can we observe gas at lower EM?

Observability of Diffuse Gas

- Absorption
  - Diffuse gas absorbs continuum photons from a background light source
  - Essentially only the ground-state is populated. Transitions from the ground state are called "resonance transitions."
  - Resonance transitions remove light from the beam.
  - The atoms re-radiate this light in all directions when they return to the ground state. The continuum light is scattered out of the beam.
  - The number of continuum photons absorbed is proportional to the column density, $N = \int n_e \, dl$, and the cross section for absorption.
  - Very important for studying the HIM and the Intergalactic Medium (IGM)

End of Introduction

- Give examples of places in the universe where LTE does not hold. Why does it fail?
- What physics does apply on the atomic scale?
- Why is it easier to detect diffuse plasmas in absorption than in emission?

Line Emission Processes

- Spectra reveal atomic and molecular lines
- Atomic Spectra (X-ray, UV, optical, IR)
- Molecular Spectra (optical, IR, far-IR, sub-mm, mm, radio)
- Measure these transitions to derive the physical conditions (density, temperature, composition) of diffuse astrophysical plasmas

Collisional Excitation

- At the low density and weak radiation field typical of the diffuse universe, most atoms reside in the ground state
- Low-lying excited states are populated mostly as a result of collisions of atoms with charged species (e- or p)
- Line flux ~ collision rate * $P_{excite}(E) \sim n^2$
- $P_{excite}(E)$ is significant when E ~ kT
- kT = 0.86 eV (T/1e4 K); small compared to 1 Rydberg
- Common metals (O, N, S) have energy levels a few eV above the ground state that give rise to forbidden transitions

Excitation by e- Impact

- **Collision cross section** varies roughly inversely with impact energy. Why?
- Define a **collision strength**, taking out this E^{-1} dependence [BB]
- **Principle of detailed balance.** In equilibrium, each microscopic process must be exactly balanced by the inverse process
- Apply to collisional excitation of a level and show the collision strength is symmetric between excitations and de-excitations [BB]

Critical Density

- Define a **critical density** where the collisional deexcitation rate matches the radiative depopulation rate.

$$n_{crit} = A_{21}\ g_2\ T^{1/2}\ /\ \beta\ \Omega_{12}$$

- Represents the transition from the LDL to LTE level populations for that transition
- The line emissivity (vs. n) flattens from slope 2 to slope 1 (in log space)
- Compare the critical densities for forbidden, intercombination, and resonance lines.

Low Density Limit

- Collision rate between atoms and electrons << radiatve deexcitation rate
- Then line flux scales with the collision rate, n_e^2, and reaches a maximum at T = E_{12}/k [BB]

High Density Limit

- Level populations described by Boltzmann equilibrium
- Line flux scales as n_e and tends to a constant at high temperature

Temperature diagnostics via p^2 and p^4 ions

- **Three (five) Level Atom** describes many of the strong lines in spectra of ionized nebulae (and late-type galaxies) used to infer physical conditions.
- The states have different spin-orbit interactions but the same principle quantum number, so these are forbidden lines
- **E_{32} ~ E_{21} and Low Density** [BB]
- **$F_{32}/F_{21} = E_{32}/E_{21} * A_{32}/(A_{32} + A_{31}) * \Omega_{12}/\Omega_{12} * \exp(-E_{32}/kT)$**

Density diagnostics via p^3 ions

- $E_{32} << E_{21}$
- LDL -- all collisional excitations result in radiative decays
- [BB] $F_{31}/F_{21} = \Omega_{13}/\Omega_{12}$
- See sum rule for collision strengths
- HDL -- Radiative decay rate matters because collision deexcitation may occur
- [BB] $F_{31}/F_{21} = A_{13}/A_{12} * g_3/g$
- At what densities is the line ratio a good indicator of the electron density? [BB]

Infrared Line Diagnostics

- Transitions between fine-structure levels of p^2 and p^4 ions are dominant coolants of gas at 100 - 3000 K. See DS Table 3.3.
- E.g., [CII] 158 um; [OIII] 88.36, 51.81 um
- Atmospheric water vapor blocks 25-300 um
- Infrared Space Observatory (ISO)
- Spitzer IRS
- Herschel
- Sofia
- ALMA (these lines for high redshift galaxies)

Addendum (T & n diagnostics)

- HII regions: T and density diagnostics
  - Real ISM is clumpy; measure $n_{e,c}$
  - Emission measure ~ $\langle n_e^2 \rangle$ * length
  - Define a volume filling factor for the clumps such that $\langle n_e^2 \rangle \sim f * n_{e,c}^2$
- f ~ 0.01 to 0.1 typically

Atomic Spectra

- Governed by rules of quantum mechanics
  - Wave function of individual electrons (in a spherically symmetric potential)

    $\Psi(r,\theta,\phi) = R_{nl}(r)\, \Theta_{lm}(\theta)\, \Phi_m(\phi)$
  - Principal n = 1, 2, 3, …
  - Electron spin s = 1/2
  - Angular momentum l = 0, 1, 2, …, (n-1)
  - Magnetic m = -l, -(l-1), …, 0, …, (l-1), l

Atomic Spectra

- Resonance Lines
  - Electric dipole transition selection rules
    - Only 1 electron involved in the transition
    - Initial and final states have different parity
    - Emitted photon carries 1 unit of angular momentum, so $\Delta l = +/- 1$
    - Electron spin does not change
    - Change in the total angular momentum of the active electron is $\Delta J = +/- 1, 0$ (with J=0 to J=0 forbidden)
  - Statistical weight of any level is $g = 2J + 1$

Resonance Lines II: Einstein Coefficients

- Two-level atom model [BB]
  - Transition probability (from the excited state) given by Einstein relation $A_{21} \sim 10^{8-9}\ s^{-1}$ [BB]
  - A_{21} calculated from the overlap of the wave functions for the initial and final states
  - Favors the bluer transition when there are multiple paths for decay, $A_{21} \sim$ (Dipole matrix element) * ν_{12}^3
  - Einstein coefficients
  - Stimulated emission important when the upper level has a high population compared to the ground level
  - Probability of photon absorption (by an atom in the ground state) depends on the energy density of the electromagnetic field, $B_{12}U(\nu_{12})$, where B_{12} is the Einstein coefficient for absorption

Resonance Lines III: Oscillator Strength

- Conceptually useful to treat the active electron as oscillating between states.
- The "Oscillator Strength f" is the effective number of classical electrons involved in the transition.
- Strongest transitions have f~1.
- The sum of the f values for all the transitions in the atom cannot exceed the number of optically active electrons.

Pure Recombination Lines

- Recombination of an ion and electron forms an ion an excited state.
- The electron cascades through many possible energy levels back down to the ground state
- Why is this more complicated than the two-level resonance line transitions?
- Calculation of the cascade process requires precision; See [DS] 2.1.2 for the QM approach.
- Hydrogen recombination spectrum [BB]

Spectroscopic Notation

- Notation for ions
- Electron configuration
- Spectroscopic term
- Pauli exclusion principle
  - One active electron (e.g., Mg II, Na I)
  - Two electrons with LS coupling (e.g., Mg I)
  - Heavier ions can have JJ coupling
  - Real coupling usually intermediate to these limiting cases

Selection Rules

		Electric dipole (E1)	Magnetic dipole (M1)	Electric quadrupole (E2)	Mag quadru
Rigorous rules	(1)	$\Delta J = 0, \pm 1$ $(J = 0 \not\leftrightarrow 0)$		$\Delta J = 0, \pm 1, \pm$ $(J = 0 \not\leftrightarrow 0, 1;\ \frac{1}{2}$	
	(2)	$\Delta M_J = 0, \pm 1$		$\Delta M_J = 0, \pm 1,$	
	(3)	$\pi_f = -\pi_i$	$\pi_f = \pi_i$		
LS coupling	(4)	One electron jump $\Delta l = \pm 1$	No electron jump $\Delta l = 0$, $\Delta n = 0$	None or one electron jump $\Delta l = 0, \pm 2$	One ele Δl
	(5)	If $\Delta S = 0$ $\Delta L = 0, \pm 1$ $(L = 0 \not\leftrightarrow 0)$	If $\Delta S = 0$ $\Delta L = 0$	If $\Delta S = 0$ $\Delta L = 0, \pm 1, \pm$ $(L = 0 \not\leftrightarrow 0, 1$	
Intermediate coupling	(6)	If $\Delta S = \pm 1$ $\Delta L = 0, \pm 1, \pm 2$		If $\Delta S = \pm 1$ $\Delta L = 0, \pm 1, \pm 2, \pm 3$ $(L = 0 \not\leftrightarrow 0)$	If Δ $\Delta L =$ $(L =$

Intercombination or Semi-forbidden Lines

- Departure from pure LS coupling means that electric quadrupole transitions between states of different multiplicity can occur
- But at much lower probability, A of 10^3 s^{-1}
- At typical ISM density and temperature, these transitions are still more probable than a collision with another atom.
- Example: CIII]

Forbidden Lines

- Magnetic dipole transitions
- A of 10^{-2} s^{-1}
- How long do electons involved in forbidden transitions rest in their excited states?
- Clearly the densities must be very low indeed for the atom to avoid a collision on this timescale.
- The emitted photon is very unlikely to be reabsorbed by another ion. Why?
- Forbidden line photons usually escape from a nebula, so they are very important coolants.
- Examples [OIII] 4959, 5007; [OII] 3726,29

Molecular Spectra

- Rotating Molecules
- Vibrating Molecules
- Ro-Vibrational Spectra
- Electronic Molecular Spectra

Rotating Molecules

- Quantized rotational energy levels related to the moments of inertia of the molecules along the various axes of symmetry
- Example: Linear (Diatomic) molecules
  - Roughly a rigid rotator with constant spacing between atoms
  - Solutions quantized in the z component of the angular momentum, m_j, and the rotational quantum number, J, analogous to "l" in one electron atoms [BB]
  - $E_J \sim J(J+1)$
  - Lines are linearly spaced

Molecular Hydrogen

- Does H2 radiate strongly?
- To produce electric dipole line emission in these rotational transitions requires a heterogeneous linear molecule like CO.
- Transitions occur via electric quadrupole interaction. The least energetic transition is J=0 to J=2
- Lifetimes of excited states are much, much longer than for the ions, e.g., about 1000 years for the J=2 level.
- Hence the rotational levels are populated by collisions

Vibrating Molecules

- Equilibrium distance r0 at potential minimum, I.e., repulsion of the nuclei vs. attractive force of the bond [BB]
- Stretching
- 1 frequency!
- Higher E

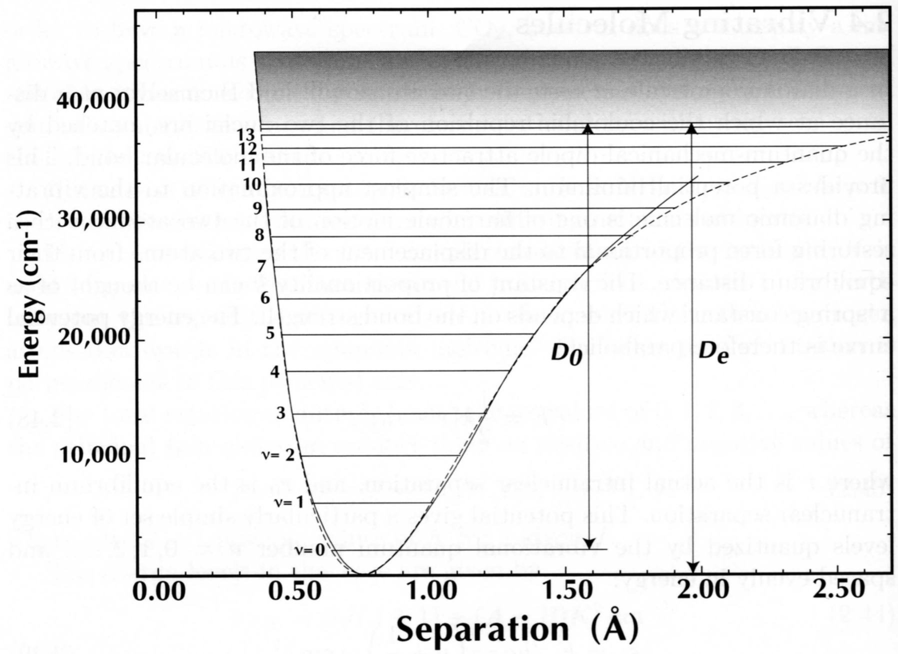


Fig. 2.6. The Morse potential for H_2. The actual potential inferred from detailed spectroscopy is the solid curve, the Morse potential is given by the dashed curve.

Ro-vibrational levels

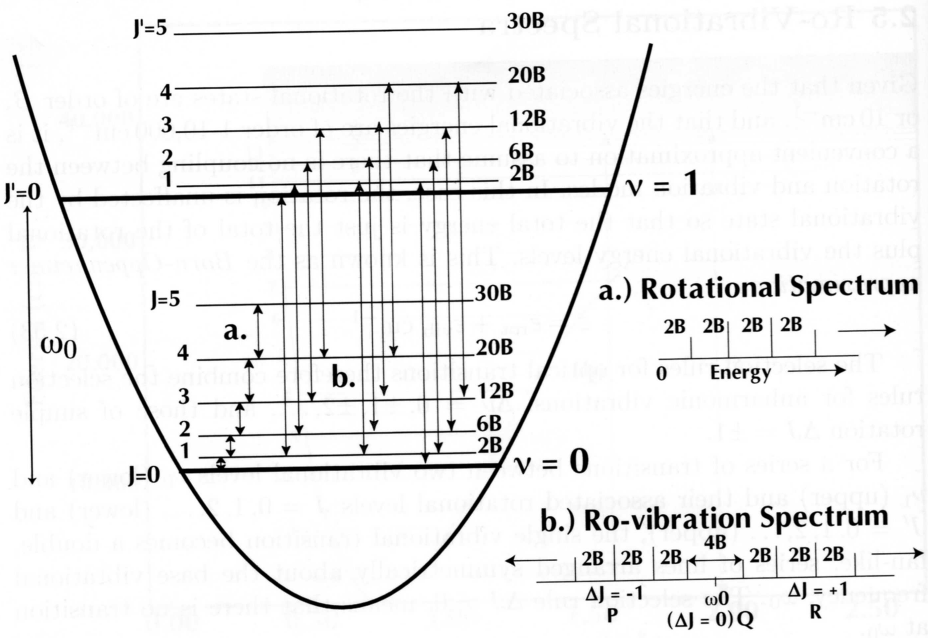


Fig. 2.7. Rotational-vibrational levels. Within each vibrational level of a diatomic molecule, a series of rotational levels occur, here magnified by a factor of several hundred for illustrative purposes. Transitions within the rotational levels (a.) produce a microwave spectrum with a spacing of $2B$ in energy. Transitions between rotational levels across vibrational levels, (b.), produce an infrared line at ω_0 that is split into rotational series spectral lines (P- and R- branches) that are also separated by an energy of $2B$. If $\Delta J = 0$ is permitted by out-of-line bending vibrations, a series of lines called the Q branch can appear at ω_0 with zero spacing because the energy differences for $\Delta J = 0$ are constant.

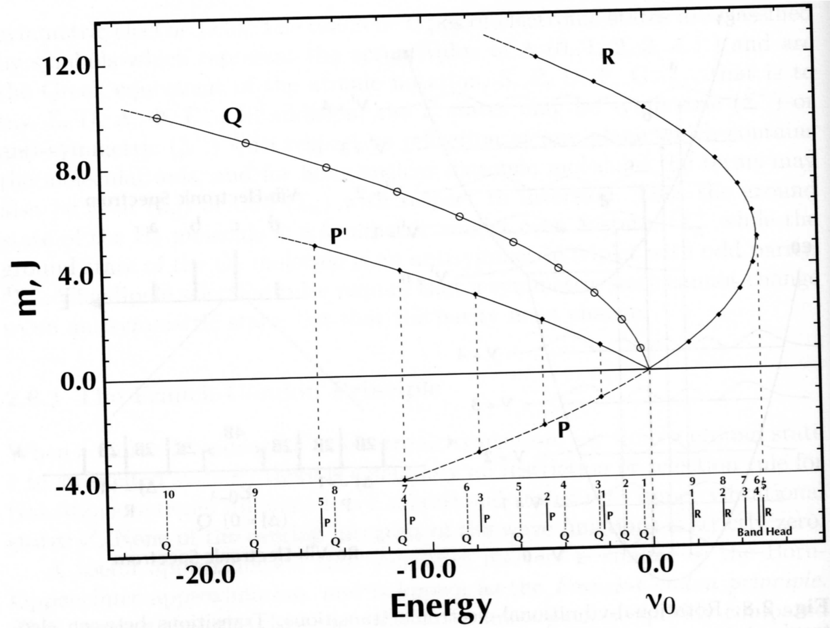


Fig. 2.9. Fortrat diagram for a Rotational-vibrational-electronic spectrum.

Electronic Transitions

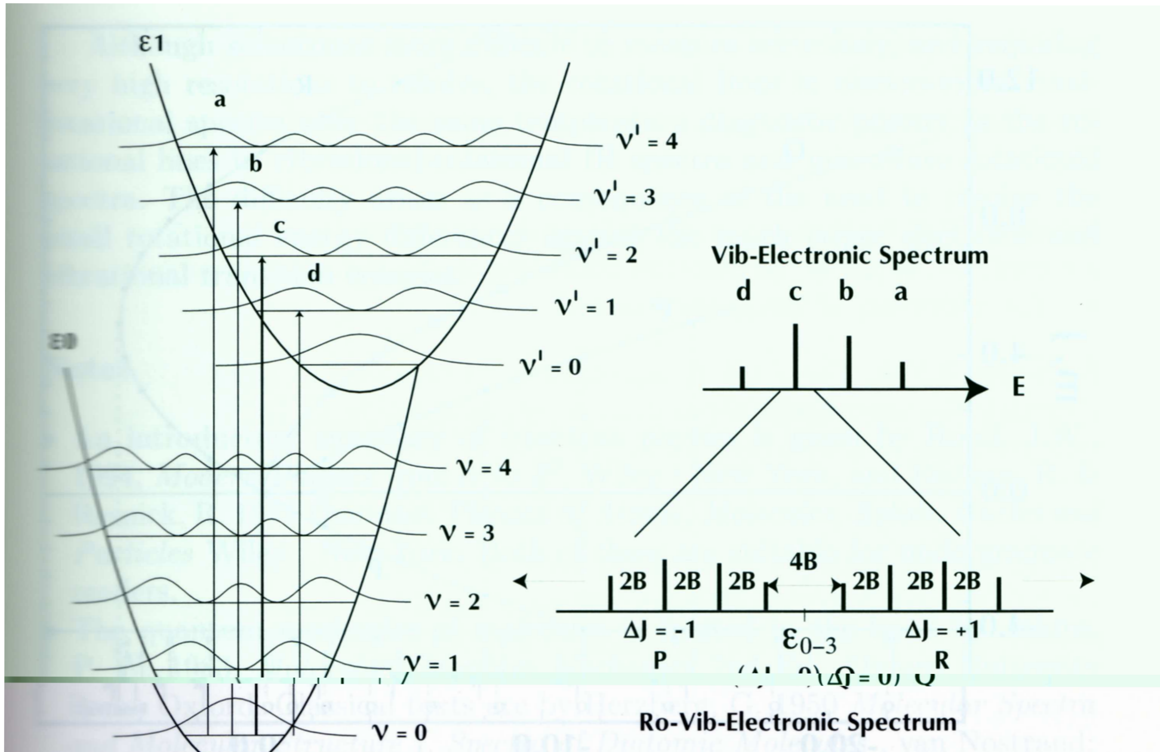


Fig. 2.8. Rotational-vibrational-electronic transitions. Transitions between electronic states are essentially instantaneous compared to vibrational or rotational timescales, and therefore are vertical transitions with no change in the separation of the atoms. Transitions are possible between different ν levels, with transition probabilities determined by the overlap of the vibrational wave functions. The line intensities are determined by the level populations and the transition probabilities. Each vibrational-electronic transition is also split by rotational levels, resulting in a ro-vib-electronic spectrum with P-, Q- and R- rotational branches.

"I ask you to look both ways. For the road to a knowledge of the stars leads through the atom; and important knowledge of the atom has been reached through the stars."

-- Sir Arthur Eddington (Stars & Atoms, 1928)

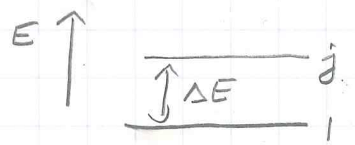
$\tau_{coll} \sim \frac{1}{\sigma v n}$

Collision Rate

		τ_{coll}
Earth's Atmosphere	$n = 2 \times 10^{18}\ cm^{-3}$	few $\times 10^{-9}$ s
Molecular Cloud	$= 10^{6}\ cm^{-3}$	several days few $\times 10^{5}$ s
ISM	$= 1\ cm^{-3}$	few $\times 10^{11}$ s, every 10^{4} yr

X 1) Boltzmann describes T_{ex}

$$\frac{N_j}{N_1} = \frac{g_j}{g_1} e^{-\Delta E_m / k T_{ex}}$$



$g = 2J + 1$

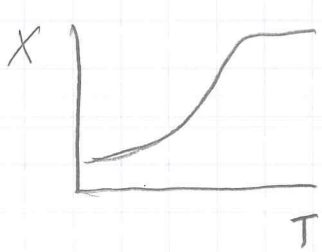
✓ 2) Maxwell describes T_{ke}

$$n(E) = \frac{2N}{\sqrt{\pi}\,(kT)^{3/2}} E^{1/2} e^{-E/kT_K}\, dE$$

X 3) Saha describes ionization balance T_{ion}

$h\nu_0 + H \rightleftarrows p + e^-$

$$\frac{n_p n_e}{n_{1s}} = \left(\frac{2\pi m k T}{h^2}\right)^{3/2} e^{-h\nu_0 / k T_{ion}}$$



✓ 4) Planck function T_γ specific Intensity

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\frac{1}{e^{h\nu/kT_\gamma} - 1}\right)$$

energy/area/time/solid angle/unit freq.

Energy Density $U(\nu) = \frac{4\pi}{c} B$ $\frac{\text{energy}}{\text{volume} \cdot \text{freq}}$

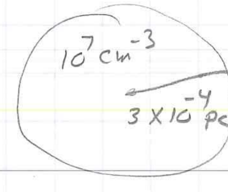
$$T_{ex} \neq T_K \neq T_{ion} \neq T_\gamma$$

Observability

$$EM = \int n_e^2 \, dl$$

Examples:

a) nova shell - ejected from WD at 1000 km/s
about $10^{-4} M_\odot$



@ 100 days
$$R = (10^8 \tfrac{cm}{s})(8.64 \times 10^6 s)$$
$$= 8.64 \times 10^{14} cm = 2.8 \times 10^{-4} pc$$

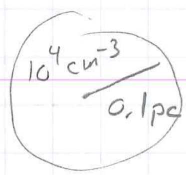
and $n = \dfrac{\rho}{4 m_H} = \dfrac{10^{-4} M_\odot (1.99 \times 10^{33} g)}{\frac{4}{3}\pi (8.64 \times 10^{14} cm)^3 \, 4 \, (1.67 \times 10^{-24})}$

$$n = 1.1 \times 10^7 cm^{-3}$$

$$EM = (10^7 cm^{-3})^2 (3 \times 10^{-4} pc) = 3 \times 10^{10} cm^{-6} pc$$

b) PNe (RG → WD, ionized envelope of a dying star)
$v = 10-30$ km/s for $\sim 10^3$ yr

$$EM \sim (10^4 cm^{-3})^2 \, 0.1 pc \sim 10^7 cm^{-6} pc$$

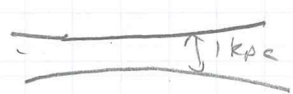


c) HII Regions (Ionized by hot, young stars)

$R_S \sim 10 - 100$ pc

$$EM \sim (10 cm^{-3})^2 \, 10 \sim 10^{3-4} cm^{-6} pc$$

d) Diffuse Ionized ISM



$$EM \sim (0.1 cm^{-3})^2 (10^3 pc)$$
$$\sim 10 \, pc \, cm^{-6}$$

Challenge to Detect

e) Circumgalactic Gas ($\sim$10 kpc)
$$EM \sim (10^{-3})^2 (10^4 pc) \sim 10^{-2} pc \, cm^{-6}$$

Physics 233 — 1/8/2014

Lecture 2 - New Outline

Collisionally Excited Lines. Part 1

Review Elastic / Inelastic Scattering [Draine ch. 2]
- collisional ionization is elastic
- collisional excitation (followed by radiative decay) is inelastic

Collision Rates
- Earth's Atmosphere
- Molecular Cloud
- ISM

Emission Measure - observability
- Had students estimate EM for nova, PNe, HII regions

Principle of Detailed Balance
- Follows from the Law of Mass Action (Draine ch. 3)
- Prove that $\Omega_{12} = \Omega_{21}$ for 2-level atom.
  * Derive it in LTE, but it holds out of LTE.
- Assign derivation of Milne Relation.

Two Level Atom
- LDL
- HDL

Lecture 3 - New Outline

Collisionally Excited Lines. Part 2

Three Level Atom
- Statistical Equilibrium
- T diagnostics
- ρ diagnostics

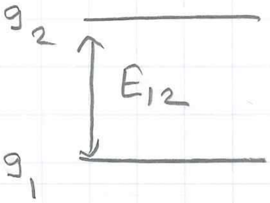
Spectroscopic Notation
- e^- configuration
- terms
- selection rules
- recombination Lines

Collisional Excitation by e^- impact

Collision cross section

$$\sigma_{12}(E) = \frac{h^2}{8\pi m_e E} \frac{\Omega_{12}(E)}{g_1}$$

← collision strength "dimensionless"



$$cm^2 = \frac{g\, cm^2\, s^2}{s^2\, g}$$

Show $\Omega_{12} = \Omega_{21}$

Detailed balance: $R_{12} = R_{21}$

$$n_1 n_e \alpha_{12} = n_2 n_e \alpha_{21}$$

↑ collisional excitation/de-excitation coefficients

In general $\alpha_{12} \neq \alpha_{21}$ because of the different populations of the two levels.

$$R_{12} = n_1 n_e \int_{E_{12}}^{\infty} \sigma_{12}(v) \cdot v \cdot f(v)\, dv$$

Notice int. limits!

$$R_{21} = n_2 n_e \int_0^{\infty} \sigma_{21}(v) \cdot v \cdot f(v)\, dv$$

Can show $n_1 \frac{\Omega_{12}}{g_1} e^{-E_{12}/kT} = n_2 \frac{\Omega_{21}}{g_2}$

$$\frac{n_2}{n_1} = \frac{\alpha_{12}}{\alpha_{21}} = \frac{\Omega_{12}}{g_1}\frac{g_2}{\Omega_{21}} e^{-E_{12}/kT}$$

Now at high density, where collisions completely determine level populations (via Boltzmann) $\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-E_{12}/kT}$

$\therefore \Omega_{12} = \Omega_{21}$

Low Density Limit

$$n_e n_1 \alpha_{12} = n_2 A_{21}$$

$$n_2 = n_e n_1 \frac{\alpha_{12}}{A_{21}}$$

$$= n_e n_1 \frac{1}{A_{21}} \left(\frac{2\pi\hbar^4}{k m_e^3}\right)^{1/2} \frac{1}{\sqrt{T}} \frac{\Omega_{12}}{g_1} e^{-E_{12}/kT}$$

Line Flux

$$F_{12} = h\nu_{12} A_{21} n_2$$

note cancellation of A_{21}'s

All collisional excitation's result in a radiative decay, so all that matters is the collision rate.

$$F_{12} = h\nu_{12} n_e n_1 (8.62942\times10^{-6}) \frac{1}{\sqrt{T}} \frac{\Omega_{12}}{g_1} e^{-E_{12}/kT}$$

At low T, most collisions have too low E to excite line.

$$\max F_{12} \quad @ \quad T = \frac{E_{12}}{k}$$

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-\Delta E/kT}$$

High Density Limit

$$F_{12} = h\nu_{21} A_{21} \left[n_1 \frac{g_2}{g_1} e^{-E_{12}/kT} \right]$$

Boltzmann

Three-Level Atom

g_3 ________

g_2 ________

g_1 ________

Let $C_{ij} = n_e \alpha_{ij}$ the collision rate per atom in state i

Write down the statistical equilibrium

(3) $$n_1 C_{13} + n_2 \cancel{C_{23}} = n_3 (\cancel{C_{31}} + \cancel{C_{32}} + A_{32} + A_{31})$$

(2) $$n_1 C_{12} + n_3 (\cancel{C_{32}} + A_{32}) = n_2 (\cancel{C_{23}} + \cancel{C_{21}} + A_{21})$$

$$n_1 + n_2 + n_3 = n$$

Can solve for level populations

Special Cases

1) Temperature Diagnostic

$E_{12} \sim E_{23}$ and $C_{13} << C_{12}$

LDL $\rightarrow$ Ignore collisional deexcitation

$\rightarrow n_1 >> n_2 >> n_3$

Reduce to
$$n_1 C_{13} = n_3 (A_{32} + A_{31})$$
$$n_1 C_{12} + n_3 A_{32} = n_2 (A_{21})$$
$$n_1 + n_2 + n_3 = n$$

$$\Rightarrow n_3 = \frac{n_1 C_{13}}{A_{32} + A_{31}}$$

$$n_2 = \frac{1}{A_{21}} \left[n_1 C_{12} + \frac{n_1 C_{13} A_{32}}{A_{32} + A_{31}} \right]$$

$$\approx \frac{n_1 C_{12}}{A_{21}} \quad \text{because } C_{13} << C_{12}$$

$\rightarrow$

$$\frac{F_{32}}{F_{21}} = \frac{E_{32} A_{32} n_3}{E_{21} A_{21} n_2}$$

$$= \frac{E_{32} A_{32}}{E_{21} A_{21}} \frac{n_1 C_{13}}{A_{32} + A_{31}} \frac{A_{21}}{n_1 C_{12}}$$

$$= \frac{E_{32} A_{32} C_{13}}{E_{21} (A_{32} + A_{31}) C_{12}}$$

$$C_{13} = n_e \alpha_{13}$$

$$= n_e \left(\frac{2\pi \hbar^4}{k m_e^3}\right)^{1/2} \frac{1}{\sqrt{T}} \frac{\Omega_{13}}{g_1} e^{-E_{13}/kT}$$

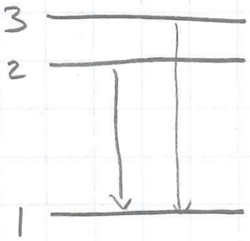
$$C_{12} = n_e \alpha_{12} = n_e \left(\frac{2\pi \hbar^4}{k m_e^3}\right)^{1/2} \frac{1}{\sqrt{T}} \frac{\Omega_{12}}{g_1} e^{-E_{12}/kT}$$

$$\frac{C_{13}}{C_{12}} = \frac{\Omega_{13}}{\Omega_{12}} e^{-E_{23}/kT}$$

✓ $$\frac{F_{32}}{F_{21}} = \frac{E_{32} A_{32}}{E_{21} (A_{32} + A_{31})} \frac{\Omega_{13}}{\Omega_{12}} e^{-E_{23}/kT}$$

[N II] ✓
[O III] ✓
[Ne V] quite blue
[S III] quite red

2) Density Diagnostics $\qquad E_{23} \ll E_{12}$



$$\frac{F_{31}}{F_{21}} = \frac{E_{31}\, A_{31}\, N_3}{E_{21}\, A_{21}\, N_2}$$

Note: small A_{32} $\qquad$ Rem: $A_{21} \sim \nu^3 S_{21}$

small E_{32}

a) LDL

Eqn. 2 $\quad n_1 C_{12} + n_3 (C_{32} + A_{32}) = n_2 (C_{23} + C_{21} + A_{21})$ (small; $\to d$)

because:

$$n_1 C_{12} = n_2 A_{21} - n_3 A_{32} \quad (\to d)$$

Eqn. 1 $\quad n_1 C_{13} + n_2 C_{23} = n_3 (C_{31} + C_{32} + A_{32} + A_{31})$ ($\to d$)

$$n_1 C_{13} = n_3 A_{31} + n_3 A_{32} \quad (\to d)$$

why $A_{32} \ll A_{31}$?

$$\frac{F_{31}}{F_{21}} = \frac{E_{31}}{E_{21}} \frac{A_{31}}{A_{21}} \underbrace{\frac{n_1 C_{13}}{A_{31}}}_{n_3} \cdot \underbrace{\frac{A_{21}}{n_1 C_{12}}}_{n_2^{-1}}$$

$$= \underbrace{\frac{E_{31}}{E_{21}}}_{\approx 1} \frac{C_{13}}{C_{12}} = \frac{\Omega_{13}}{\Omega_{12}} \frac{e^{-E_{13}/kT}}{e^{-E_{12}/kT}}$$

$\Delta E_{23} \approx 0$

$$= \frac{\Omega_{13}}{\Omega_{12}} = \frac{g_3}{g_2}$$

QM sum rule (see 3.6)

b) HDL

Boltzmann Ratios $\frac{n_3}{n_2} = \frac{g_3}{g_2} e^{-E_{32}/kT} \approx \frac{g_3}{g_2}$

$$\frac{F_{31}}{F_{21}} = \frac{E_{31}}{E_{21}} \frac{A_{31}}{A_{21}} \frac{n_3}{n_2}$$

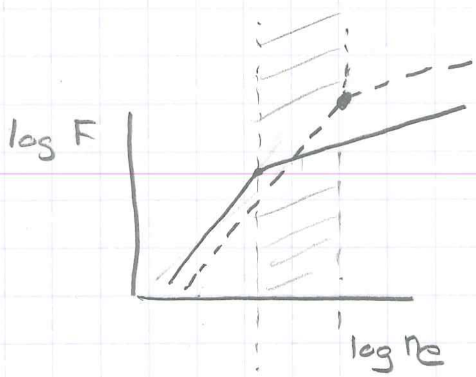
$$= \underbrace{\frac{E_{31}}{E_{21}}}_{\approx 1} \frac{A_{31}}{A_{21}} \frac{g_3}{g_2}$$

$$= \frac{A_{31}\, g_3}{A_{21}\, g_2}$$

Why?

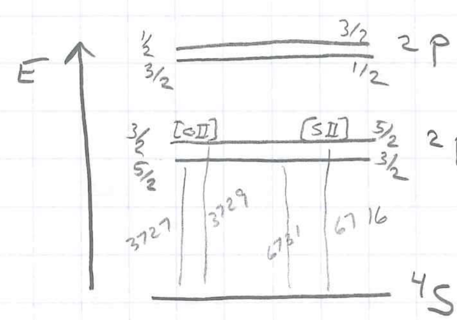
For $2 \to 1$ $\quad n_{crit, 2\to1} = \frac{A_{21}\, g_2\, T^{1/2}}{\beta\, \Omega_{12}}$

$3 \to 1$ $\quad n_{crit, 3\to1} = \frac{A_{31}\, g_3\, T^{1/2}}{\beta\, \Omega_{13}}$



Note: Last time, we showed $F_{12} \propto n^2$ in LDL. And $F_{12} \propto n$ in HDL.

p^3 ions

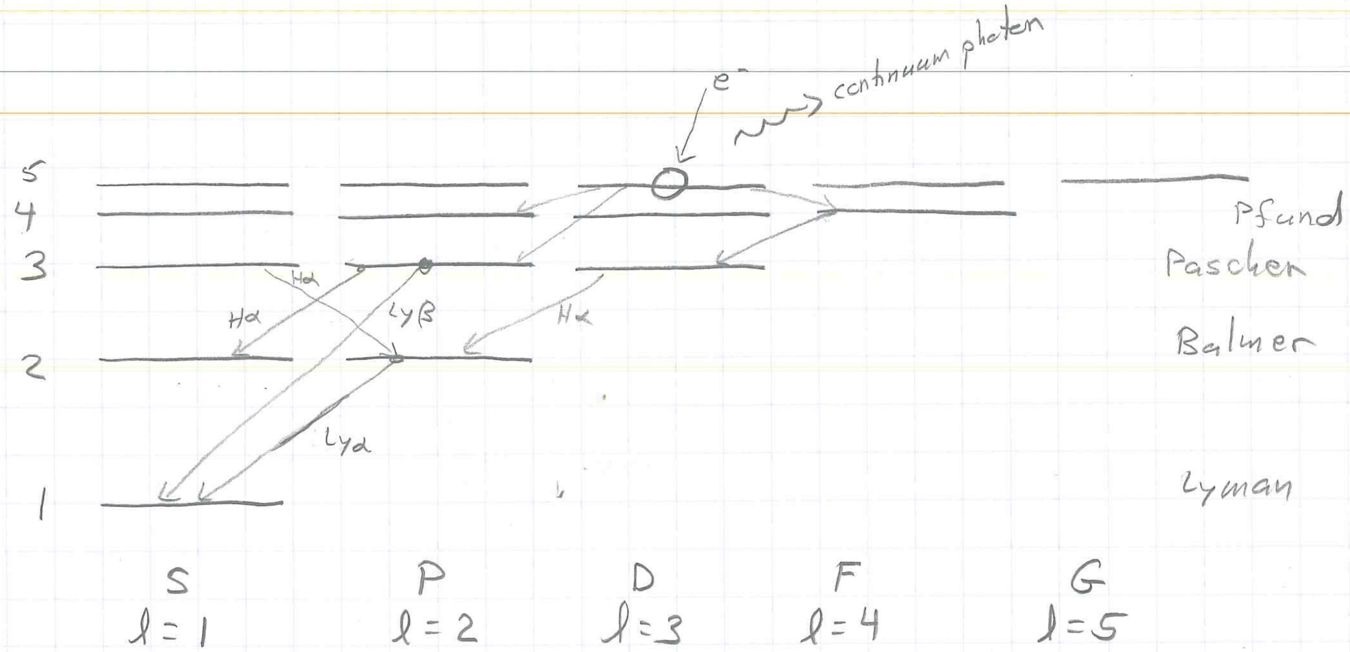


Examples: [OII], [SII], [NeIV], [ArIV]

NUV

Recombination Lines

- Require a model with many energy levels
- e^-'s tend to recombine to high l levels
- Each emitted photon carries away $\Delta l = \pm 1$ units of angular momentum, so there can be many steps in the cascade



E1 (Electric Dipole) Selection Rules

$\Delta J = 0, \pm 1$ $\quad\quad J=0 \not\to J=0$

$\Delta l = \pm 1$ $\quad\quad$ parity change

Δn arbitrary

$\Delta L = 0, \pm 1$ $\quad\quad L=0 \not\to L=0$

$\Delta S = 0$

$^{2S+1}L_J$

Examples

- 1 Active Electron

			1s	2s	2p	3s	
Na I	Z = 11	(11 e⁻)	⊗	⊗	⊗⊗⊗	⊘	$1s^2 2s^2 2p^6 3s^1$
Mg II	Z = 12	(11 e⁻)	⊗	⊗	⊗⊗⊗	⊘	

$s = \frac{1}{2}$ $S = \frac{1}{2}$ $2S + 1 = 2$

$l = 0$ $L = 0$

$\bar{J} = \bar{L} + \bar{S} = \frac{1}{2}$ ⟹ $^2S_{1/2}$

- Multiple active electrons

The two e⁻'s can't share the same wave function.

O III Z = 8 (6 e⁻) ⊗ ⊗ ⊘⊘○ $1s^2 2s^2 2p^2$

Ground configuration has even parity. $\sum_i l_i$ is even

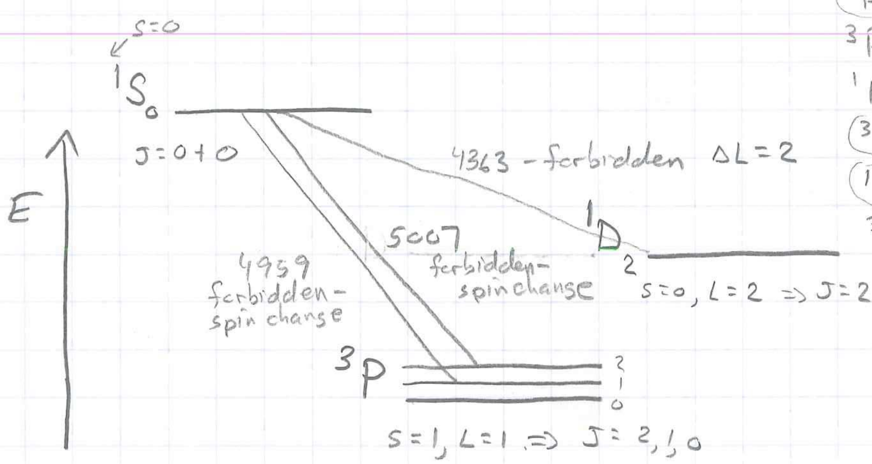
$s = \frac{1}{2}, s = \frac{1}{2}$ $S = 0, 1$ $2S + 1 = 2$ or 3

↓↑ ↑↑

$l = 1, l = 1$ $L = 0, 1, 2$

↓↑ ↖ ↑↑

(¹D)	L = 2; S = 0	→	J = 2
³D	L = 2; S = 1	→	J = 3, 2, 1
¹P	L = 1; S = 0		J = 1
(³P)	L = 1; S = 1		J = 2, 1, 0
(¹S)	L = 0; S = 0		J = 0
³S	L = 0; S = 1		J = 1



There are 3 ways that the two 2p electron orbits and spins can be organized into an overall wave function that is antisymmetric under e⁻ exchange as required by exclusion principle.

- Why no triplet S or D?
- Why no singlet P?

The L=2 configuration requires two $l=1$ vectors be aligned.

↑↑ [same L_z, so same m]

Since n and m are the same, the s values must differ.

when L = 0, $\downarrow_l \uparrow_l$ $\downarrow_l \uparrow_l$

$\downarrow_s \uparrow_s$ $\uparrow_s \uparrow_s$